Lesson 21: Transformations of the Quadratic Parent Function,

$f(x) = x^2$

Classwork

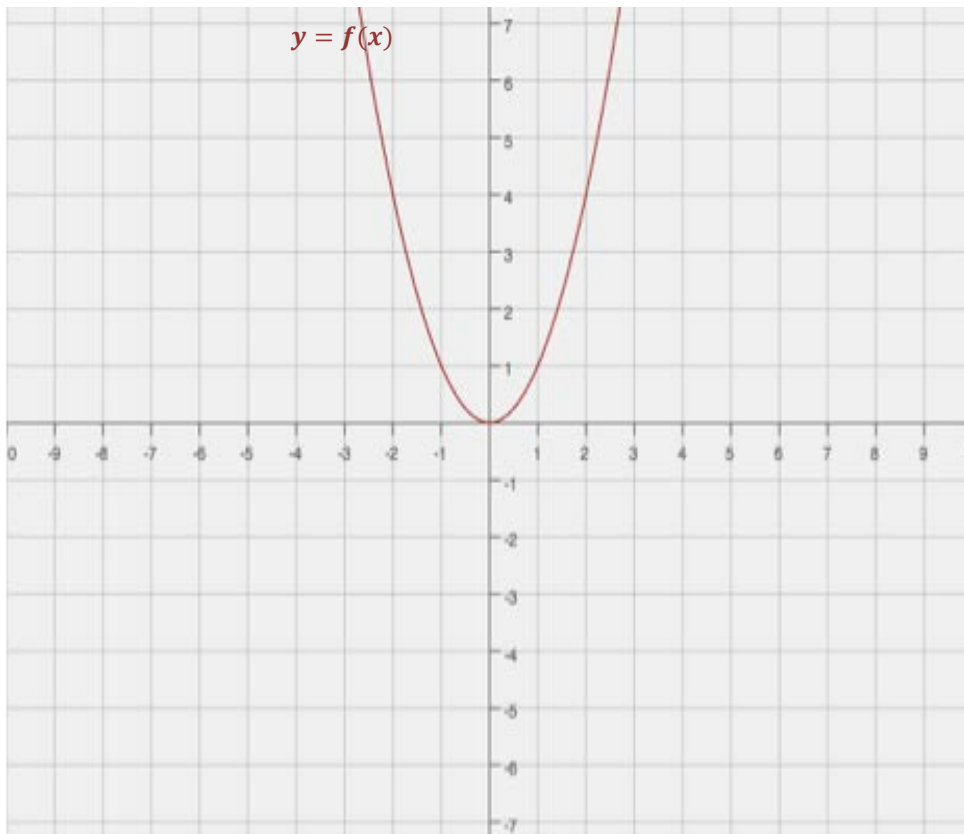### Example 1: Quadratic Expression Representing a Function

a. A quadratic function is defined by $g(x) = 2x^2 + 12x + 1$. Write this in the completed-square (vertex) form and show all the steps.

b. Where is the vertex of the graph of this function located?

c. Look at the completed-square form of the function. Can you name the parent function? How do you know?

d. What transformations have been applied to the parent function to arrive at function g? Be specific.

e. How does the completed-square form relate to the quadratic parent function $f(x) = x^2$?

Example 2

The graph of a quadratic function $f(x) = x^2$ has been translated 3 units to the right, vertically stretched by a factor of 4, and moved 2 units up. Write the formula for the function that defines the transformed graph.

Exercises

1. Without using a graphing calculator, sketch the graph of the following quadratic functions on the same coordinate plane, using transformations of the graph of the parent function $f(x) = x^2$.

   a. $g(x) = -2(x-3)^2 + 4$

   b. $h(x) = -3(x+5)^2 + 1$

   c. $k(x) = 2(x+4)^2 - 3$

   d. $p(x) = x^2 - 2x$

   e. $t(x) = x^2 - 2x + 3$

2. Write a formula for the function that defines the described transformation of the graph of the quadratic parent function $f(x) = x^2$.

    a. 3 units shift to the right

    b. Vertical shrink by a factor of 0.5

    c. Reflection across the x-axis

    d. 4 units shift up

    Then, graph both the parent and the transformed functions on the same coordinate plane.
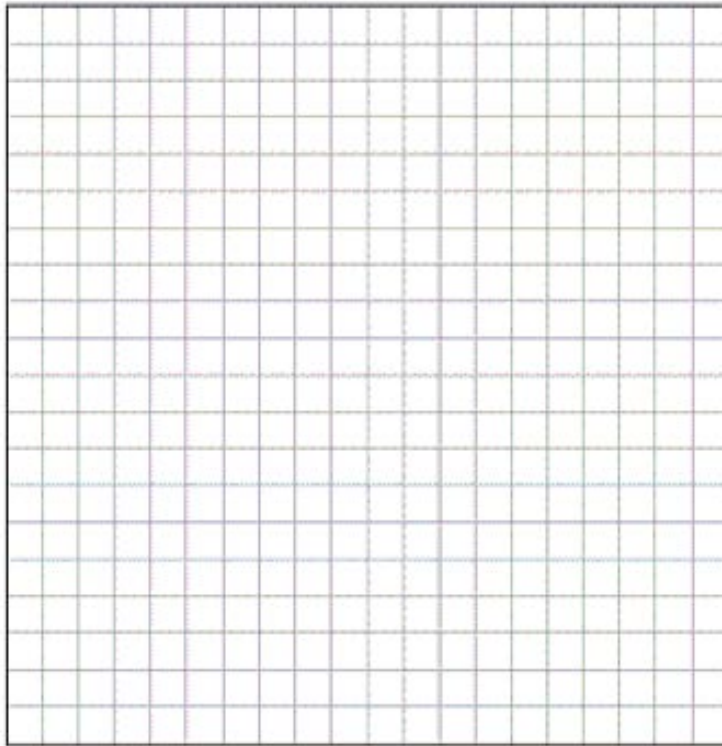
3. Describe the transformation of the quadratic parent function $f(x) = x^2$ that results in the quadratic function $g(x) = 2x^2 + 4x + 1$.

4. Sketch the graphs of the following functions based on the graph of the function $f(x) = x^2$. If necessary, rewrite some of the functions in the vertex (completed-square) form. Label your graphs.

a. $g(x) = -(x-4)^2 + 3$

b. $h(x) = 3(x-2)^2 - 1$

c. $k(x) = 2x^2 + 8x$

d. $p(x) = x^2 + 6x + 5$

$y = f(x)$
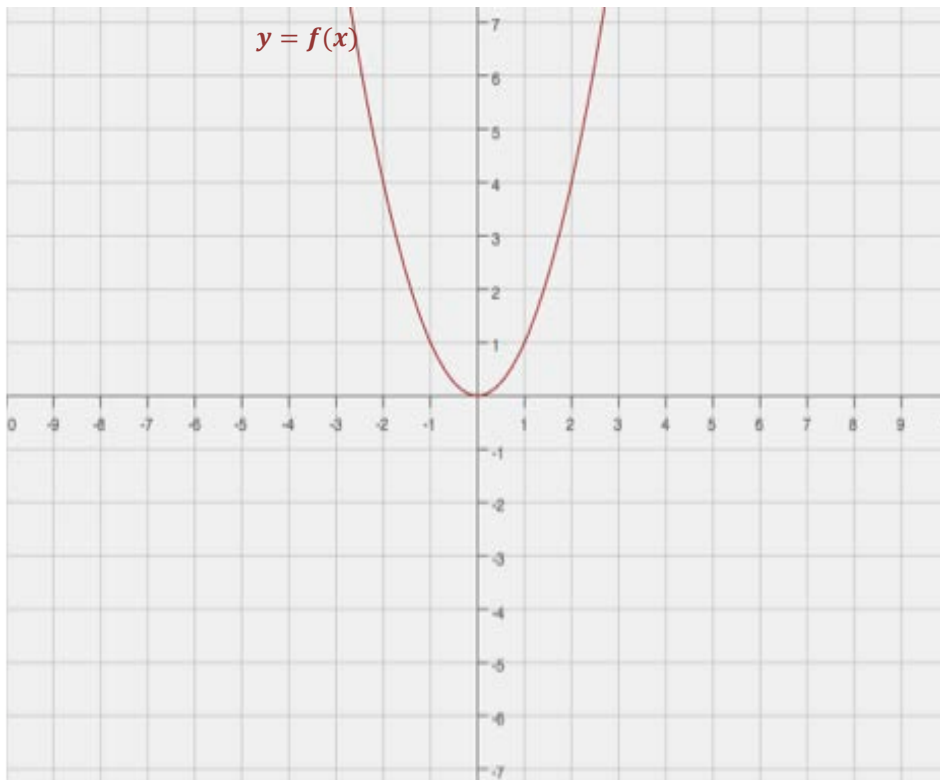
Lesson Summary

Transformations of the quadratic parent function, $f(x) = x^2$, can be rewritten in form $g(x) = a(x - h)^2 + k$, where (h, k) is the vertex of the translated and scaled graph of f, with the scale factor of a, the leading coefficient. We can then quickly and efficiently (without the use of technology) sketch the graph of any quadratic function in the form $f(x) = a(x - h)^2 + k$ using transformations of the graph of the quadratic parent function, $f(x) = x^2$.

Problem Set

1. Write the function $g(x) = -2x^2 - 20x - 53$ in completed-square form. Describe the transformations of the graph of the parent function $f(x) = x^2$ that result in the graph of g.

2. Write the formula for the function whose graph is the graph of $f(x) = x^2$ translated 6.25 units to the right, vertically stretched by a factor of 8, and translated 2.5 units up.

3. Without using a graphing calculator, sketch the graphs of the functions below based on transformations of the graph of the parent function $f(x) = x^2$. Use your own graph paper and label your graphs.

    a. $g(x) = (x + 2)^2 - 4$
    b. $h(x) = -(x - 4)^2 + 2$
    c. $k(x) = 2x^2 - 12x + 19$
    d. $p(x) = -2x^2 - 4x - 5$
    e. $q(x) = 3x^2 + 6x$