Lesson 21: Comparing Linear and Exponential Functions Again

Classwork

Opening Exercise

	Linear Model	Exponential Model
General Form		
Meaning of Parameters a and b		
Example		
Rule for Finding $f(x+1)$ from $f(x)$		
Table		
Graph		
Story Problem Example		

Exercise 1–2

1. For each table below, assume the function f is defined for all real numbers. Calculate $\Delta f = f(x+1) - f(x)$ in the last column in the tables below and show your work (the symbol Δ in this context means "change in"). What do you notice about Δf? Could the function be linear or exponential? Write a linear or exponential function formula that generates the same input-output pairs as given in the table.

x	$f(x)$	$\Delta f = f(x+1) - f(x)$
1	-3	
2	1	
3	5	
4	9	
5	13	

x	$f(x)$	$\Delta f = f(x+1) - f(x)$
0	2	
1	6	
2	18	
3	54	
4	162	

2. Terence looked down the second column of the table and noticed that $\frac{3}{1} = \frac{9}{3} = \frac{27}{3} = \frac{81}{27}$. Because of his observation, he claimed that the input-output pairs in this table could be modeled with an exponential function. Explain why Terence is correct or incorrect. If he is correct, write a formula for the exponential function that generates the input-output pairs given in the table. If he is incorrect, determine and write a formula for a function that generates the input-output pairs given in the table.

x	$T(x)$

3. A river has an initial minnow population of $40{,}000$ that is growing at 5% per year. Due to environmental conditions, the amount of algae that minnows use for food is decreasing, supporting $1{,}000$ fewer minnows each year. Currently, there is enough algae to support $50{,}000$ minnows. Is the minnow population increasing linearly or exponentially? Is the amount of algae decreasing at a linear or exponential rate? In what year will the minnow population exceed the amount of algae available?

4. Using a calculator, Joanna made the following table, and then made the following conjecture: $3x$ is always greater than $(1.02)^x$. Is Joanna correct? Explain.

x	$(1.02)^x$	$3x$
1	1.02	3
2	1.0404	6
3	1.0612	9
4	1.0824	12
5	1.1041	15

Lesson Summary

- Suppose that the input-output pairs of a bivariate dataset have the following property: For every two inputs that are a given difference apart, the difference in their corresponding outputs is constant. Then an appropriate model for that dataset could be a linear function.
- Suppose that the input-output pairs of a bivariate dataset have the following property: For every two inputs that are a given difference apart, the quotient of their corresponding outputs is constant. Then an appropriate model for that dataset could be an exponential function.
- An increasing exponential function will eventually exceed any linear function. That is, if $f(x) = ab^x$ is an exponential function with $a > 0$ and $b > 1$, and $g(x) = mx + k$ is any linear function, then there is a real number M such that for all $x > M$, then $f(x) > g(x)$. Sometimes this is not apparent in a graph displayed on a graphing calculator; that is because the graphing window does not show enough of the graph to show the sharp rise of the exponential function in contrast with the linear function.

Problem Set

For each table in Problems 1–6, classify the data as describing a linear relationship, an exponential growth relationship, an exponential decay relationship, or neither. If the relationship is linear, calculate the constant rate of change (slope), and write a formula for the linear function that models the data. If the function is exponential, calculate the common quotient for input values that are distance one apart, and write the formula for the exponential function that models the data. For each linear or exponential function found, graph the equation $y = f(x)$.

1.

x	$f(x)$
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$
5	$\frac{1}{32}$

2.

x	$f(x)$
1	1.4
2	2.5
3	3.6
4	4.7
5	5.8

3.

x	$f(x)$
1	-1
2	0
3	2
4	5
5	9

4.

x	$f(x)$
1	20
2	40
3	80
4	160
5	320

5.

x	$f(x)$
1	-5
2	-12
3	-19
4	-26
5	-33

6.

x	$f(x)$
1	$\frac{1}{2}$
2	$\frac{1}{3}$
3	$\frac{1}{4}$
4	$\frac{1}{5}$
5	$\frac{1}{6}$

7. Here is a variation on a classic riddle: Jayden has a dog-walking business. He has two plans. Plan 1 includes walking a dog once a day for a rate of $5/day. Plan 2 also includes one walk a day, but charges 1 cent for 1 day, 2 cents for 2 days, 4 cents for 3 days, 8 cents for 4 days, and continues to double for each additional day. Mrs. Maroney needs Jayden to walk her dog every day for two weeks. Which plan should she choose? Show the work to justify your answer.

8. Tim deposits money in a Certificate of Deposit account. The balance (in dollars) in his account t years after making the deposit is given by $T(t) = 1{,}000(1.06)^t$ for $t \geq 0$.
   a. Explain, in terms of the structure of the expression used to define $T(t)$, why Tim's balance can never be $999.
   b. By what percent does the value of $T(t)$ grow each year? Explain by writing a recursive formula for the sequence $T(1)$, $T(2)$, $T(3)$, etc.
   c. By what percentages does the value of $T(t)$ grow every two years? (Hint: Use your recursive formula to write $T(n + 2)$ in terms of $T(n)$.)

9. Your mathematics teacher asks you to sketch a graph of the exponential function $f(x) = \left(\frac{3}{2}\right)^x$ for x, a number between 0 and 40 inclusively, using a scale of 10 units to one inch for both the x- and y-axes.
   a. What are the dimensions in feet of the roll of paper you will need to sketch this graph?
   b. How many more feet of paper would you need to add to the roll in order to graph the function on the interval $0 \leq x \leq 41$?
   c. Find an m so that the linear function $g(x) = mx + 2$ is greater than $f(x)$ for all x such that $0 \leq x \leq 40$, but $f(41) > g(41)$.