Lesson 24: Multiplying and Dividing Rational Expressions

Classwork

If a, b, c, and d are rational expressions with $b \neq 0$, $d \neq 0$, then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

Example 1

Make a conjecture about the product $\frac{x^3}{4y} \cdot \frac{y^2}{x}$. What will it be? Explain your conjecture and give evidence that it is correct.

Example 2

Find the following product:

$$\left(\frac{3x-6}{2x+6}\right) \cdot \left(\frac{5x+15}{4x+8}\right).$$

Exercises 1–3

1. Summarize what you have learned so far with your neighbor.

2. Find the following product and reduce to lowest terms: $\left(\frac{2x+6}{x^2+x-6}\right) \cdot \left(\frac{x^2-4}{2x}\right)$.

3. Find the following product and reduce to lowest terms: $\left(\frac{4n-12}{3m+6}\right)^{-2} \cdot \left(\frac{n^2-2n-3}{m^2+4m+4}\right)$.

If a, b, c, and d are rational expressions with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}.$$

Example 3

Find the quotient and reduce to lowest terms: $\frac{x^2-4}{3x} \div \frac{x-2}{2x}$.

Exercises 4–5

4. Find the quotient and reduce to lowest terms: $\frac{x^2-5x+6}{x+4} \div \frac{x^2-9}{x^2+5x+4}$.

5. Simplify the rational expression.

$$\frac{\left(\frac{x+2}{x^2-2x-3}\right)}{\left(\frac{x^2-x-6}{x^2+6x+5}\right)}$$

Lesson Summary

In this lesson we extended multiplication and division of rational numbers to multiplication and division of rational expressions.

- To multiply two rational expressions, multiply the numerators together and multiply the denominators together, and then reduce to lowest terms.
- To divide one rational expression by another, multiply the first by the multiplicative inverse of the second, and reduce to lowest terms.
- To simplify a complex fraction, apply the process for dividing one rational expression by another.

Problem Set

1. Complete the following operations:

   a. Multiply $\frac{1}{3}(x-2)$ by 9.

   b. Divide $\frac{1}{4}(x-8)$ by $\frac{1}{12}$.

   c. Multiply $\frac{1}{4}\left(\frac{1}{3}x+2\right)$ by 12.

   d. Divide $\frac{1}{3}\left(\frac{2}{5}x-\frac{1}{5}\right)$ by $\frac{1}{15}$.

   e. Multiply $\frac{2}{3}\left(2x+\frac{2}{3}\right)$ by $\frac{9}{4}$.

   f. Multiply $0.03(4-x)$ by 100.

2. Simplify each of the following expressions.

   a. $\left(\frac{a^3b^2}{c^2d^2}\cdot\frac{c}{ab}\right)\div\frac{a}{c^2d^3}$

   b. $\frac{a^2+6a+9}{a^2-9}\cdot\frac{3a-9}{a+3}$

   c. $\frac{6x}{4x-16}\div\frac{4x}{x^2-16}$

   d. $\frac{3x^2-6x}{3x+1}\cdot\frac{x+3x^2}{x^2-4x+4}$

   e. $\frac{2x^2-10x+12}{x^2-4}\cdot\frac{2+x}{3-x}$

   f. $\frac{a-2b}{a+2b}\div\left(4b^2-a^2\right)$

   g. $\frac{d+c}{c^2+d^2}\div\frac{c^2-d^2}{d^2-dc}$

   h. $\frac{12a^2-7ab+b^2}{9a^2-b^2}\div\frac{16a^2-b^2}{3ab+b^2}$

   i. $\left(\frac{x-3}{x^2-4}\right)^{-1}\cdot\left(\frac{x^2-x-6}{x-2}\right)$

   j. $\left(\frac{x-2}{x^2+1}\right)^{-3}\div\left(\frac{x^2-4x+4}{x^2-2x-3}\right)$

   k. $\frac{6x^2-11x-10}{6x^2-5x-6}\cdot\frac{6-4x}{25-20x+4x^2}$

   l. $\frac{3x^3-3a^2x}{x^2-2ax+a^2}\cdot\frac{a-x}{a^3x+a^2x^2}$

3. Simplify the following complex rational expressions.

   a. $\dfrac{\left(\frac{4a}{6b^2}\right)}{\left(\frac{20a^3}{12b}\right)}$

   b. $\dfrac{\left(\frac{x-2}{x^2-1}\right)}{\left(\frac{x^2-4}{x-6}\right)}$

   c. $\dfrac{\left(\frac{x^2+2x-3}{x^2+3x-4}\right)}{\left(\frac{x^2+x-6}{x+4}\right)}$

4. Suppose that $x = \frac{t^2+3t-4}{3t^2-3}$ and $y = \frac{t^2+2t-8}{2t^2-2t-4}$, for $t \neq 1$, $t \neq -1$, $t \neq 2$, and $t \neq -4$. Show that the value of x^2y^{-2} does not depend on the value of t.

5. Determine which of the following numbers is larger without using a calculator, $\frac{15^{16}}{16^{15}}$ or $\frac{20^{24}}{24^{20}}$.

   (Hint: We can compare two positive quantities a and b by computing the quotient $\frac{a}{b}$. If $\frac{a}{b} > 1$, then $a > b$. Likewise, if $0 < \frac{a}{b} < 1$, then $a < b$.)

6. One of two numbers can be represented by the rational expression $\frac{x-2}{x}$, where $x \neq 0$ and $x \neq 2$.

   a. Find a representation of the second number if the product of the two numbers is 1.

   b. Find a representation of the second number if the product of the two numbers is 0.