Lesson 18: Equations Involving a Variable Expression in the Denominator

Classwork

Opening Exercise

Nolan says that he checks the answer to a division problem by performing multiplication. For example, he says that $20 \div 4 = 5$ is correct because 5×4 is 20, and $\frac{3}{\frac{1}{2}} = 6$ is correct because $6 \times \frac{1}{2}$ is 3.

a. Using Nolan's reasoning, explain why there is no real number that is the answer to the division problem $5 \div 0$.

b. Quentin says that $\frac{0}{0} = 17$. What do you think?

c. Mavis says that the expression $\frac{5}{x+2}$ has a meaningful value for whatever value one chooses to assign to x. Do you agree?

d. Bernoit says that the expression $\frac{3x-6}{x-2}$ always has the value 3 for whichever value one assigns to x. Do you agree?

Exercises 1–2

1. Rewrite $\frac{10}{x+5}$ as a compound statement.

2. Consider $\frac{x^2-25}{x^2-9 \quad x+4}$.
   a. Is it permissible to let $x = 5$ in this expression?

   b. Is it permissible to let $x = 3$ in this expression?

   c. Give all the values of x that are ***not*** permissible in this expression.

Example 1

Consider the equation $\frac{1}{x} = \frac{3}{x-2}$.

a. Rewrite the equation into a system of equations.

b. Solve the equation for x, excluding the value(s) of x that lead to a denominator of zero.

Example 2

Consider the equation $\frac{x+3}{x-2} = \frac{5}{x-2}$.

a. Rewrite the equation into a system of equations.

b. Solve the equation for x, excluding the value(s) of x that lead to a denominator of zero.

Exercises 3–11

Rewrite each equation into a system of equations excluding the value(s) of x that lead to a denominator of zero; then, solve the equation for x.

3. $\frac{5}{x} = 1$

4. $\frac{1}{x-5} = 3$

5. $\frac{x}{x+1} = 4$

6. $\frac{2}{x} = \frac{3}{x-4}$

7. $\frac{x}{x+6} = -\frac{6}{x+6}$

8. $\frac{x-3}{x+2} = 0$

9. $\frac{x+3}{x+3} = 5$

10. $\frac{x+3}{x+3} = 1$

11. A baseball player's batting average is calculated by dividing the number of times a player got a hit by the total number of times the player was at bat. It is expressed as a decimal rounded to three places. After the first ten games of the season, Samuel had 12 hits off of 33 "at bats."

    a. What is his batting average after the first ten games?

    b. How many hits in a row would he need to get to raise his batting average to above 0.500?

    c. How many "at bats" in a row without a hit would result in his batting average dropping below 0.300?

Problem Set

1. Consider the equation $\frac{10\ x^2-49}{3x\ x^2-4\ \ x+1} = 0$. Is $x = 7$ permissible? Which values of x are excluded? Rewrite as a system of equations.

2. Rewrite each equation as a system of equations excluding the value(s) of x that lead to a denominator of zero. Then solve the equation for x.
   a. $25x = \frac{1}{x}$
   b. $\frac{1}{5x} = 10$
   c. $\frac{x}{7-x} = 2x$
   d. $\frac{2}{x} = \frac{5}{x+1}$
   e. $\frac{3+x}{3-x} = \frac{3+2x}{3-2x}$

3. Ross wants to cut a 40-foot rope into two pieces so that the length of the first piece divided by the length of the second piece is 2.
   a. Let x represent the length of the first piece. Write an equation that represents the relationship between the pieces as stated above.
   b. What values of x are not permissible in this equation? Describe within the context of the problem, what situation is occurring if x were to equal this value(s). Rewrite as a system of equations to exclude the value(s).
   c. Solve the equation to obtain the lengths of the two pieces of rope. (Round to the nearest tenth if necessary.)

4. Write an equation with the restrictions $x \neq 14$, $x \neq 2$, and $x \neq 0$.

5. Write an equation that has no solution.