Lesson 19: Four Interesting Transformations of Functions

Classwork

Exploratory Challenge 1

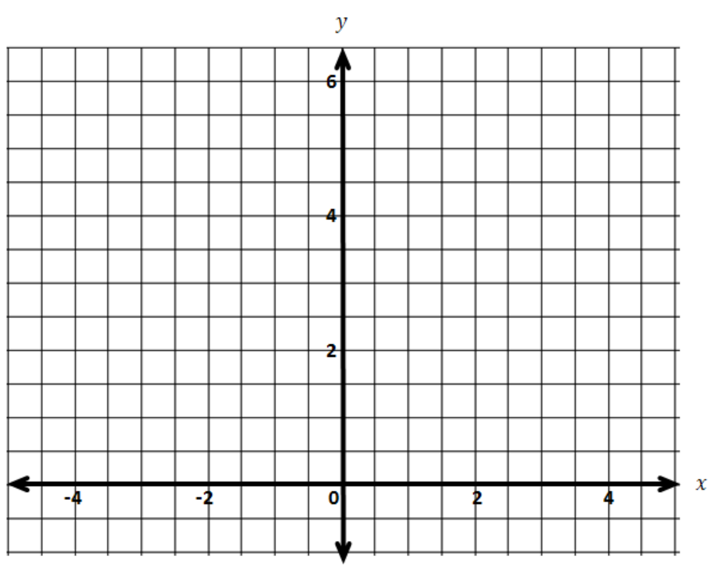
Let $f(x) = x^2$ and $g(x) = f(2x)$, where x can be any real number.

a. Write the formula for g in terms of x^2 (i.e., without using $f(x)$ notation).

b. Complete the table of values for these functions.

x	$f(x) = x^2$	$g(x) = f(2x)$
-3		
-2		
-1		
0		
1		
2		
3		

c. Graph both equations: $y = f(x)$ and $y = f(2x)$.



d. How does the graph of $y = g(x)$ relate to the graph of $y = f(x)$?

e. How are the values of f related to the values of g?

Exploratory Challenge 2

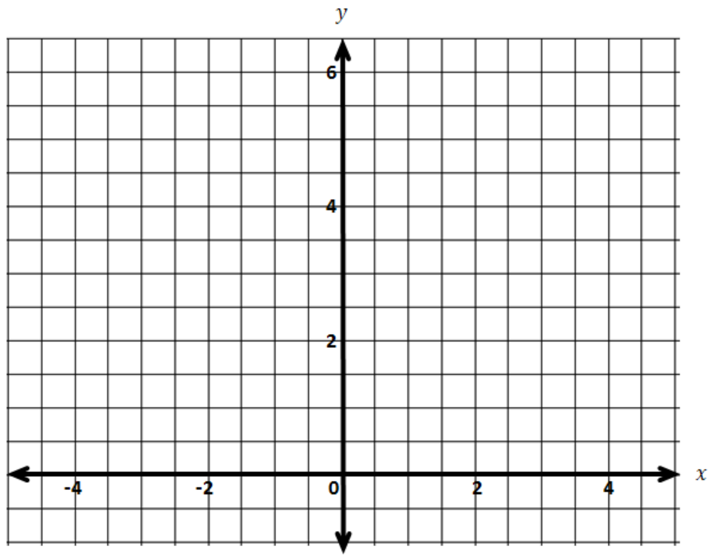
Let $f(x) = x^2$ and $h(x) = f\left(\frac{1}{2}x\right)$, where x can be any real number.

a. Rewrite the formula for h in terms of x^2 (i.e., without using $f(x)$ notation).

b. Complete the table of values for these functions.

x	$f(x) = x^2$	$h(x) = f\left(\frac{1}{2}x\right)$
-3		
-2		
-1		
0		
1		
2		
3		

c. Graph both equations: $y = f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



d. How does the graph of $y = f(x)$ relate to the graph of $y = h(x)$?

e. How are the values of f related to the values of h?

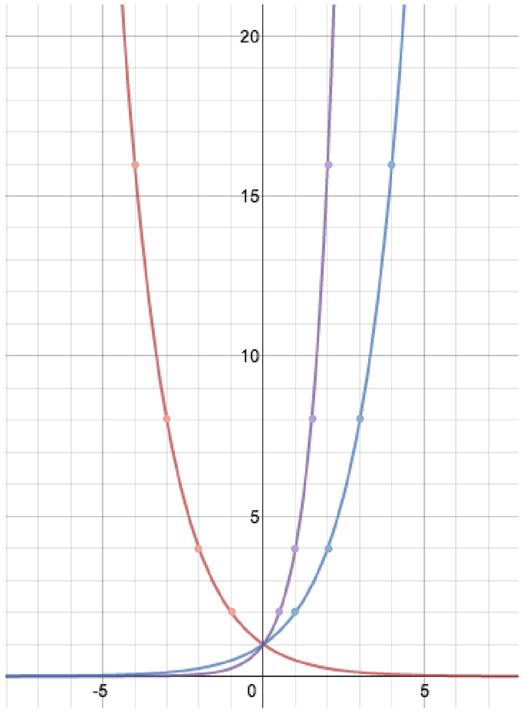
Exercise

Complete the table of values for the given functions.

a.

x	$f(x) = 2^x$	$g(x) = 2^{(2x)}$	$h(x) = 2^{(-x)}$
-2			
-1			
0			
1			
2			

b. Label each of the graphs with the appropriate functions from the table.



c. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = g(x)$.

d. Consider $y = f(x)$ and $y = h(x)$. What does negating the input do to the graph of f?

e. Write the formula of an exponential function whose graph would be a horizontal stretch relative to the graph of g.

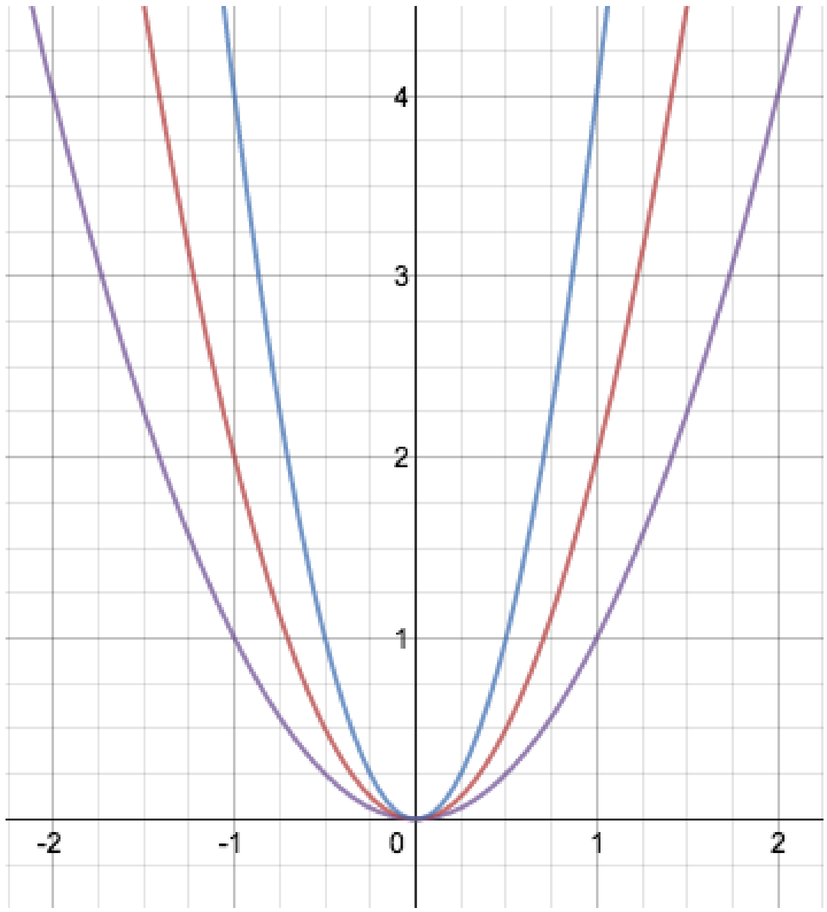
Exploratory Challenge 3

a. Look at the graph of $y = f(x)$ for the function $f(x) = x^2$ in Exploratory Challenge 1 again. Would we see a difference in the graph of $y = g(x)$ if -2 was used as the scale factor instead of 2? If so, describe the difference. If not, explain why not.

b. A reflection across the y-axis takes the graph of $y = f(x)$ for the function $f(x) = x^2$ back to itself. Such a transformation is called a *reflection symmetry*. What is the equation for the graph of the reflection symmetry of the graph of $y = f(x)$?

c. Deriving the answer to the following question is fairly sophisticated; do only if you have time. In Lessons 17 and 18, we used the function $f(x) = |x|$ to examine the graphical effects of transformations of a function. In this lesson, we use the function $f(x) = x^2$ to examine the graphical effects of transformations of a function. Based on the observations you made while graphing, why would using $f(x) = x^2$ be a better option than using the function $f(x) = |x|$?

Problem Set



Let $f(x) = x^2$, $g(x) = 2x^2$, and $h(x) = (2x)^2$, where x can be any real number. The graphs above are of the functions $y = f(x)$, $y = g(x)$, and $y = h(x)$.

1. Label each graph with the appropriate equation.

2. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = g(x)$. Use coordinates to illustrate an example of the correspondence.

3. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = h(x)$. Use coordinates to illustrate an example of the correspondence.