Lesson 4: Advanced Factoring Strategies for Quadratic Expressions

Classwork

Opening Exercises

Factor the following quadratic expressions.

1. $2x^2 + 10x + 12$

2. $6x^2 + 5x - 6$

Example 1: Splitting the Linear Term

How might we find the factors of $6x^2 + 5x - 6$?

1. Consider the product $(a)(c)$: $(6)(-6) = -36$.
2. Discuss the possibility that a and c are also multiplied when the leading coefficient is 1.
3. List all possible factor pairs of $(a)(c)$: $(1, -36)$, $(-1, 36)$, $(2, -18)$, $(-2, 18)$, $(3, -12)$, $(-3, 12)$, $(4, -9)$, $(-4, 9)$, and $(-6, 6)$.
4. Find the pair that satisfies the requirements of the product-sum method (i.e., a pair of numbers whose product equals ac and whose sum is b): $(-4) + 9 = 5$.
5. Rewrite the expression with the same first and last term but with an expanded b term using that pair of factors as coefficients: $6x^2 - 4x + 9x - 6$.
6. We now have four terms that can be entered into a tabular model or factored by grouping.
7. Factoring by grouping: Take the four terms above and pair the first two and the last two; this makes two groups.

   $[6x^2 - 4x] + [9x - 6]$ [Form two groups by pairing the first two and the last two.]

   $[2x(3x - 2)] + [3(3x - 2)]$ [Factor out the GCF from each pair.]

   The common binomial factor is now visible as a common factor of each group. Now rewrite by carefully factoring out the common factor, $3x - 2$, from each group: $(3x - 2)(2x + 3)$.

Note that we can factor difficult quadratic expressions, such as $6x^2 + 5x - 6$, using a tabular model or by splitting the linear term algebraically. Try both ways to see which one works best for you.

Exercises 1–4

Factor the following expressions using your method of choice. After factoring each expression completely, check your answers using the distributive property. Remember to always look for a GCF prior to trying any other strategies.

1. $2x^2 - x - 10$

2. $6x^2 + 7x - 20$

3. $-4x^2 + 4x - 1$

4. The area of a particular triangle can be represented by $x^2 + \frac{3}{2}x - \frac{9}{2}$. What are its base and height in terms of x?

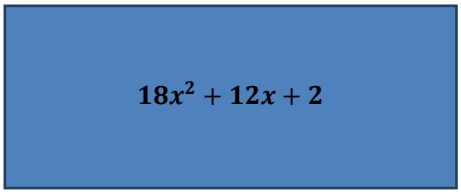
Lesson Summary

While there are several steps involved in splitting the linear term, it is a relatively more efficient and reliable method for factoring trinomials in comparison to simple guess-and-check.

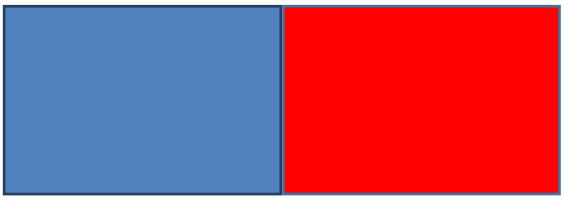
Problem Set

1. Factor completely.
   a. $6x^2 + 7x + 2$
   b. $8x^2 + 20x + 8$
   c. $3x^2 + 10x + 7$
   d. $x^2 + \frac{11}{2}x + \frac{5}{2}$
   e. $6x^3 - 2x^2 - 4x$ [Hint: Look for a GCF first.]

2. The area of the rectangle below is represented by the expression $18x^2 + 12x + 2$ square units. Write two expressions to represent the dimensions, if the length is known to be twice the width.



3. Two mathematicians are neighbors. Each owns a separate rectangular plot of land that shares a boundary and has the same dimensions. They agree that each has an area of $2x^2 + 3x + 1$ square units. One mathematician sells his plot to the other. The other wants to put a fence around the perimeter of his new combined plot of land. How many linear units of fencing will he need? Write your answer as an expression in x.



Note: This question has two correct approaches and two different correct solutions. Can you find them both?