Lesson 15: Structure in Graphs of Polynomial Functions

Classwork

Opening Exercise

Sketch the graph of $f(x) = x^2$. What will the graph of $g(x) = x^4$ look like? Sketch it on the same coordinate plane. What will the graph of $h(x) = x^6$ look like?

Example 1

Sketch the graph of $f(x) = x^3$. What will the graph of $g(x) = x^5$ look like? Sketch this on the same coordinate plane. What will the graph of $h(x) = x^7$ look like? Sketch this on the same coordinate plane.

Exercise 1

a. Consider the following function, $f(x) = 2x^4 + x^3 - x^2 + 5x + 3$, with a mixture of odd and even degree terms. Predict whether its end behavior will be like the functions in the Opening Exercise or more like the functions from Example 1. Graph the function f using a graphing utility to check your prediction.

b. Consider the following function, $f(x) = 2x^5 - x^4 - 2x^3 + 4x^2 + x + 3$, with a mixture of odd and even degree terms. Predict whether its end behavior will be like the functions in the Epening Exercise or more like the functions from Example 1. Graph the function f using a graphing utility to check your prediction.

c. Thinking back to our discussion of x-intercepts of graphs of polynomial functions from the previous lesson, sketch a graph of an even degree polynomial function that has no x-intercepts.

d. Similarly, can you sketch a graph of an odd degree polynomial function with no x-intercepts?

Exercise 2

The Center for Transportation Analysis (CTA) studies all aspects of transportation in the United States, from energy and environmental concerns to safety and security challenges. A 1997 study compiled the following data of the fuel economy in miles per gallon (mpg) of a car or light truck at various speeds measured in miles per hour (mph). The data is compiled in the table below.

Fuel Economy by Speed

Speed (mph)	Fuel Economy (mpg)
15	24.4
20	27.9
25	30.5
30	31.7
35	31.2
40	31.0
45	31.6
50	32.4
55	32.4
60	31.4
65	29.2
70	26.8
75	24.8

Source: Transportation Energy Data Book, Table 4.28. http://cta.ornl.gov/data/chapter4.shtml

a. Plot the data using a graphing utility. Which variable is the independent variable?

b. This data can be modeled by a polynomial function. Determine if the function that models the data would have an even or odd degree.

c. Is the leading coefficient of the polynomial that can be used to model this data positive or negative?.

d. List two possible reasons the data might have the shape that it does.

Relevant Vocabulary

Even Function: Let f be a function whose domain and range is a subset of the real numbers. The function f is called *even* if the equation $f(x) = f(-x)$ is true for every number x in the domain.

Even-degree polynomial functions are sometimes even functions, like $f(x) = x^{10}$, and sometimes not, like $g(x) = x^2 - x$.

Odd Function: Let f be a function whose domain and range is a subset of the real numbers. The function f is called *odd* if the equation $f(-x) = -f(x)$ is true for every number x in the domain.

Odd-degree polynomial functions are sometimes odd functions, like $f(x) = x^{11}$, and sometimes not, like $h(x) = x^3 - x^2$.

Problem Set

1. Graph the functions from the Opening Exercise simultaneously using a graphing utility and zoom in at the origin.

   a. At $x = 0.5$, order the values of the functions from least to greatest.

   b. At $x = 2.5$, order the values of the functions from least to greatest.

   c. Identify the x-value(s) where the order reverses. Write a brief sentence on why you think this switch occurs.

2. The National Agricultural Statistics Service (NASS) is an agency within the USDA that collects and analyzes data covering virtually every aspect of agriculture in the United States. The following table contains information on the amount (in tons) of the following vegetables produced in the U.S. from 1988–1994 for processing into canned, frozen, and packaged foods: lima beans, snap beans, beets, cabbage, sweet corn, cucumbers, green peas, spinach, and tomatoes.

   Vegetable Production by Year

Year	Vegetable Production (tons)
1988	11,393,320
1989	14,450,860
1990	15,444,970
1991	16,151,030
1992	14,236,320
1993	14,904,750
1994	18,313,150

   Source: NASS Statistics of Vegetables and Melons, 1995, Table 191.

   http://www.nass.usda.gov/Publications/Ag_Statistics/1995-1996/agr95_4.pdf

   a. Plot the data using a graphing utility.

   b. Determine if the data displays the characteristics of an odd- or even-degree polynomial.

   c. List two possible reasons the data might have the shape that it does.

3. The U.S. Energy Information Administration (EIA) is responsible for collecting and analyzing information about energy production and use in the United States and for informing policy makers and the public about issues of energy, the economy, and the environment. The following table contains data from the EIA about natural gas consumption from 1950–2010, measured in millions of cubic feet.
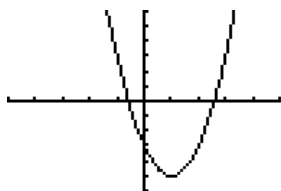
U.S. Natural Gas Consumption by Year

Year	U.S. natural gas total consumption (millions of cubic feet)
1950	5.77
1955	8.69
1960	11.97
1965	15.28
1970	21.14
1975	19.54
1980	19.88
1985	17.28
1990	19.17
1995	22.21
2000	23.33
2005	22.01
2010	24.09

Source: U.S. Energy Information Administration. http://www.eia.gov/dnav/ng/hist/n9140us2a.htm
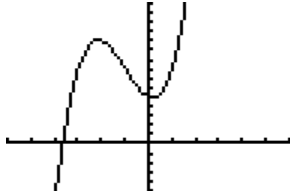
  a. Plot the data using a graphing utility.

  b. Determine if the data displays the characteristics of an odd- or even-degree polynomial function.

  c. List two possible reasons the data might have the shape that it does.

4. We use the term *even function* when a function f satisfies the equation $f(-x) = f(x)$ for every number x in its domain. Consider the function $f(x) = -3x^2 + 7$. Note that the degree of the function is even, and each term is of an even degree (the constant term is degree 0.

  a. Graph the function using a graphing utility.

  b. Does this graph display any symmetry?

  c. Evaluate $f(-x)$.

  d. Is f an even function? Explain how you know.

5. We use the term *odd function* when a function f satisfies the equation $f(-x) = -f(x)$ for every number x in its domain. Consider the function $f(x) = 3x^3 - 4x$. The degree of this function is odd, and each term is of an odd degree.

  a. Graph the function using a graphing utility.

  b. Does this graph display any symmetry?

  c. Evaluate $f(-x)$.

  d. Is f an odd function? Explain how you know.

6. We have talked about x-intercepts of the graph of a function in both this lesson and the previous one. The x-intercepts correspond to the zeros of the function. Consider the following examples of polynomial functions and their graphs to determine an easy way to find the y-intercept of the graph of a polynomial function

$f(x) = 2x^2 - 4x - 3$

$f(x) = x^3 + 3x^2 - x + 5$

$f(x) = x^4 - 2x^3 - x^2 + 3x - 6$