

- c. Suppose the second line is replaced by the line with equation $2x = 12 - 2y$. Plot the lines on the same set of axes, and solve the pair of equations algebraically to verify your graphical solution.

- d. We have seen that a pair of lines can intersect in 1, 0, or an infinite number of points. Are there any other possibilities?

Exploratory Challenge 2

- a. Suppose that instead of equations for a pair of lines, you were given an equation for a circle and an equation for a line. What possibilities are there for the two figures to intersect? Sketch a graph for each possibility.

- b. Graph the parabola with equation $y = x^2$. What possibilities are there for a line to intersect the parabola? Sketch each possibility.
- c. Sketch the circle given by $x^2 + y^2 = 1$ and the line given by $y = 2x + 2$ on the same set of axes. One solution to the pair of equations has a value of y that is easily identifiable from the sketch. What is it?
- d. Solve $x^2 + (2x + 2)^2 = 1$.

Exercises 1–6

1. Draw a graph of the circle with equation $x^2 + y^2 = 9$.
 - a. What are the solutions to the system of circle and line when the circle is given by $x^2 + y^2 = 9$ and the line is given by $y = 2$?
 - b. What happens when the line is given by $y = 3$?
 - c. What happens when the line is given by $y = 4$?

2. By solving the equations as a system, find the points common to the line with equation $x - y = 6$ and the circle with equation $x^2 + y^2 = 26$. Graph the line and the circle to show those points.
3. Graph the line given by $5x + 6y = 12$ and the circle given by $x^2 + y^2 = 1$. Find all solutions to the system of equations.
4. Graph the line given by $3x + 4y = 25$ and the circle given by $x^2 + y^2 = 25$. Find all solutions to the system of equations. Verify your result both algebraically and graphically.

5. Graph the line given by $2x + y = 1$ and the circle given by $x^2 + y^2 = 10$. Find all solutions to the system of equations. Verify your result both algebraically and graphically.
6. Graph the line given by $x + y = -2$ and the quadratic curve given by $y = x^2 - 4$. Find all solutions to the system of equations. Verify your result both algebraically and graphically.

Lesson Summary

Here are some steps to consider when solving systems of equations that represent a line and a quadratic curve.

1. Solve the linear equation for y in terms of x . This is equivalent to rewriting the equation in slope-intercept form. Note that working with the quadratic equation first would likely be more difficult and might cause the loss of a solution.
2. Replace y in the quadratic equation with the expression involving x from the slope-intercept form of the linear equation. That will yield an equation in one variable.
3. Solve the quadratic equation for x .
4. Substitute x into the linear equation to find the corresponding value of y .
5. Sketch a graph of the system to check your solution.

Problem Set

1. Where do the lines given by $y = x + b$ and $y = 2x + 1$ intersect?

2. Find all solutions to the following system of equations.

$$\begin{aligned}(x - 2)^2 + (y + 3)^2 &= 4 \\ x - y &= 3\end{aligned}$$

Illustrate with a graph.

3. Find all solutions to the following system of equations.

$$\begin{aligned}x + 2y &= 0 \\ x^2 - 2x + y^2 - 2y - 3 &= 0\end{aligned}$$

Illustrate with a graph.

4. Find all solutions to the following system of equations.

$$\begin{aligned}x + y &= 4 \\ (x + 3)^2 + (y - 2)^2 &= 10\end{aligned}$$

Illustrate with a graph.

5. Find all solutions to the following system of equations.

$$\begin{aligned}y &= -2x + 3 \\ y &= x^2 - 6x + 3\end{aligned}$$

Illustrate with a graph.

6. Find all solutions to the following system of equations.

$$\begin{aligned}-y^2 + 6y + x - 9 &= 0 \\ 6y &= x + 27\end{aligned}$$

Illustrate with a graph.

7. If the following system of equations has two solutions, what is the value of k ?

$$\begin{aligned}x^2 + y^2 &= 25 \\ y &= k\end{aligned}$$

Illustrate with a graph.

8. If the following system of equations has exactly one solution, what is the value of k ?

$$\begin{aligned}y &= 5 - (x - 3)^2 \\ y &= k\end{aligned}$$

Illustrate with a graph.

9. If the following system of equations no solutions, what is the value of k ?

$$\begin{aligned}x^2 + (y - k)^2 &= 36 \\ y &= 5x + k\end{aligned}$$

Illustrate with a graph.