# Lesson 2: The Multiplication of Polynomials

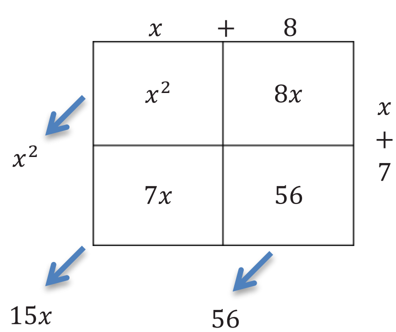
## Classwork

### Opening Exercise

Show that $28 \times 27 = (20 + 8)(20 + 7)$ using an area model. What do the numbers you placed inside the four rectangular regions you drew represent?

## Example 1

Use tabular method to multiply $(x + 8)(x + 7)$ and combine like terms.



**Exercises 1–2**

1. Use the tabular method to multiply $(x^2 + 3x + 1)(x^2 - 5x + 2)$ and combine like terms.

2. Use the tabular method to multiply $(x^2 + 3x + 1)(x^2 - 2)$ and combine like terms.

## Example 2

Multiply the polynomials $(x-1)(x^4+x^3+x^2+x+1)$ using a table. Generalize the pattern that emerges by writing down an identity for $(x-1)(x^n+x^{n-1}+\cdots+x^2+x+1)$ for $n$ a positive integer.

| | $x$ | $-1$ | |
|---|---|---|---|
| $x^5$ | $x^5$ | $-x^4$ | $x^4$ |
| $0x^4$ | $x^4$ | $-x^3$ | $x^3$ |
| $0x^3$ | $x^3$ | $-x^2$ | $x^2$ |
| $0x^2$ | $x^2$ | $-x$ | $x$ |
| | $x$ | $-1$ | $1$ |
| $0x$ | $-1$ | | |

### Exercises 3–4

3. Multiply $(x-y)(x^3+x^2y+xy^2+y^3)$ using the distributive property and combine like terms. How is this calculation similar to Example 2?

4. Multiply $(x^2-y^2)(x^2+y^2)$ using the distributive property and combine like terms. Generalize the pattern that emerges to write down an identity for $(x^n-y^n)(x^n+y^n)$ for positive integers $n$.

## Relevant Vocabulary

**Equivalent Polynomial Expressions:** Two polynomial expressions in one variable are *equivalent* if, whenever a number is substituted into all instances of the variable symbol in both expressions, the numerical expressions created are equal.

**Polynomial Identity:** A *polynomial identity* is a statement that two polynomial expressions are equivalent. For example, $(x+3)^2 = x^2 + 6x + 9$ for any real number $x$ is a polynomial identity.

**Coefficient of a Monomial:** The *coefficient of a monomial* is the value of the numerical expression found by substituting the number $1$ into all the variable symbols in the monomial. The coefficient of $3x^2$ is $3$, and the coefficient of the monomial $(3xyz) \cdot 4$ is $12$.

**Terms of a Polynomial:** When a polynomial is expressed as a monomial or a sum of monomials, each monomial in the sum is called a *term* of the polynomial.

**Like Terms of a Polynomial:** Two terms of a polynomial that have the same variable symbols each raised to the same power are called *like terms*.

**Standard Form of a Polynomial in One Variable:** A polynomial expression with one variable symbol, $x$, is in *standard form* if it is expressed as

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where $n$ is a non-negative integer, and $a_0, a_1, a_2 \ldots, a_n$ are constant coefficients with $a_n \neq 0$.

A polynomial expression in $x$ that is in standard form is often just called a *polynomial in* $x$ or a *polynomial*.

The *degree of the polynomial in standard form* is the highest degree of the terms in the polynomial, namely $n$. The term $a_n x^n$ is called the *leading term* and $a_n$ (thought of as a specific number) is called the *leading coefficient.* The *constant term* is the value of the numerical expression found by substituting $0$ into all the variable symbols of the polynomial, namely $a_0$.

## Problem Set

1. Complete the following statements by filling in the blanks.
   a. $(a+b)(c+d+e) = ac + ad + ae + \_\_\_ + \_\_\_ + \_\_\_$
   b. $(r-s)^2 = (\_\_\_\_)^2 - (\_\_\_\_)rs + s^2$
   c. $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (\_\_\_\_)^2$
   d. $(w-1)(1+w+w^2) = \_\_\_\_ - 1$
   e. $a^2 - 16 = (a + \_\_\_\_)(a - \_\_\_\_)$
   f. $(2x+5y)(2x-5y) = \_\_\_\_ - \_\_\_\_$
   g. $(2^{21}-1)(2^{21}+1) = \_\_\_\_ - 1$
   h. $[(x-y)-3][(x-y)+3] = (\_\_\_\_)^2 - 9$

2. Use the tabular method to multiply and combine like terms.
   a. $(x^2 - 4x + 4)(x + 3)$
   b. $(11 - 15x - 7x^2)(25 - 16x^2)$
   c. $(3m^3 + m^2 - 2m - 5)(m^2 - 5m - 6)$
   d. $(x^2 - 3x + 9)(x^2 + 3x + 9.$

3. Multiply and combine like terms to write as the sum or difference of monomials.
   a. $2a(5 + 4a)$
   b. $x^3(x + 6) + 9$
   c. $\frac{1}{8}(96z + 24z^2)$
   d. $2^{23}(2^{84} - 2^{81})$
   e. $(x - 4)(x + 5)$
   f. $(10w - 1)(10w + 1)$
   g. $(3z^2 - 8)(3z^2 + 8)$
   h. $(-5w - 3)w^2$
   i. $8y^{1000}(y^{12200} + 0.125y)$
   j. $(2r + 1)(2r^2 + 1)$
   k. $(t - 1)(t + 1)(t^2 + 1)$
   l. $(w - 1)(w^5 + w^4 + w^3 + w^2 + w + 1)$
   m. $(x+2)(x+2)(x+2)$
   n. $n(n + 1)(n + 2)$
   o. $n(n + 1)(n + 2)(n + 3)$
   p. $n(n + 1)(n + 2)(n + 3)(n + 4)$
   q. $(x + 1)(x^3 - x^2 + x - 1)$
   r. $(x + 1)(x^5 - x^4 + x^3 - x^2 + x - 1)$
   s. $(x + 1)(x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x - 1)$
   t. $(m^3 - 2m + 1)(m^2 - m + 2)$

4. Polynomial expressions can be thought of as a generalization of place value.
   a. Multiply $214 \times 112$ using the standard paper-and-pencil algorithm.
   b. Multiply $(2x^2 + x + 4)(x^2 + x + 2)$ using the tabular method and combine like terms.
   c. Put $x = 10$ into your answer from part (b).
   d. Is the answer to part (c) equal to the answer from part (a)? Compare the digits you computed in the algorithm to the coefficients of the entries you computed in the table. How do the place-value units of the digits compare to the powers of the variables in the entries?

5. Jeremy says $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ must equal $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ because when $x = 10$, multiplying by $x - 9$ is the same as multiplying by $1$.
   a. Multiply $(x - 9)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$.
   b. Put $x = 10$ into your answer.
   c. Is the answer to part (b) the same as the value of $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ when $x = 10$?
   d. Was Jeremy right?

6. In the diagram, the side of the larger square is $x$ units and the side of the smaller square is $y$ units. The area of the shaded region is $(x^2 - y^2)$ square units. Show how the shaded area might be cut and rearranged to illustrate that the area is $(x - y)(x + y)$ square units.