Lesson 17: Four Interesting Transformations of Functions

Classwork

Exploratory Challenge 1

Let $f(x) = |x|$, $g(x) = f(x) - 3$, $h(x) = f(x) + 2$ for any real number x.

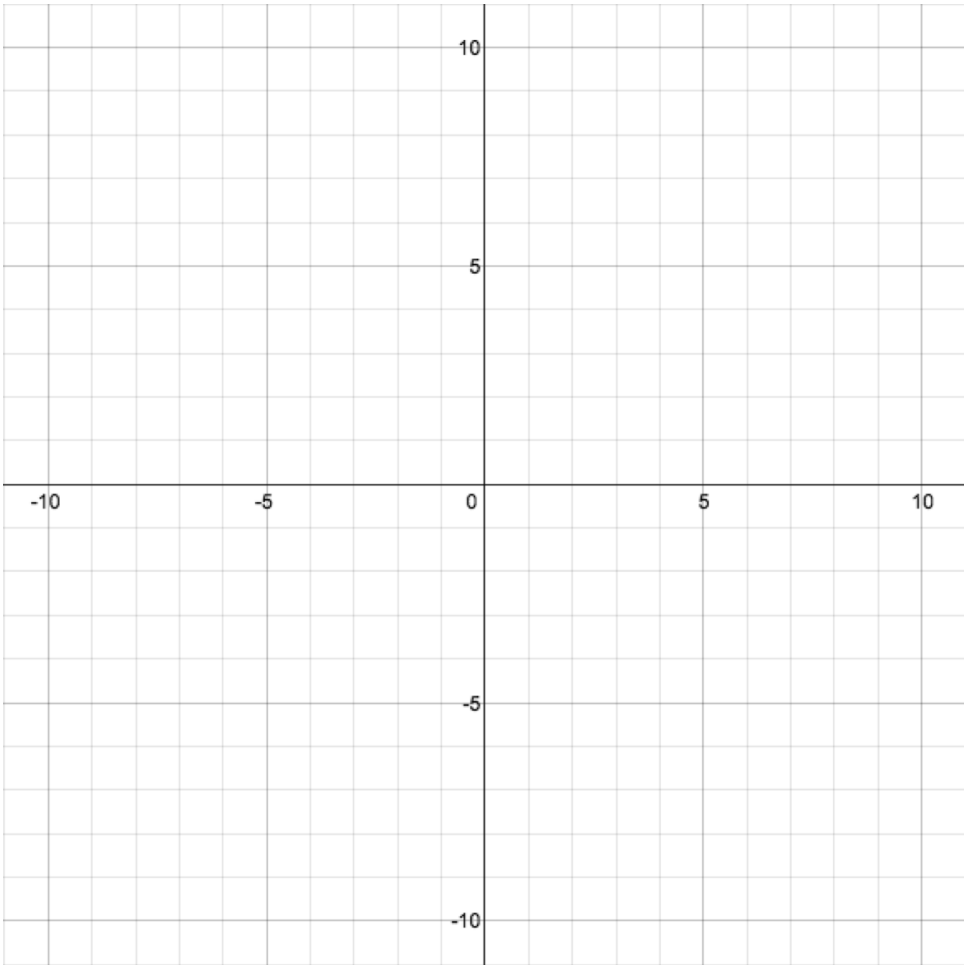
a. Write an explicit formula for $g(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

b. Write an explicit formula for $h(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

c. Complete the table of values for these functions.

x	$f(x) = \|x\|$	$g(x) = f(x) - 3$	$h(x) = f(x) + 2$
-3			
-2			
-1			
0			
1			
2			
3			

d. Graph all three equations: $y = f(x)$, $y = f(x) - 3$, and $y = f(x) + 2$.



e. What is the relationship between the graph of $y = f(x)$ and the graph of $y = f(x) + k$?

f. How do the values of g and h relate to the values of f?

Exploratory Challenge 2

Let $f(x) = |x|$, $g(x) = 2f(x)$, $h(x) = \frac{1}{2}f(x)$ for any real number x.

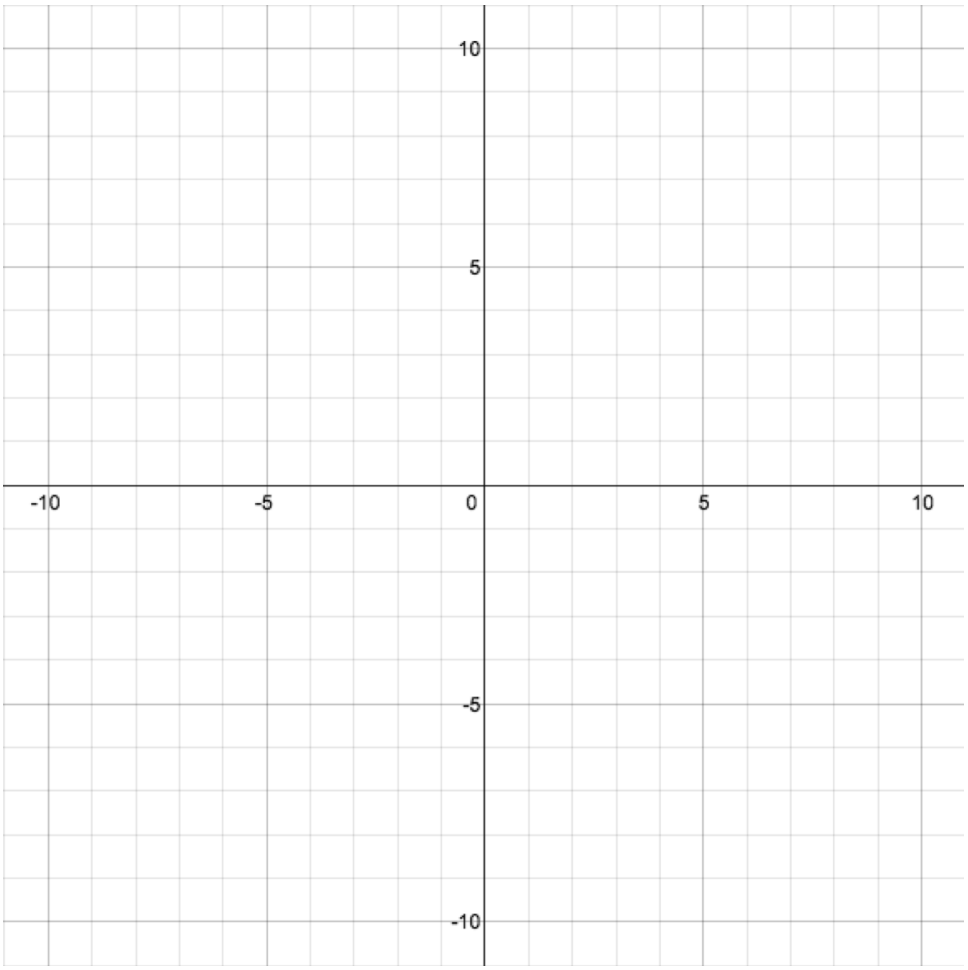
a. Write a formula for $g(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

b. Write a formula for $h(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

c. Complete the table of values for these functions.

x	$f(x) = \|x\|$	$g(x) = 2f(x)$	$h(x) = \frac{1}{2}f(x)$
-3			
-2			
-1			
0			
1			
2			
3			

d. Graph all three equations: $y = f(x)$, $y = 2f(x)$, and $y = \frac{1}{2}f(x)$.



Given $f(x) = |x|$, let $p(x) = -|x|$, $q(x) = -2f(x)$, $r(x) = -\frac{1}{2}f(x)$ for any real number x.

e. Write the formula for $q(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

f. Write the formula for $r(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

g. Complete the table of values for the functions $p(x) = -|x|$, $q(x) = -2f(x)$, $r(x) = -\frac{1}{2}f(x)$.

x	$p(x) = -\lvert x\rvert$	$q(x) = -2f(x)$	$r(x) = -\frac{1}{2}f(x)$
-3			
-2			
-1			
0			
1			
2			
3			

h. Graph all three functions on the same graph that was created in part (d). Label the graphs as $y = p(x)$, $y = q(x)$, and $y = r(x)$.

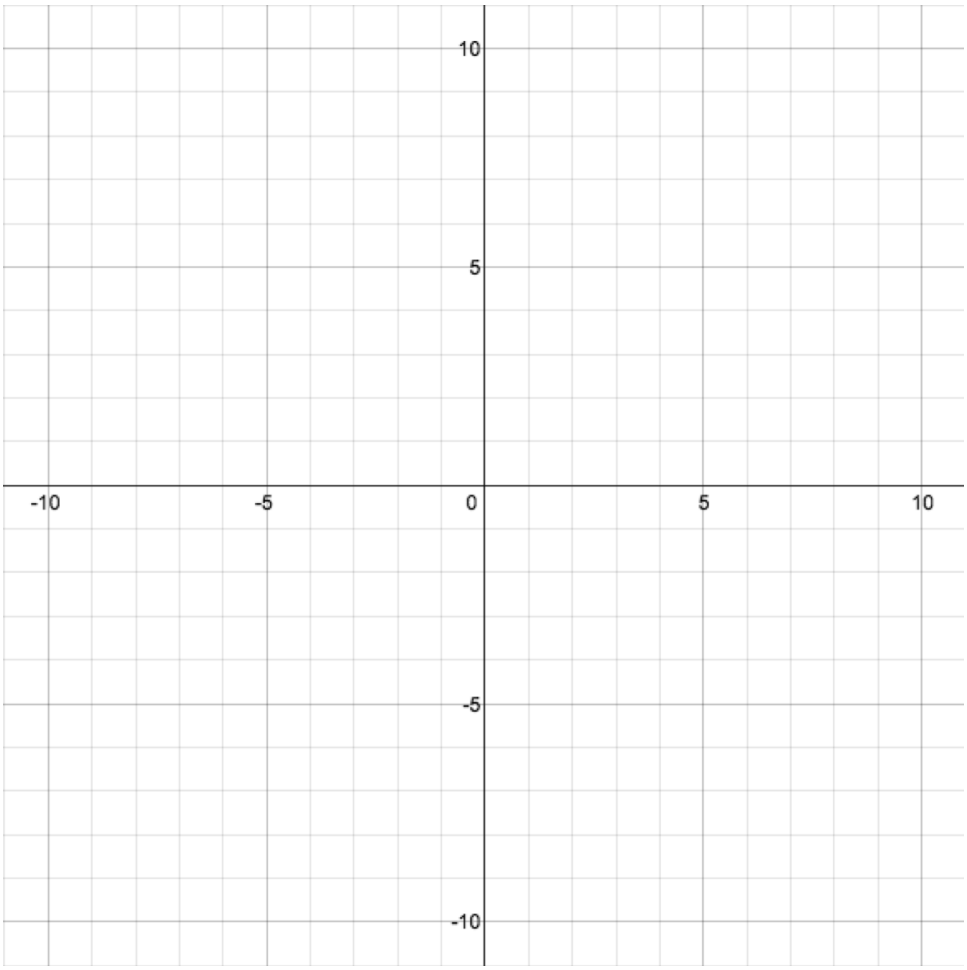
i. How is the graph of $y = f(x)$ related to the graph of $y = kf(x)$ when $k > 1$?

j. How is the graph of $y = f(x)$ related to the graph of $y = kf(x)$ when $0 < k < 1$?

k. How do the values of functions p, q, and r relate to the values of functions f, g, and h, respectively? What transformation of the graphs of f, g, and h represents this relationship?

Exercise

Make up your own function f by drawing the graph of it on the Cartesian plane below. Label it as the graph of the equation, $y = f(x)$. If $b(x) = f(x) - 4$ and $c(x) = \frac{1}{4}f(x)$ for every real number x, graph the equations $y = b(x)$ and $y = c(x)$ on the same Cartesian plane.



Problem Set

Let $f(x) = |x|$ for every real number x. The graph of $y = f(x)$ is shown below. Describe how the graph for each function below is a transformation of the graph of $y = f(x)$. Then use this same set of axes to graph each function for Problems 1–5. Be sure to label each function on your graph (by $y = a(x)$, $y = b(x)$, etc.).

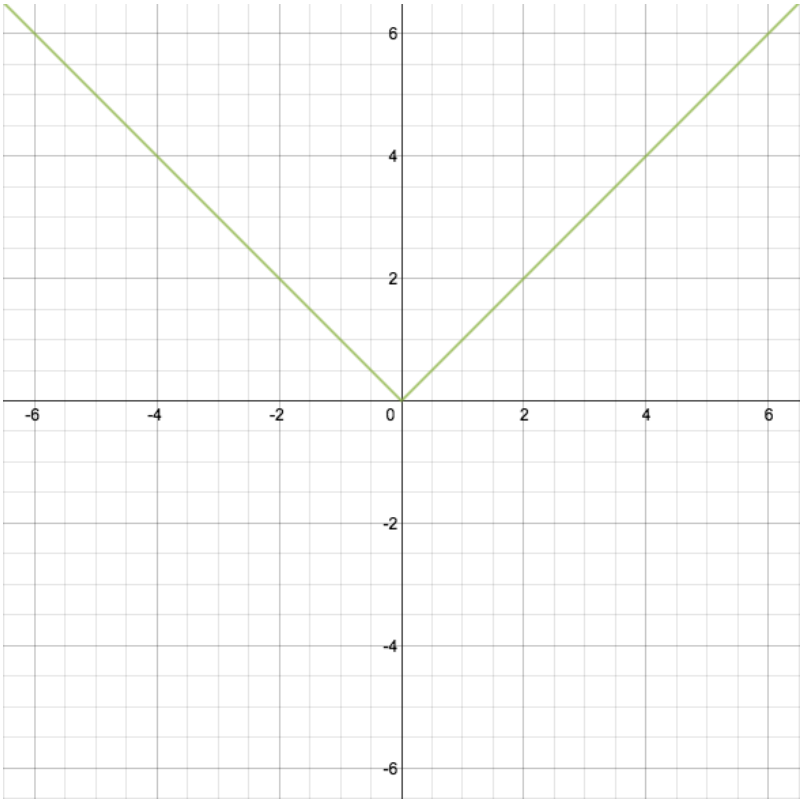
1. $a(x) = |x| + \frac{3}{2}$

2. $b(x) = -|x|$

3. $c(x) = 2|x|$

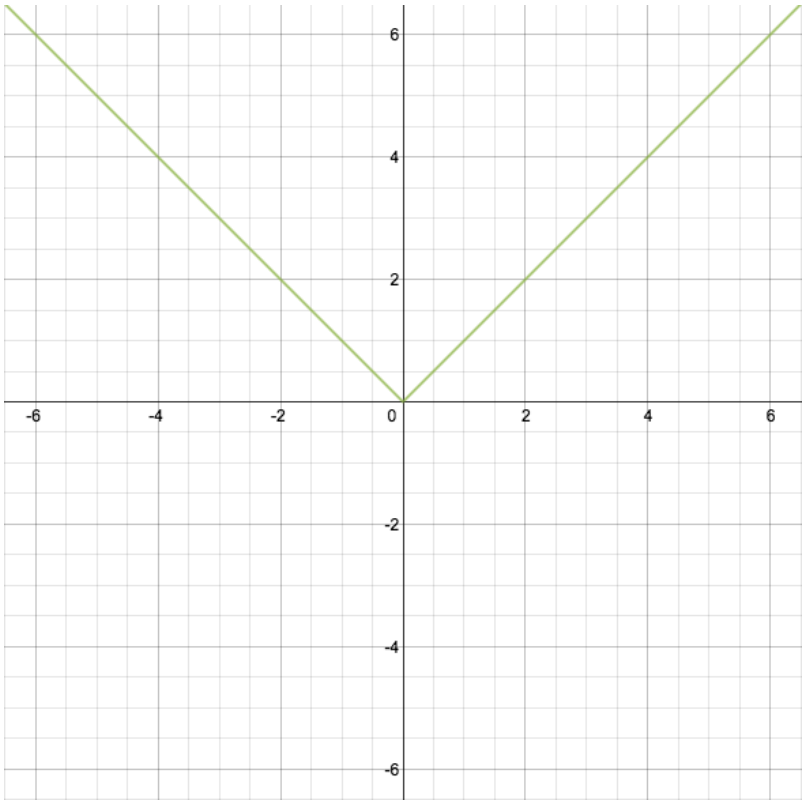
4. $d(x) = \frac{1}{3}|x|$

5. $e(x) = |x| - 3$



6. Let $r(x) = |x|$ and $t(x) = -2|x| + 1$ for every real number x. The graph of $y = r(x)$ is shown below. Complete the table below to generate output values for the function t, and then graph the equation $y = t(x)$ on the same set of axes as the graph of $y = r(x)$.

x	$r(x) = \|x\|$	$t(x) = -2\|x\| + 1$
-2		
-1		
0		
1		
2		



7. Let $f(x) = |x|$ for every real number x. Let m and n be functions found by transforming the graph of $y = f(x)$. Use the graphs of $y = f(x), y = m(x)$, and $y = n(x)$ below to write the functions m and n in terms of the function f. (Hint: What is the k?)