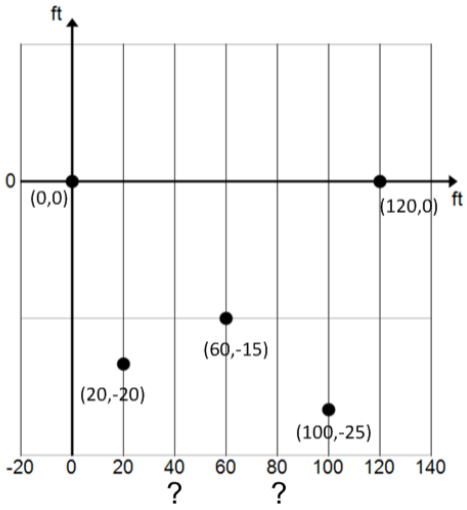
Lesson 20: Modeling Riverbeds with Polynomials

Classwork

Discussion

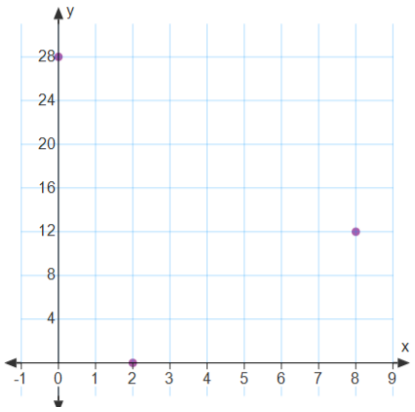
The Environmental Protection Agency (EPA) is studying the flow of a river in order to establish flood zones. The EPA hired a surveying company to determine the flow rate of the river, measured as volume of water per minute. The firm set up a coordinate system and found the depths of the river at five locations as shown on the graph below. After studying the data, the firm decided to model the riverbed with a polynomial function and divide the area into six regions that are either trapezoidal or triangular so that the overall area can be easily estimated. The firm will need to approximate the depth of the river at two more data points in order to do this.



Draw the four trapezoids and two triangles that will be used to estimate the area.

Example 1

Find a polynomial P such that $P(0) = 28$, $P(2) = 0$, and $P(8) = 12$.



Example 2

Find a degree 3 polynomial P such that $P(-1) = -3$, $P(0) = -2$, $P(1) = -1$, and $P(2) = 6$.

Value	Substitute point	Remainder Theorem for $a(x)$, $b(x)$, and $c(x)$	Substitute into $P(x)$
$P(-1) = -3$			
$P(0) = -2$			
$P(1) = -1$			
$P(2) = 6$			

Lesson Summary

A linear polynomial is determined by 2 points on its graph.

A degree 2 polynomial is determined by 3 points on its graph.

A degree 3 polynomial is determined by 4 points on its graph.

A degree 4 polynomial is determined by 5 points on its graph.

The Remainder Theorem can be used to find a polynomial P whose graph will pass through a given set of points.

Problem Set

1. Suppose a polynomial P is such that $P(2) = 5$ and $P(3) = 12$.
   a. What is the largest degree polynomial that can be uniquely determined given the information?
   b. Is this the only polynomial that satisfies $P(2) = 5$ and $P(3) = 12$?
   c. Use the Remainder Theorem to find the polynomial P of least degree that satisfies the two points given.
   d. Verify that your equation is correct by demonstrating that it satisfies the given points.

2. Write a quadratic equation P such that $P(0) = -10$, $P(5) = 0$, and $P(7) = 18$ using the specified method.
   a. Setting up a system of equations
   b. Using the Remainder Theorem

3. Find a degree three polynomial function P such that $P(-1) = 0$, $P(0) = 2$, $P(2) = 12$, and $P(3) = 32$. Use the table below to organize your work. Write your answer in standard form, and verify by showing that each point satisfies the equation.

Value	Substitute point	Remainder Theorem for $a(x)$, $b(x)$, and $c(x)$	Substitute into $P(x)$
$P(-1) = 0$			
$P(0) = 2$			
$P(2) = 12$			
$P(3) = 32$			

4. The method used in Problem 3 is based on the Lagrange Interpolation method. Research Joseph-Louis Lagrange and write a paragraph about his mathematical work.