

## Lesson 38: Complex Numbers as Solutions to Equations

### Classwork

#### Opening Exercises

1. Use the quadratic formula to solve the following quadratic equations. Calculate the discriminant for each equation.

a.  $x^2 - 9 = 0$

b.  $x^2 - 6x + 9 = 0$

c.  $x^2 + 9 = 0$

2. How does the value of the discriminant for each equation relate the number of solutions you found?

**Example 1**

Consider the equation  $3x + x^2 = -7$ .

What does the value of the discriminant tell us about number of solutions to this equation?

Solve the equation. Does the number of solutions match the information provided by the discriminant? Explain.

**Exercise**

Compute the value of the discriminant of the quadratic equation in each part. Use the value of the discriminant to predict the number and type of solutions. Find all real and complex solutions.

a.  $x^2 + 2x + 1 = 0$

b.  $x^2 + 4 = 0$

c.  $9x^2 - 4x - 14 = 0$

d.  $3x^2 + 4x + 2 = 0$

e.  $x = 2x^2 + 5$

f.  $8x^2 + 4x + 32 = 0$

**Lesson Summary**

- A quadratic equation with real coefficients and a real constant may have real or complex solutions.
- Given a quadratic equation  $ax^2 + bx + c = 0$ , the discriminant  $b^2 - 4ac$  indicates whether the equation has two distinct real solutions, one real solution, or two complex solutions.
  - If  $b^2 - 4ac > 0$ , there are two real solutions to  $ax^2 + bx + c = 0$ .
  - If  $b^2 - 4ac = 0$ , there is one real solution to  $ax^2 + bx + c = 0$ .
  - If  $b^2 - 4ac < 0$ , there are two complex solutions to  $ax^2 + bx + c = 0$ .

**Problem Set**

1. Give an example of a quadratic equation in standard form that has...
  - a. Exactly two distinct real solutions.
  - b. Exactly one distinct real solution.
  - c. Exactly two complex (non-real) solutions.
2. Suppose we have a quadratic equation  $ax^2 + bx + c = 0$  so that  $a + c = 0$ . Does the quadratic equation have one solution or two distinct solutions? Are they real or complex? Explain how you know.
3. Solve the equation  $5x^2 - 4x + 3 = 0$ .
4. Solve the equation  $2x^2 + 8x = -9$ .
5. Solve the equation  $9x - 9x^2 = 3 + x + x^2$ .
6. Solve the equation  $3x^2 - x + 1 = 0$ .
7. Solve the equation  $6x^4 + 4x^2 - 3x + 2 = 2x^2(3x^2 - 1)$ .
8. Solve the equation  $25x^2 + 100x + 200 = 0$ .
9. Write a quadratic equation in standard form such that  $-5$  is its only solution.
10. Is it possible that the quadratic equation  $ax^2 + bx + c = 0$  has a positive real solution if  $a$ ,  $b$ , and  $c$  are all positive real numbers?
11. Is it possible that the quadratic equation  $ax^2 + bx + c = 0$  has a positive real solution if  $a$ ,  $b$ , and  $c$  are all negative real numbers?

Extension:

12. Show that if  $k > 3.2$ , the solutions of  $5x^2 - 8x + k = 0$  are not real numbers.
13. Let  $k$  be a real number, and consider the quadratic equation  $(k + 1)x^2 + 4kx + 2 = 0$ .
- Show that the discriminant of  $(k + 1)x^2 + 4kx + 2 = 0$  defines a quadratic function of  $k$ .
  - Find the zeros of the function in part (a) and make a sketch of its graph.
  - For what value of  $k$  are there two distinct real solutions to the given quadratic equation?
  - For what value of  $k$  are there two complex solutions to the given quadratic equation?
  - For what value of  $k$  is there one solution to the given quadratic equation?
14. We can develop two formulas that can help us find errors in calculated solutions of quadratic equations.
- Find a formula for the sum  $S$  of the solutions of the quadratic equation  $ax^2 + bx + c = 0$ .
  - Find a formula for the product  $R$  of the solutions of the quadratic equation  $ax^2 + bx + c = 0$ .
  - June calculated the solutions 7 and  $-1$  to the quadratic equation  $x^2 - 6x + 7 = 0$ . Do the formulas from parts (a) and (b) detect an error in her solutions? If not, determine if her solution is correct.
  - Paul calculated the solutions  $3 - i\sqrt{2}$  and  $3 + i\sqrt{2}$  to the quadratic equation  $x^2 - 6x + 7 = 0$ . Do the formulas from parts (a) and (b) detect an error in his solutions? If not, determine if his solutions are correct.
  - Joy calculated the solutions  $3 - \sqrt{2}$  and  $3 + \sqrt{2}$  to the quadratic equation  $x^2 - 6x + 7 = 0$ . Do the formulas from parts (a) and (b) detect an error in her solutions? If not, determine if her solutions are correct.
  - If you find solutions to a quadratic equations that match the results from parts (a) and (b), does that mean your solutions are correct?
  - Summarize the results of this exercise.