# Lesson 2: Recursive Formulas for Sequences

## Classwork

### Example 1

Consider Akelia's sequence $5, 8, 11, 14, 17, \ldots$.

a. If you believed in patterns, what might you say is the next number in the sequence?

b. Write a formula for Akelia's sequence.

c. Explain how each part of the formula relates to the sequence.

d. Explain Johnny's formula.

### Exercises 1–2

1. Akelia, in a playful mood, asked Johnny: What would happen if we change the "+" sign in your formula to a "−" sign? To a "×" sign? To a "÷" sign?

   a. What sequence does $A(n+1) = A(n) - 3$ for $n \geq 1$ and $A(1) = 5$ generate?

   b. What sequence does $A(n+1) = A(n) \cdot 3$ for $n \geq 1$ and $A(1) = 5$ generate?

   c. What sequence does $A(n+1) = A(n) \div 3$ for $n \geq 1$ and $A(1) = 5$ generate?

2. Ben made up a recursive formula and used it to generate a sequence. He used $B(n)$ to stand for the $n^{\text{th}}$ term of his recursive sequence.

   a. What does $B(3)$ mean?

   b. What does $B(m)$ mean?

   c. If $B(n+1) = 33$ and $B(n) = 28$, write a possible recursive formula involving $B(n+1)$ and $B(n)$ that would generate 28 and 33 in the sequence.

   d. What does $2B(7) + 6$ mean?

   e. What does $B(n) + B(m)$ mean?

   f. Would it necessarily be the same as $B(n+m)$?

   g. What does $B(17) - B(16)$ mean?

## Example 2

Consider a sequence given by the formula $a_n = a_{n-1} - 5$, where $a_1 = 12$ and $n \geq 2$.

a. List the first five terms of the sequence.

b. Write an explicit formula.

c. Find $a_6$ and $a_{100}$ of the sequence.

## Exercises 3–6

3. One of the most famous sequences is the Fibonacci sequence:

   $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots.$

   $f(n+1) = f(n) + f(n-1)$, where $f(1) = 1$, $f(2) = 1$, and $n \geq 2$.

   How is each term of the sequence generated?

4. For each sequence below, an explicit formula is given. Write the first five terms of each sequence. Then, write a recursive formula for the sequence.

   a. $a_n = 2n + 10$ for $n \geq 1$

b. $a_n = \left(\frac{1}{2}\right)^{n-1}$ for $n \geq 1$

5. For each sequence, write *either* an explicit or recursive formula.

   a. $1, -1, 1, -1, 1, -1, \ldots$

   b. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

6. Lou opens a bank account. The deal he makes with his mother is that if he doubles the amount that was in the account at the beginning of each month by the end of the month, she will add an additional \$5 to the account at the end of the month.

   a. Let $A(n)$ represent the amount in the account at the beginning of the $n^{\text{th}}$ month. Assume that he does, in fact, double the amount every month. Write a recursive formula for the amount of money in his account at the beginning of the $(n+1)^{\text{th}}$ month.

   b. What is the least amount he could start with in order to have \$300 by the beginning of the $3^{\text{rd}}$ month?

### Lesson Summary

**Recursive Sequence**: An example of a *recursive sequence* is a sequence that (1) is defined by specifying the values of one or more initial terms and (2) has the property that the remaining terms satisfy a recursive formula that describes the value of a term based upon an expression in numbers, previous terms, or the index of the term.

An explicit formula specifies the $n^{\text{th}}$ term of a sequence as an expression in $n$.

A recursive formula specifies the $n^{\text{th}}$ term of a sequence as an expression in the previous term (or previous couple of terms).

## Problem Set

For Problems 1–4, list the first five terms of each sequence.

1. $a_{n+1} = a_n + 6$, where $a_1 = 11$ for $n \geq 1$

2. $a_n = a_{n-1} \div 2$, where $a_1 = 50$ for $n \geq 2$

3. $f(n+1) = -2f(n) + 8$ and $f(1) = 1$ for $n \geq 1$

4. $f(n) = f(n-1) + n$ and $f(1) = 4$ for $n \geq 2$

For Problems 5–10, write a recursive formula for each sequence given or described below.

5. It follows a "plus one" pattern: $8, 9, 10, 11, 12, \ldots$.

6. It follows a "times $10$" pattern: $4, 40, 400, 4000, \ldots$.

7. It has an explicit formula of $f(n) = -3n + 2$ for $n \geq 1$.

8. It has an explicit formula of $f(n) = -1(12)^{n-1}$ for $n \geq 1$.

9. Doug accepts a job where his starting salary will be $\$30{,}000$ per year, and each year he will receive a raise of $\$3{,}000$.

10. A bacteria culture has an initial population of $10$ bacteria, and each hour the population triples in size.