Lesson 4: Comparing Methods—Long Division, Again?

Opening Exercises

1. Use the reverse tabular method to determine the quotient $\frac{2x^3+11x^2+7x+10}{x+5}$.

2. Use your work from Exercise 1 to write the polynomial $2x^3 + 11x^2 + 7x + 10$ in factored form, and then multiply the factors to check your work above.

Example 1

If $x = 10$, then the division $1573 \div 13$ can be represented using polynomial division.

$$x+3\overline{\smash{)}x^3+5x^2+7x+3}$$

Example 2

Use the long division algorithm for polynomials to evaluate

$$\frac{2x^3-4x^2+2}{2x-2}.$$

Exercises 1–8

Use the long division algorithm to determine the quotient. For each problem, check your work by using the reverse tabular method.

1. $\frac{x^2+6x+9}{x+3}$

2. $(7x^3 - 8x^2 - 13x + 2) \div (7x - 1)$

3. $(x^3 - 27\,) \div (x - 3)$

4. $\frac{2x^4+14x^3+x^2-21x-6}{2x^2-3}$

5. $\frac{5x^4-6x^2+1}{x^2-1}$

6. $\frac{x^6+4x^4-4x-1}{x^3-1}$

7. $(2x^7 + x^5 - 4x^3 + 14x^2 - 2x + 7) \div (2x^2 + 1)$

8. $\frac{x^6-64}{x+2}$

Lesson Summary

The long division algorithm to divide polynomials is analogous to the long division algorithm for integers. The long division algorithm to divide polynomials produces the same results as the reverse tabular method.

Problem Set

Use the long division algorithm to determine the quotient.

1. $\frac{2x^3-13x^2-x+3}{2x+1}$

2. $\frac{3x^3+4x^2+7x+22}{x+2}$

3. $\frac{x^4+6x^3-7x^2-24x+12}{x^2-4}$

4. $(12x^4+2x^3+x-3)\div(2x^2+1)$

5. $(2x^3+2x^2+2x)\div(x^2+x+1)$

6. Use long division to find the polynomial, p, that satisfies the equation below.

$$2x^4-3x^2-2=(2x^2+1)(p(x))$$

7. Given $q(x)=3x^3-4x^2+5x+k$.
   a. Determine the value of k so that $3x-7$ is a factor of the polynomial q.
   b. What is the quotient when you divide the polynomial q by $3x-7$?

8. In parts (a)–(b) and (d)–(e), use long division to evaluate each quotient. Then, answer the remaining questions.
   a. $\frac{x^2-9}{x+3}$
   b. $\frac{x^4-81}{x+3}$
   c. Is $x+3$ a factor of x^3-27? Explain your answer using the long division algorithm.
   d. $\frac{x^3+27}{x+3}$
   e. $\frac{x^5+243}{x+3}$
   f. Is $x+3$ a factor of x^2+9? Explain your answer using the long division algorithm.
   g. For which positive integers n is $x+3$ a factor of x^n+3^n? Explain your reasoning.
   h. If n is a positive integer, is $x+3$ a factor of x^n-3^n? Explain your reasoning.