Lesson 18: Four Interesting Transformations of Functions

Classwork

Example

Let $f(x) = |x|$, $g(x) = f(x - 3)$, $h(x) = f(x + 2)$ where x can be any real number.
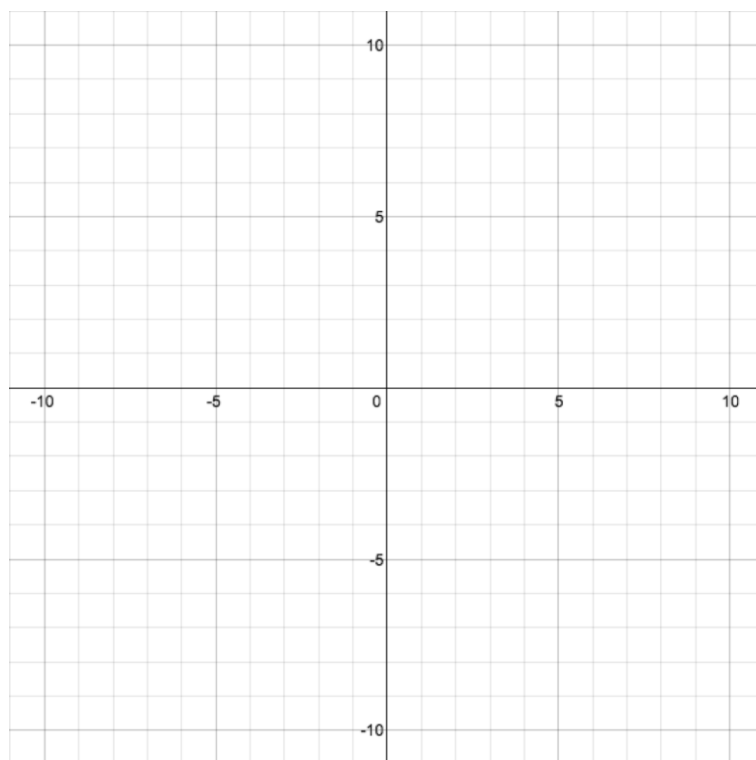
　　a.　Write the formula for $g(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

　　b.　Write the formula for $h(x)$ in terms of $|x|$ (i.e., without using $f(x)$ notation).

　　c.　Complete the table of values for these functions.

x	$f(x) = |x|$	$g(x) =$	$h(x) =$
-3			
-2			
-1			
0			
1			
2			
3			

d. Graph all three equations: $y = f(x)$, $y = f(x - 3)$, and $y = f(x + 2)$.

e. How does the graph of $y = f(x)$ relate to the graph of $y = f(x - 3)$?

f. How does the graph of $y = f(x)$ relate to the graph of $y = f(x + 2)$?

g. How does the graph of $y = |x| - 3$ and the graph of $y = |x - 3|$ relate differently to the graph of $y = |x|$?

h.   How do the values of g and h relate to the values of f?

Exercises

1. Karla and Isamar are disagreeing over which way the graph of the function $g(x) = |x + 3|$ is translated relative to the graph of $f(x) = |x|$. Karla believes the graph of g is "to the right" of the graph of f; Isamar believes the graph is "to the left." Who is correct? Use the coordinates of the vertex of f and g and to support your explanation.

2. Let $f(x) = |x|$ where x can be any real number. Write a formula for the function whose graph is the transformation of the graph of f given by the instructions below.

    a.   A translation right 5 units.

    b.   A translation down 3 units.

    c.   A vertical scaling (a vertical stretch) with scale factor of 5.
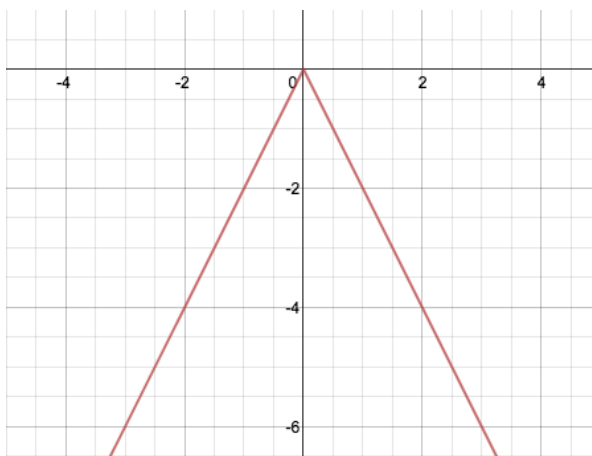
    d.   A translation left 4 units.

e. A vertical scaling (a vertical shrink) with scale factor of $\frac{1}{3}$.
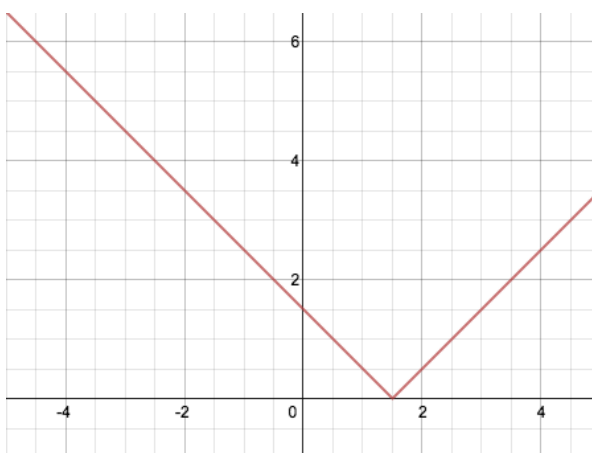
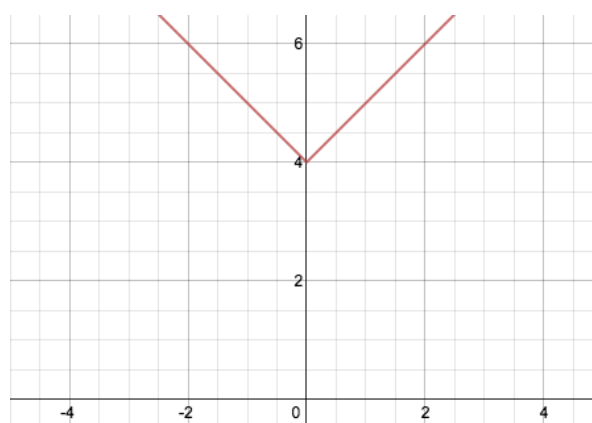3. Write the formula for the function depicted by the graph.
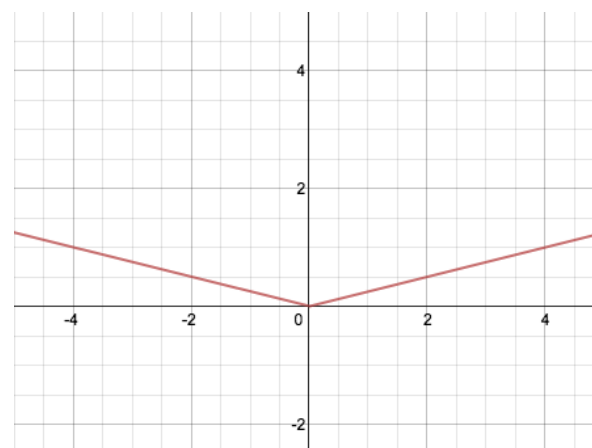
a. $y =$
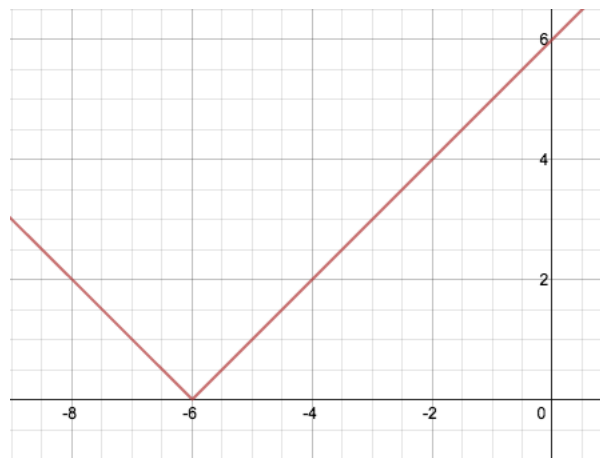
b. $y =$

c. $y =$

d. $y =$

e. $y =$

4. Let $f(x) = |x|$ where x can be any real number. Write a formula for the function whose graph is the described transformation of the graph of f.

    a. A translation 2 units left and 4 units down.

    b. A translation 2.5 units right and 1 unit up.

    c. A vertical scaling with scale factor $\frac{1}{2}$ and then a translation 3 units right.

    d. A translation 5 units right and a vertical scaling by reflecting across the x-axis with vertical scale factor -2.

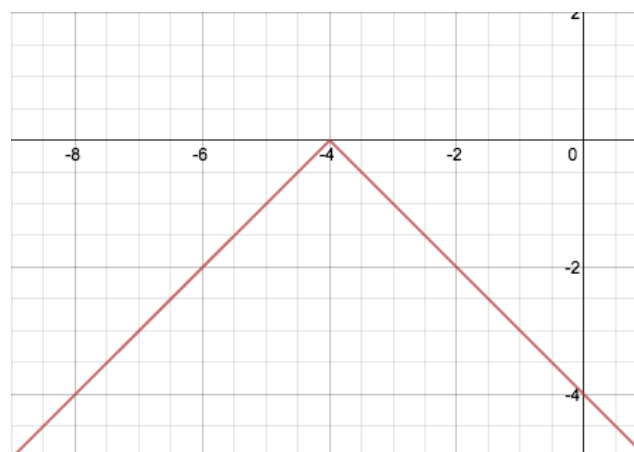5. Write the formula for the function depicted by the graph.
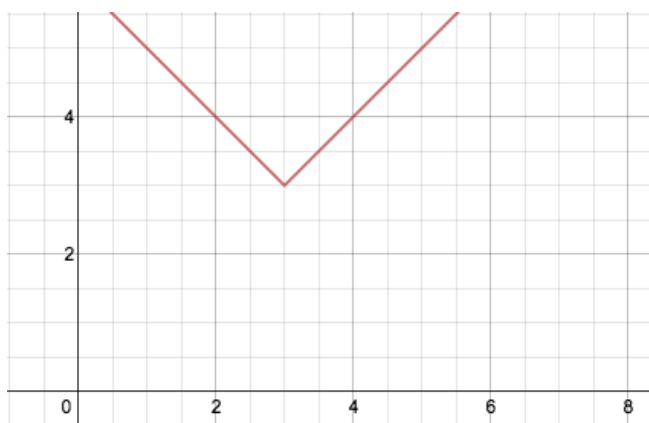
    a. $y =$
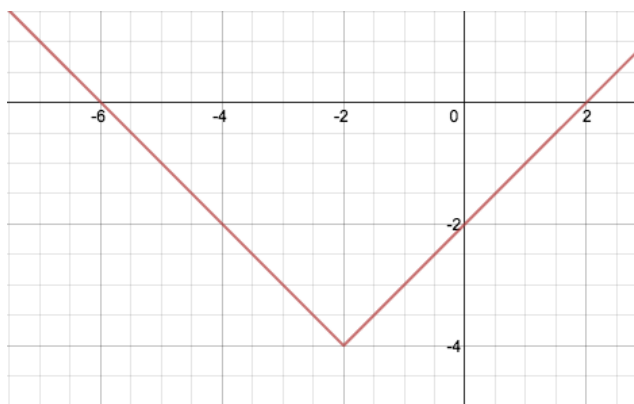
b. $y =$
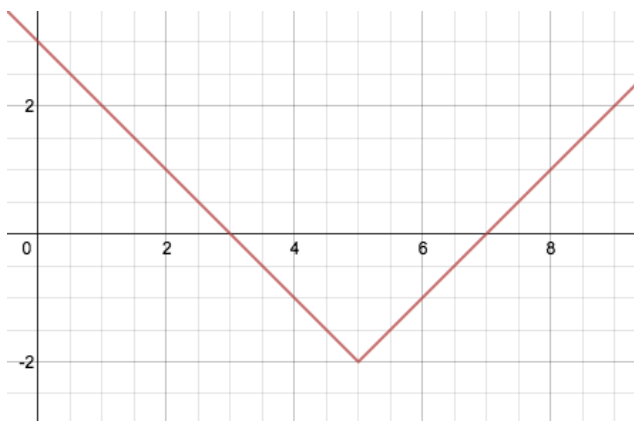
c. $y =$

d. $y =$

Problem Set

1. Working With Quadratic Functions:

    a. The vertex of the quadratic function $f(x) = x^2$ is at $(0,0)$, which is the minimum for the graph of f. Based on your work in this lesson, to where do you predict the vertex will be translated for the graphs of $g(x) = (x-2)^2$ and $h(x) = (x+3)^2$?

    b. Complete the table of values and then graph all three functions.

x	$f(x) = x^2$	$g(x) = (x-2)^2$	$h(x) = (x+3)^2$
-3			
-2			
-1			
0			
1			
2			
3			

2. Let $f(x) = |x-4|$ for every real number x. The graph of the equation $y = f(x)$ is provided on the Cartesian plane below. Transformations of the graph of $y = f(x)$ are described below. After each description, write the equation for the transformed graph. Then, sketch the graph of the equation you write for part (d).
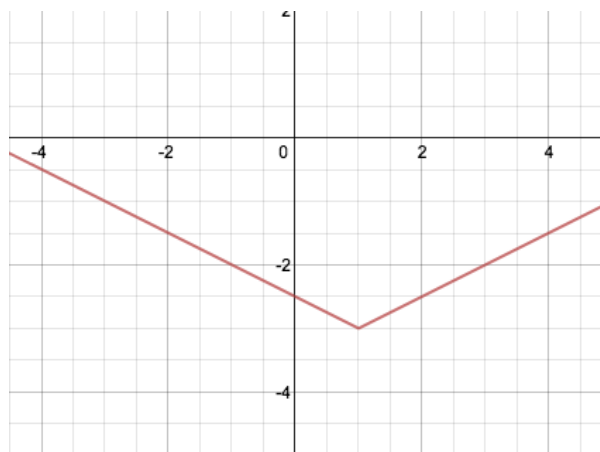
    a. Translate the graph left 6 units and down 2 units.

    b. Reflect the resulting graph from part (a) across the x-axis.

    c. Scale the resulting graph from part (b) vertically by a scale factor of $\frac{1}{2}$.

    d. Translate the resulting graph from part (c) right 3 units and up 2 units. Graph the resulting equation.

$y = |x-4|$
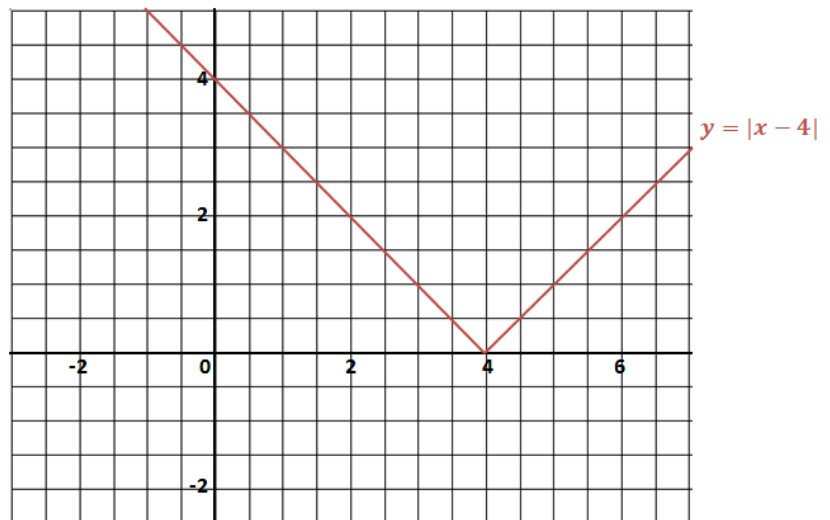
3. Let $f(x) = |x|$ for all real numbers x. Write the formula for the function represented by the described transformation of the graph of $y = f(x)$.

   a. First, a vertical stretch with scale factor $\dfrac{1}{3}$ is performed, then a translation right 3 units, and finally a translation down 1 unit.

   b. First, a vertical stretch with scale factor 3 is performed, then a reflection over the x-axis, then a translation left 4 units, and finally a translation up 5 units.

   c. First, a reflection across the x-axis is performed, then a translation left 4 units, then a translation up 5 units, and finally a vertical stretch with scale factor 3.

   d. Compare your answers to parts (b) and (c). Why are they different?

4. Write the formula for the function depicted by each graph.

   a. $a(x) =$

   b. $b(x) =$