Lesson 39: Factoring Extended to the Complex Realm

Classwork

Opening Exercise

Rewrite each expression as a polynomial in standard form.

a. $(x+i)(x-i)$

b. $(x+5i)(x-5i)$

c. $\big(x-(2+i)\big)\big(x-(2-i)\big)$

Exercises 1–4

Completely factor the following polynomial expressions.

1. x^2+9

2. x^2+5

3. Consider the polynomial $P(x) = x^4 - 3x^2 - 4$.
   a. What are the solutions to $x^4 - 3x^2 - 4 = 0$?

   b. How many x-intercepts does the graph of the equation $y = x^4 - 3x^2 - 4$ have? What are the coordinates of the x-intercepts?

   c. Are solutions to the polynomial equation $P(x) = 0$ the same as the x-intercepts of the graph of $y = P(x)$? Justify your reasoning.

4. Write a polynomial P with the lowest possible degree that has the given solutions. Explain how you generated each answer.

   a. $-2, 3, -4i, 4i$

   b. $\mathbf{-1, 3i}$

   c. $\mathbf{0, 2, 1+i, 1-i}$

   d. $\mathbf{\sqrt{2}, -\sqrt{2}, 3, 1+2i}$

   e. $\mathbf{\sqrt{2}, -\sqrt{2}, 3, 1+2i}$

Lesson Summary

- Polynomial equations with real coefficients can have real or complex solutions or they can have both.
- Complex solutions to polynomial equations with real coefficients are always members of a conjugate pair.
- Real solutions to polynomial equations are the same as $\boldsymbol{x}$-intercepts of the associated graph, but complex solutions are not.

Problem Set

1. Rewrite each expression in standard form.
   a. $(x + 3i)(x - 3i)$
   b. $(\boldsymbol{x} - \boldsymbol{a} + \boldsymbol{bi})(\boldsymbol{x} - (\boldsymbol{a} + \boldsymbol{bi}))$
   c. $(\boldsymbol{x} + \boldsymbol{2i})(\boldsymbol{x} - \boldsymbol{i})(\boldsymbol{x} + \boldsymbol{i})(\boldsymbol{x} - \boldsymbol{2i})$
   d. $(\boldsymbol{x} + \boldsymbol{i})^{\boldsymbol{2}} \cdot (\boldsymbol{x} - \boldsymbol{i})^{\boldsymbol{2}}$

2. Suppose in Problem 1 that you had no access to paper, writing utensils, or technology. How do you know that the expressions in parts (a)–(d) are polynomials with real coefficients?

3. Write a polynomial equation of degree 4 in standard form that has the solutions $i, -i, 1, -1$.

4. Explain the difference between x-intercepts and solutions to an equation. Give an example of a polynomial with real coefficients that has twice as many solutions as x-intercepts. Write it in standard form.

5. Find the solutions to $x^4 - 5x^2 - 36 = 0$ and the x-intercepts of the graph of $y = x^4 - 5x^2 - 36$.

6. Find the solutions to $2x^4 - 24x^2 + 40 = 0$ and the x-intercepts of the graph of $y = 2x^4 - 24x^2 + 40$.

7. Find the solutions to $x^4 - 64 = 0$ and the x-intercepts of the graph of $y = x^4 - 64$.

8. Use the fact that $x^4 + 64 = (x^2 - 4x + 8)(x^2 + 4x + 8)$ to explain how you know that the graph of $y = x^4 + 64$ has no x-intercepts. You need not find the solutions.