Lesson 10: Basic Trigonometric Identities from Graphs

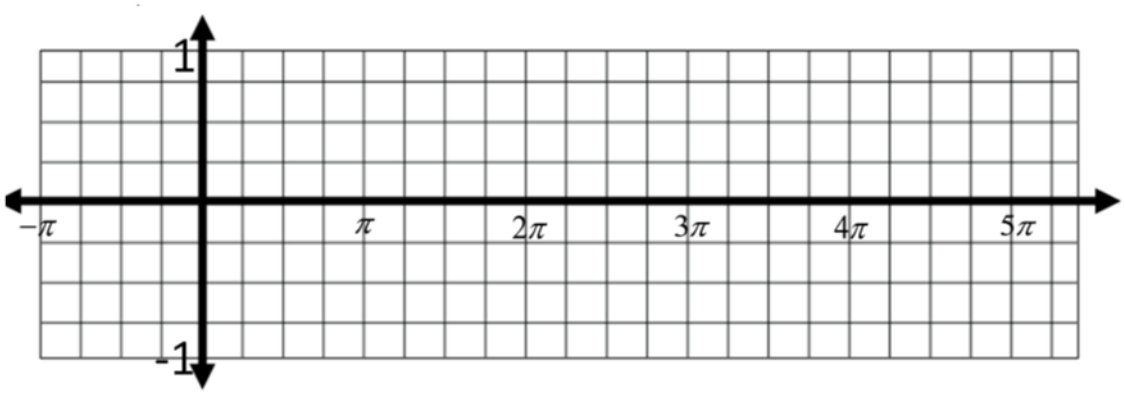
Classwork

Exploratory Challenge 1

Consider the function $f(x) = \sin(x)$ where x is measured in radians.

Graph $f(x) = \sin(x)$ on the interval $[-\pi, 5\pi]$ by constructing a table of values. Include all intercepts, relative maximum points, and relative minimum points of the graph. Then, use the graph to answer the questions that follow.

x	
$f(x)$	



a. Using one of your colored pencils, mark the point on the graph at each of the following pairs of x-values.

$x = -\frac{\pi}{2}$ and $x = -\frac{\pi}{2} + 2\pi$

$x = \pi$ and $x = \pi + 2\pi$

$x = \frac{7\pi}{4}$ and $x = \frac{7\pi}{4} + 2\pi$

b. What can be said about the y-values for each pair of x-values marked on the graph?

c. Will this relationship hold for any two x-values that differ by 2π? Explain how you know.

d. Based on these results, make a conjecture by filling in the blank below.

For any real number x, $\sin(x + 2\pi) =$ ____________________.

e. Test your conjecture by selecting another x-value from the graph and demonstrating that the equation from part (d) holds for that value of x.

f. How does the conjecture in part (d) support the claim that the sine function is a periodic function?

g. Use this identity to evaluate $\sin\left(\frac{13\pi}{6}\right)$.

h. Using one of your colored pencils, mark the point on the graph at each of the following pairs of x-values.

$x = -\frac{\pi}{4}$ and $x = -\frac{\pi}{4} + \pi$

$x = 2\pi$ and $x = 2\pi + \pi$

$x = \frac{5\pi}{2}$ and $x = \frac{5\pi}{2} + \pi$

i. What can be said about the y-values for each pair of x-values marked on the graph?

j. Will this relationship hold for any two x-values that differ by π? Explain how you know.

k. Based on these results, make a conjecture by filling in the blank below.

For any real number x, $\sin(x + \pi) =$ __________________.

l. Test your conjecture by selecting another x-value from the graph and demonstrating that the equation from part (k) holds for that value of x.

m. Is the following statement true or false? Use the conjecture from (k) to explain your answer.

$$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$

n. Using one of your colored pencils, mark the point on the graph at each of the following pairs of x-values.

$x = -\frac{3\pi}{4}$ and $x = \frac{3\pi}{4}$

$x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

o. What can be said about the y-values for each pair of x-values marked on the graph?

p. Will this relationship hold for any two x-values with the same magnitude but opposite sign? Explain how you know.

q. Based on these results, make a conjecture by filling in the blank below.

For any real number x, $\sin(-x) =$ ________________.

r. Test your conjecture by selecting another x-value from the graph and demonstrating that the equation from part (q) holds for that value of x.

s. Is the sine function an odd function, even function, or neither? Use the identity from part (q) to explain.

t. Describe the x-intercepts of the graph of the sine function.

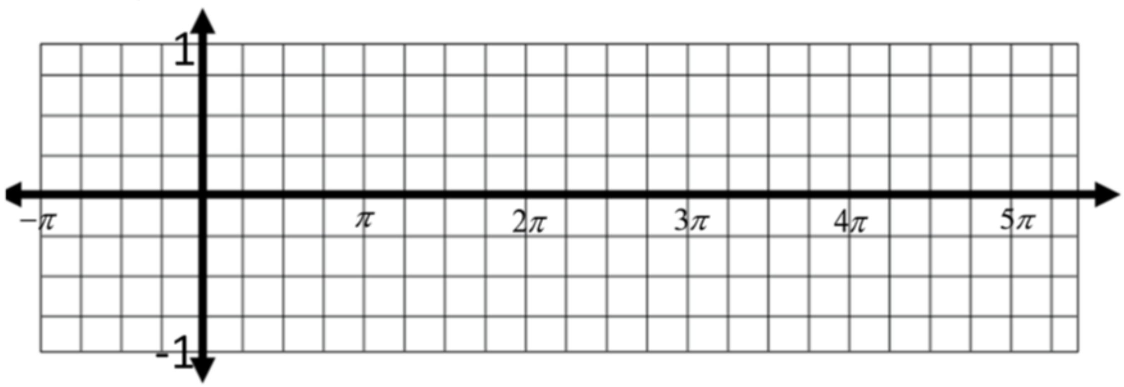
u. Describe the end behavior of the sine function.

Exploratory Challenge 2

Consider the function $g(x) = \cos(x)$ where x is measured in radians.

Graph $g(x) = \cos(x)$ on the interval $[-\pi, 5\pi]$ by constructing a table of values. Include all intercepts, relative maximum points, and relative minimum points. Then, use the graph to answer the questions that follow.

x	
$g(x)$	



a. Using one of your colored pencils, mark the point on the graph at each of the following pairs of $\boldsymbol{x}$-values.

$x = -\frac{\pi}{2}$ and $x = -\frac{\pi}{2} + 2\pi$

$x = \pi$ and $x = \pi + 2\pi$

$x = \frac{7\pi}{4}$ and $x = \frac{7\pi}{4} + 2\pi$

b. What can be said about the y-values for each pair of x-values marked on the graph?

c. Will this relationship hold for any two x-values that differ by 2π? Explain how you know.

d. Based on these results, make a conjecture by filling in the blank below.

For any real number x, $\cos(x + 2\pi) =$ ___________.

e. Test your conjecture by selecting another x-value from the graph and demonstrating that the equation from part (d) holds for that value of x.

f. How does the conjecture from part (d) support the claim that the cosine function is a periodic function?

g. Use this identity to evaluate $\cos\left(\frac{9\pi}{4}\right)$.

h. Using one of your colored pencils, mark the point on the graph at each of the following pairs of x-values.

$$x = -\frac{\pi}{4} \text{ and } x = -\frac{\pi}{4} + \pi$$

$$x = 2\pi \text{ and } x = 2\pi + \pi$$

$$x = \frac{5\pi}{2} \text{ and } x = \frac{5\pi}{2} + \pi$$

i. What can be said about the y-values for each pair of x-values marked on the graph?

j. Will this relationship hold for any two x-values that differ by π? Explain how you know.

k. Based on these results, make a conjecture by filling in the blank below.

For any real number x, $\cos(x + \pi) =$ ___________.

l. Test your conjecture by selecting another x-value from the graph and demonstrating that the equation from part (k) holds for that value of x.

m. Is the following statement true or false? Use the identity from part (k) to explain your answer.

$$\cos\left(\frac{5\pi}{3}\right) = -\cos\left(\frac{2\pi}{3}\right)$$

n. Using one of your colored pencils, mark the point on the graph at each of the following pairs of x-values.

$x = -\frac{3\pi}{4}$ and $x = \frac{3\pi}{4}$

$x = -\pi$ and $x = \pi$

o. What can be said about the y-values for each pair of x-values marked on the graph?

p. Will this relationship hold for any two x-values with the same magnitude and same sign? Explain how you know.

q. Based on these results, make a conjecture by filling in the blank below.

For any real number , $\cos(-x) =$ ___________.

r. Test your conjecture by selecting another x-value from the graph and demonstrating that the identity is true for that value of x.

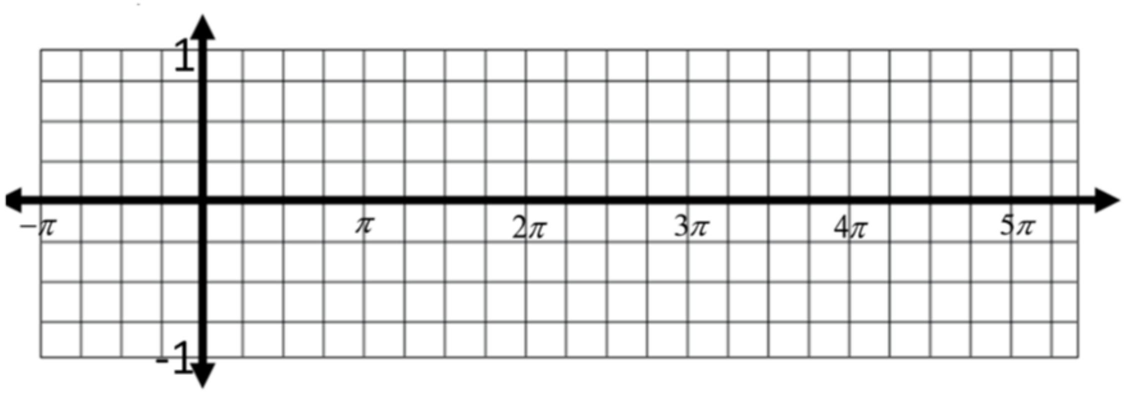
s. Is the cosine function an odd function, even function, or neither? Use the identity from part (n) to explain.

t. Describe the x-intercepts of the graph of the cosine function.

u. Describe the end behavior of $g(x) = \cos(x)$.

Exploratory Challenge 3

Graph both $f(x) = \sin(x)$ and $g(x) = \cos(x)$ on the graph below. Then, use the graphs to answer the questions that follow.



a. List ways in which the graphs of the sine and cosine functions are alike.

b. List ways in which the graphs of the sine and cosine functions are different.

c. What type of transformation would be required to make the graph of the sine function coincide with the graph of the cosine function?

d. What is the smallest possible horizontal translation required to make the graph of $f(x) = \sin(x)$ coincide with the graph of $g(x) = \cos(x)$?

e. What is the smallest possible horizontal translation required to make the graph of $g(x) = \cos(x)$ coincide with the graph of $f(x) = \sin(x)$?

f. Use your answers from parts (d) and (e) to fill in the blank below.

For any real number x, ____________ $= \cos\left(x - \frac{\pi}{2}\right)$.

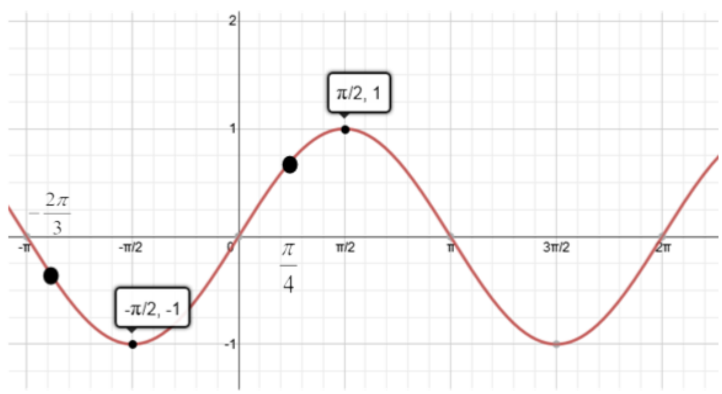
For any real number x, ____________ $\sin\left(x + \frac{\pi}{2}\right)$.

Problem Set

1. Describe the values of x for which each of the following is true.
   a. The cosine function has a relative maximum.
   b. The sine function has a relative maximum.

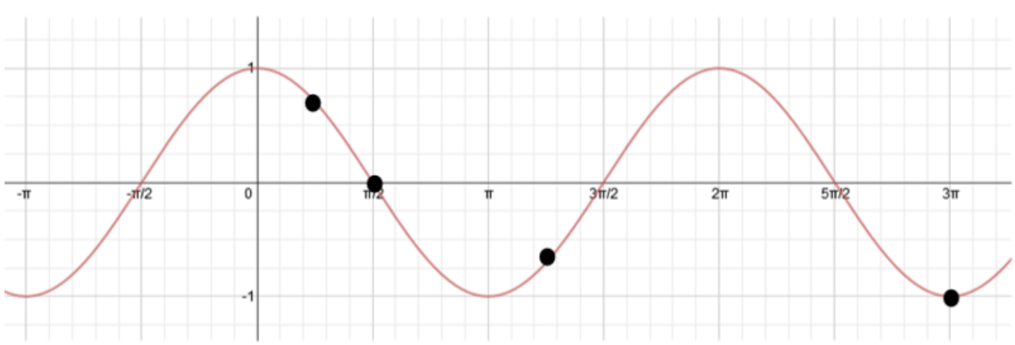
2. Without using a calculator, rewrite each of the following in order from least to greatest. Use the graph to explain your reasoning.

   $\sin\left(\frac{\pi}{2}\right)$ $\sin\left(-\frac{2\pi}{3}\right)$ $\sin\left(\frac{\pi}{4}\right)$ $\sin\left(-\frac{\pi}{2}\right)$



3. Without using a calculator, rewrite each of the following in order from least to greatest. Use the graph to explain your reasoning.

   $\cos\left(\frac{\pi}{2}\right)$ $\cos\left(\frac{5\pi}{4}\right)$ $\cos\left(\frac{\pi}{4}\right)$ $\cos(5\pi)$



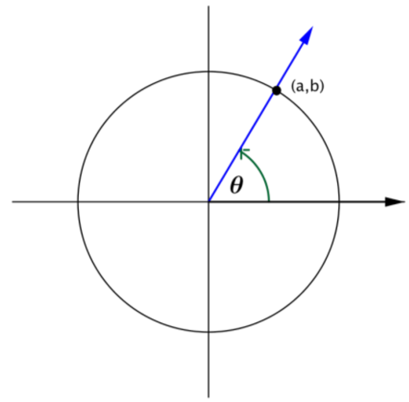
4. Evaluate each of the following without a calculator using a trigonometric identity when needed.

   $\cos\left(\frac{\pi}{6}\right)$ $\cos\left(-\frac{\pi}{6}\right)$ $\cos\left(\frac{7\pi}{6}\right)$ $\cos\left(\frac{13\pi}{6}\right)$

5. Evaluate each of the following without a calculator using a trigonometric identity when needed.

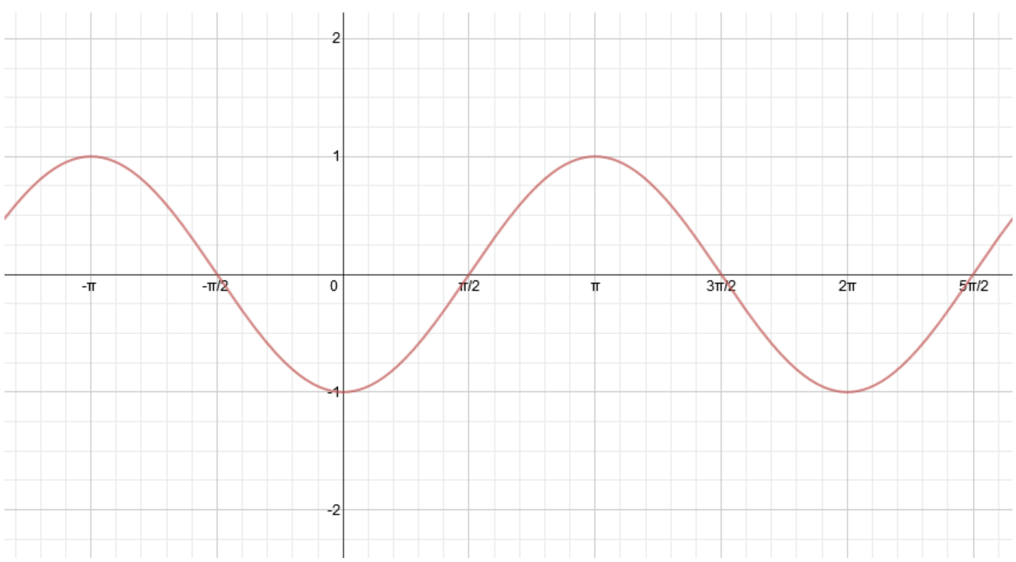
$\sin\left(\frac{3\pi}{4}\right)$ $\sin\left(\frac{11\pi}{4}\right)$ $\sin\left(\frac{7\pi}{4}\right)$ $\sin\left(\frac{-5\pi}{4}\right)$

6. Use the rotation through θ radians shown to answer each of the following questions.
   a. Explain why $\sin(-\theta) = -\sin(\theta)$ for all real numbers θ.
   b. What symmetry does this identity demonstrate about the graph of $y = \sin(x)$?



7. Use the same rotation shown in Problem 6 to answer each of the following questions.
   a. Explain why $\cos(-\theta) = \cos(\theta)$.
   b. What symmetry does this identity demonstrate about the graph of $y = \cos(x)$?

8. Find equations of two different functions that can be represented by the graph shown below—one sine and one cosine—using different horizontal transformations.



9. Find equations of two different functions that can be represented by the graph shown below—one sine and one cosine—using different horizontal translations.