

## Lesson 30: Linear Systems in Three Variables

### Classwork

#### Exercises 1–3

Determine the value of  $x$  and  $y$  in the following systems of equations.

1. 
$$\begin{aligned} 2x + 3y &= 7 \\ 2x + y &= 3 \end{aligned}$$

2. 
$$\begin{aligned} 5x - 2y &= 4 \\ -2x + y &= 2 \end{aligned}$$

3. A scientist wants to create 120 ml of a solution that is 30% acidic. To create this solution, she has access to a 20% solution and a 45% solution. How many milliliters of each solution should she combine to create the 30% solution?

**Example 1**

Determine the values for  $x$ ,  $y$ , and  $z$  in the following system.

$$2x + 3y - z = 5$$

$$4x - y - z = -1$$

$$x + 4y + z = 12$$

**Exercises 4–5**

4. Given the system below, determine the values of  $r$ ,  $s$ , and  $u$  that satisfy all three equations.

$$r + 2s - u = 8$$

$$s + u = 4$$

$$r - s - u = 2$$

5. Find the equation of the form  $y = ax^2 + bx + c$  that satisfies the points  $(1, 6)$ ,  $(3, 20)$ , and  $(-2, 15)$ .

## Problem Set

Solve the following systems.

1.  $x + y = 3$   
 $y + z = 6$   
 $x + z = 5$

2.  $r = 2(s - t)$   
 $2t = 3(s - r)$   
 $r + t = 2s - 3$

3.  $2a + 4b + c = 5$   
 $a - 4b = -6$   
 $2b + c = 7$

4.  $2x + y - z = -5$   
 $4x - 2y + z = 10$   
 $2x + 3y + 2z = 3$

5.  $r + 3s + t = 3$   
 $2r - 3s + 2t = 3$   
 $-r + 3s - 3t = 1$

6.  $x - y = 1$   
 $2y + z = -4$   
 $x - 2z = -6$

7.  $x = 3(y - z)$   
 $y = 5(z - x)$   
 $x + y = z + 4$

8.  $p + q + 3r = 4$   
 $2q + 3r = 7$   
 $p - q - r = -2$

9.  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5$   
 $\frac{1}{x} + \frac{1}{y} = 2$   
 $\frac{1}{x} - \frac{1}{z} = -2$

10.  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 6$   
 $\frac{1}{b} + \frac{1}{c} = 5$   
 $\frac{1}{a} - \frac{1}{b} = -1$

11. Find the equation of the form  $y = ax^2 + bx + c$  whose graph passes through the points  $(1, -1)$ ,  $(3, 23)$ , and  $(-1, 7)$ .

12. Show that for any number  $t$ , the values  $x = t + 2$ ,  $y = 1 - t$ , and  $z = t + 1$  are solutions to the system of equations below.

$$\begin{aligned}x + y &= 3 \\ y + z &= 2\end{aligned}$$

(In this situation, we say that  $t$  parameterizes the solution set of the system.)

13. Some rational expressions can be written as the sum of two or more rational expressions whose denominators are the factors of its denominator (called a *partial fraction decomposition*). Find the partial fraction decomposition for the following example by filling in the blank to make the equation true for all  $n$  except 0 and  $-1$ .

$$\frac{1}{n(n+1)} = \frac{\quad}{n} - \frac{1}{n+1}$$

14. A chemist needs to make 30 ml of a 15% acid solution. He has a 5% acid solution and a 30% acid solution on hand. If he uses the 5% and 30% solutions to create the 15% solution, how many ml of each will he need?
15. An airplane makes a 400 mile trip against a head wind in 4 hours. The return trip takes 2.5 hours, the wind now being a tail wind. If the plane maintains a constant speed with respect to still air, and the speed of the wind is also constant and does not vary, find the still-air speed of the plane and the speed of the wind.
16. A restaurant owner estimates that she needs in small change the same number of dimes as pennies and nickels together and the same number of pennies as nickels. If she gets \$26 worth of pennies, nickels, and dimes, how should they be distributed?