

# Lesson 9: Radicals and Conjugates

## Classwork

## **Opening Exercise**

Which of these statements are true for all a, b > 0? Explain your conjecture.

i. 
$$2(a + b) = 2a + 2b$$
  
ii. 
$$\frac{a+b}{2} = \frac{a}{2} + \frac{b}{2}$$
  
iii. 
$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

## Example 1

Express  $\sqrt{50} - \sqrt{18} + \sqrt{8}$  in simplest radical form and combine like terms.

## Exercises 1–5

1.  $\sqrt{\frac{1}{4}} + \sqrt{\frac{9}{4}} - \sqrt{45}$ 

## 2. $\sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$





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3.  $\sqrt{\frac{3}{8}}$ 

4.  $\sqrt[3]{\frac{5}{32}}$ 

5.  $\sqrt[3]{16x^5}$ 

Example 2

Multiply and combine like terms. Then explain what you notice about the two different results.

 $(\sqrt{3} + \sqrt{2}) (\sqrt{3} + \sqrt{2})$  $(\sqrt{3} + \sqrt{2}) (\sqrt{3} - \sqrt{2})$ 







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## **Exercise 6**

6. Find the product of the conjugate radicals.  $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ 

 $(7+\sqrt{2})(7-\sqrt{2})$ 

 $(\sqrt{5}+2)(\sqrt{5}-2)$ 

Example 3

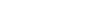
Write  $\frac{\sqrt{3}}{5-2\sqrt{3}}$  in simplest radical form.







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#### **Lesson Summary**

For real numbers  $a \ge 0$  and  $b \ge 0$ , where  $b \ne 0$  when b is a denominator,

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$
 and  $\sqrt{\frac{a}{b}} = rac{\sqrt{a}}{\sqrt{b}}$ 

For real numbers  $a \ge 0$  and  $b \ge 0$ , where  $b \ne 0$  when b is a denominator,

$$\sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$$
 and  $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ .

• Two binomials of the form  $\sqrt{a} + \sqrt{b}$  and  $\sqrt{a} - \sqrt{b}$  are called conjugate radicals:

 $\sqrt{a} + \sqrt{b}$  is the conjugate of  $\sqrt{a} - \sqrt{b}$ , and

 $\sqrt{a} - \sqrt{b}$  is the conjugate of  $\sqrt{a} + \sqrt{b}$ .

For example, the conjugate of  $2 - \sqrt{3}$  is  $2 + \sqrt{3}$ .

• To express a numeric expression with a denominator of the form  $\sqrt{a} + \sqrt{b}$  in simplest radical form, multiply the numerator and denominator by the conjugate  $\sqrt{a} - \sqrt{b}$  and combine like terms.

## **Problem Set**

- 1. Express each of the following as a rational number or in simplest radical form. Assume that the symbols *a*, *b*, and *x* represent positive numbers.
  - a.  $\sqrt{36}$
  - b.  $\sqrt{72}$
  - c.  $\sqrt{18}$
  - d.  $\sqrt{9x^3}$
  - e.  $\sqrt{27x^2}$
  - f.  $\sqrt[3]{16}$
  - g.  $\sqrt[3]{24a}$
  - h.  $\sqrt{9a^2 + 9b^2}$
- 2. Express each of the following in simplest radical form, combining terms where possible.

a. 
$$\sqrt{25} + \sqrt{45} - \sqrt{20}$$
  
b.  $3\sqrt{3} - \sqrt{\frac{3}{4}} + \sqrt{\frac{1}{3}}$   
c.  $\sqrt[3]{54} - \sqrt[3]{8} + 7\sqrt[3]{\frac{1}{4}}$ 

d.  $\sqrt[3]{\frac{5}{8}} + \sqrt[3]{40} - \sqrt[3]{\frac{8}{9}}$ 





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- Evaluate  $\sqrt{x^2 y^2}$  when x = 33 and y = 15. 3.
- Evaluate  $\sqrt{x^2 + y^2}$  when x = 20 and y = 10. 4.
- Express each of the following as a rational expression or in simplest radical form. Assume that the symbols x and y5. represent positive numbers.
  - a.  $\sqrt{3}(\sqrt{7} \sqrt{3})$
  - b.  $(3+\sqrt{2})^2$
  - c.  $(2 + \sqrt{3})(2 \sqrt{3})$
  - d.  $(2+2\sqrt{5})(2-2\sqrt{5})$
  - e.  $(\sqrt{7} 3)(\sqrt{7} + 3)$
  - f.  $(3\sqrt{2} + \sqrt{7})(3\sqrt{2} \sqrt{7})$

g. 
$$(x-\sqrt{3})(x+\sqrt{3})$$

- h.  $(2x\sqrt{2} + y)(2x\sqrt{2} y)$
- Simplify each of the following quotients as far as possible. 6.
  - a.  $(\sqrt{21} \sqrt{3}) \div \sqrt{3}$ b.  $(\sqrt{5}+4) \div (\sqrt{5}+1)$
  - c.  $(3 \sqrt{2}) \div (3\sqrt{2} 5)$
  - d.  $(2\sqrt{5} \sqrt{3}) \div (3\sqrt{5} 4\sqrt{2})$
- 7. If  $x = 2 + \sqrt{3}$ , show that  $x + \frac{1}{x}$  has a rational value.
- Evaluate  $5x^2 10x$  when the value of x is  $\frac{2-\sqrt{5}}{2}$ . 8.
- Write the factors of  $a^4 b^4$ . Use the result to obtain the factored form of  $(\sqrt{3} + \sqrt{2})^4 (\sqrt{3} \sqrt{2})^4$ . 9.
- 10. The converse of the Pythagorean Theorem is also a theorem: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Use the converse of the Pythagorean Theorem to show that for A, B, C > 0, if A + B = C, then  $\sqrt{A} + \sqrt{B} > \sqrt{C}$ , so that  $\sqrt{A} + \sqrt{B} > \sqrt{A + B}$ .





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