Lesson 3: Arithmetic and Geometric Sequences

Classwork

Exercise 2

Think of a real-world example of an arithmetic or geometric sequence? Describe it and write its formula.

Exercise 3

If we fold a rectangular piece of paper in half multiple times and count the number of rectangles created, what type of sequence are we creating? Can you write the formula?

Lesson Summary

Two types of sequences were studied:

Arithmetic Sequence: A sequence is called *arithmetic* if there is a real number d such that each term in the sequence is the sum of the previous term and d.

Geometric Sequence: A sequence is called *geometric* if there is a real number r such that each term in the sequence is a product of the previous term and r.

Problem Set

For Problems 1–4, list the first five terms of each sequence, and identify them as arithmetic or geometric.

1. $A(n+1) = A(n) + 4$ for $n \geq 1$ and $A(1) = -2$

2. $A(n+1) = \frac{1}{4} \cdot A(n)$ for $n \geq 1$ and $A(1) = 8$

3. $A(n+1) = A(n) - 19$ for $n \geq 1$ and $A(1) = -6$

4. $A(n+1) = \frac{2}{3}A(n)$ for $n \geq 1$ and $A(1) = 6$

For Problems 5–8, identify the sequence as arithmetic or geometric, and write a recursive formula for the sequence. Be sure to identify your starting value.

5. $14, \ 21, \ 28, \ 35, \ldots$

6. $4, \ 40, \ 400, \ 4000, \ldots$

7. $49, \ 7, \ \frac{1}{7}, \frac{1}{49}, \ldots$

8. $-101, \ -91, \ -81, \ -71, \ldots$

9. The local football team won the championship several years ago, and since then, ticket prices have been increasing $\$20$ per year. The year they won the championship, tickets were $\$50$. Write a recursive formula for a sequence that will model ticket prices. Is the sequence arithmetic or geometric?

10. A radioactive substance decreases in the amount of grams by one third each year. If the starting amount of the substance in a rock is $1{,}452$ g, write a recursive formula for a sequence that models the amount of the substance left after the end of each year. Is the sequence arithmetic or geometric?

11. Find an explicit form $f(n)$ for each of the following arithmetic sequences (assume a is some real number and x is some real number).
    a. $-34,\ -22,\ -10,\ 2, \ldots$
    b. $\frac{1}{5},\ \frac{1}{10},\ 0,\ -\frac{1}{10}, \ldots$
    c. $x+4,\ x+8,\ x+12,\ x+16, \ldots$
    d. $a,\ 2a+1,\ 3a+2,\ 4a+3, \ldots$

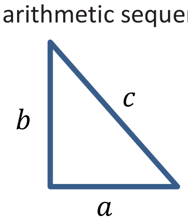
12. Consider the arithmetic sequence $13,\ 24,\ 35, \ldots$.
    a. Find an explicit form for the sequence in terms of n.
    b. Find the 40^{th} term.
    c. If the nth term is 299, find the value of n.

13. If $-2, a, b, c, 14$ forms an arithmetic sequence, find the values of a, b, and c.

14. $3+x, 9+3x, 13+4x, \ldots$ is an arithmetic sequence for some real number x.
    a. Find the value of x.
    b. Find the 10^{th} term of the sequence.

15. Find an explicit form $f(n)$ of the arithmetic sequence where the 2^{nd} term is 25 and the sum of the 3^{rd} term and 4^{th} term is 86.

16. <u>Challenge</u>: In the right triangle figure below, the lengths of the sides a cm, b cm, and c cm of the right triangle form a finite arithmetic sequence. If the perimeter of the triangle is 18 cm, find the values of a, b, and c.



17. Find the common ratio and an explicit form in each of the following geometric sequences.
    a. $4,\ 12,\ 36,\ 108, \ldots$
    b. $162,\ 108,\ 72,\ 48, \ldots$
    c. $\frac{4}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$
    d. $xz,\ x^2z^3,\ x^3z^5, x^4z^7, \ldots$

18. The first term in a geometric sequence is 54, and the 5^{th} term is $\frac{2}{3}$. Find an explicit form for the geometric sequence.

19. If 2, a, b, -54 forms a geometric sequence, find the values of a and b.

20. Find the explicit form $f(n)$ of a geometric sequence if $f(3) - f(1) = 48$ and $\frac{f(3)}{f(1)} = 9$.