Lesson 3: The Motion of the Moon, Sun, and Stars—Motivating Mathematics

Classwork

Opening

- Why does it look like the sun moves across the sky?

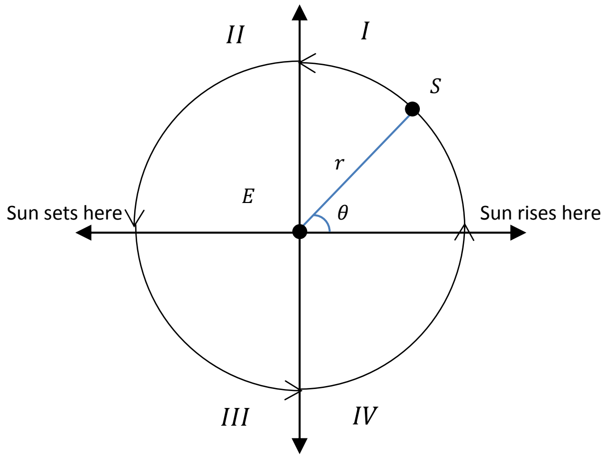
- Is the sun moving, or are you moving?

- In ancient Greek mythology, the god Helios was the personification of the sun. He rode across the sky every day in his chariot led by four horses. Why do your answers make it believable that in ancient times, people imagined the sun was pulled across the sky each day?

Discussion

In mathematics, counterclockwise rotation is considered to be the positive direction of rotation, which runs counter to our experience with a very common example of rotation: the rotation of the hands on a clock.

- Is there a connection between counterclockwise motion being considered to be positive and the naming of the quadrants on a standard coordinate system?



- What does the circle's radius, r, represent?

- How has the motion of the sun influenced the development of mathematics?

- How is measuring the "height" of the sun like measuring the Ferris wheel passenger car height in the previous lessons?

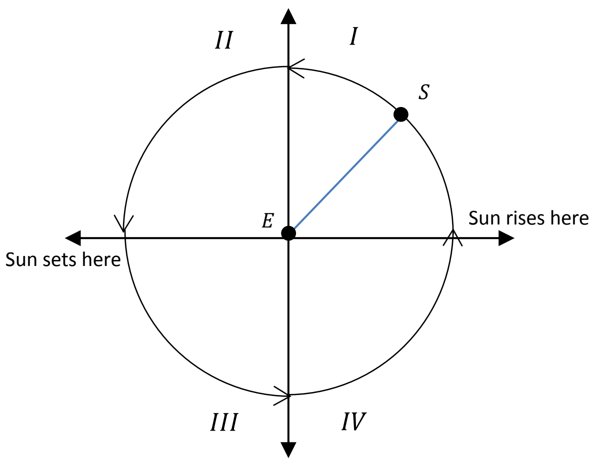
Exercises 1–4

1. Calculate jya$(7\frac{1}{2}°)$, jya$(11\frac{1}{4}°)$, jya$(15°)$, and jya$(18\frac{3}{4}°)$ using Aryabhata's formula[1], round to the nearest integer, and add your results to the table below. Leave the rightmost column blank for now.

n	$\theta°$	jya(θ)	$3438\sin(\theta)$	n	$\theta°$	jya(θ)	$3438\sin(\theta)$
1	$3\frac{3}{4}°$	225		13	$48\frac{3}{4}°$	2585	
2	$7\frac{1}{2}°$			14	$52\frac{1}{2}°$	2728	
3	$11\frac{1}{4}°$			15	$56\frac{1}{4}°$	2859	
4	$15°$			16	$60°$	2978	
5	$18\frac{3}{4}°$			17	$63\frac{3}{4}°$	3084	
6	$22\frac{1}{2}°$	1315		18	$67\frac{1}{2}°$	3177	
7	$26\frac{1}{4}°$	1520		19	$71\frac{1}{4}°$	3256	
8	$30°$	1719		20	$75°$	3321	
9	$33\frac{3}{4}°$	1910		21	$78\frac{3}{4}°$	3372	
10	$37\frac{1}{2}°$	2093		22	$82\frac{1}{2}°$	3409	
11	$41\frac{1}{4}°$	2267		23	$86\frac{1}{4}°$	3431	
12	$45°$	2431		24	$90°$	3438	

[1] In constructing the table, Aryabhata made adjustments to the values of his approximation to the jya to match his observational data. The first adjustment occurs in the calculation of jya$(30°)$. Thus, the entire table cannot be accurately constructed using this formula.

2. Label the angle θ, $\text{jya}(\theta)$, $\text{kojya}(\theta)$, and r in the diagram shown below.



a. How does this relate to something you have done before?

b. How does $\text{jya}(\theta)$ relate to a length *we* already know?

3. Use your calculator to compute $r\sin(\theta)$ for each value of θ in the table from Exercise 1, where $r = 3438$. Record this in the blank column on the right in Exercise 1, rounding to the nearest integer. How do Aryabhata's approximated values from around the year A.D. 500 compare to the value we can calculate with our modern technology?

4. We will assume that the sun rises at 6:00 am, is directly overhead at 12:00 noon, and sets at 6:00 pm. We measure the "height" of the sun by finding its vertical distance from the horizon line; the horizontal line that connects the eastern-most point, where the sun rises, to the western-most point, where the sun sets.

   a. Using $r = 3438$, as Aryabhata did, find the "height" of the sun at the times listed in the following table:

Time of day	Height
6:00 a.m.	
7:00 a.m.	
8:00 a.m.	
9:00 a.m.	
10:00 a.m.	
11:00 a.m.	
12:00 p.m.	

   b. Now, find the height of the sun at the times listed in the following table using the actual distance from the earth to the sun, $r = 93$ million miles.

Time of day	Height
6:00 a.m.	
7:00 a.m.	
8:00 a.m.	
9:00 a.m.	
10:00 a.m.	
11:00 a.m.	
12:00 p.m.	

Closing

- Our exploration of the historical development of the sine table is based on observations of the motion of the planets and stars in Babylon and India. Is it based on a geocentric or heliocentric model? What does that term mean?

- How is Aryabhata's function jya related to the sine of an angle of a triangle?

- How does the apparent motion of the sun in the sky relate to the motion of a passenger car of a Ferris wheel?

Lesson Summary

Ancient scholars in Babylon and India conjectured that celestial motion was circular; the sun and other stars orbited the earth in circular fashion. The earth was presumed the center of the sun's orbit.

The quadrant numbering in a coordinate system is consistent with the counterclockwise motion of the sun, which rises from the east and sets in the west.

The 6th century Indian scholar Aryabhata created the first sine table, using a measurement he called *jya*. The purpose of his table was to calculate the position of the sun, the stars, and the planets.

Problem Set

1. An Indian astronomer noted that the angle of his line of sight to Venus measured $52\frac{1}{2}°$. We now know that the average distance from the Earth to Venus is 162 million miles. Use Aryabhata's table to estimate the apparent height of Venus. Round your answer to the nearest million miles.

2. Later, the Indian astronomer saw that the angle of his line of sight to Mars measured $82\frac{1}{2}°$. We now know that the average distance from the Earth to Mars is 140 million miles. Use Aryabhata's table to estimate the apparent height of Mars. Round your answer to the nearest million miles.

3. The moon orbits the earth in an elongated orbit, with an average distance of the moon from the earth of roughly $239{,}000$ miles. It takes the moon 27.32 days to travel around the earth, so the moon moves with respect to the stars roughly $0.5°$ every hour. Suppose that angle of inclination of the moon with respect to the observer measures $45°$ at midnight. As in Example 1, an observer is standing still and facing north. Use Aryabhata's jya table to find the apparent height of the moon above the observerat the times listed in the table below, to the nearest thousand miles.

Time (hour:min)	Angle of elevation θ	Height
12:00 a.m.		
7:30 a.m.		
3:00 p.m.		
10:30 p.m.		
6:00 a.m.		
1:30 p.m.		
9:00 p.m.		

4. George wants to apply Aryabhata's method to estimate the height of the International Space Station, which orbits earth at a speed of about $17{,}500$ miles per hour. This means that the space station makes one full rotation around the earth roughly every 90 minutes. The space station maintains a low earth orbit, with an average distance from earth of 238 miles.

   a. George supposes that the space station is just visible on the eastern horizon at 12:00 midnight, so its apparent height at that time would be 0 miles above the horizon. Use Aryabhata's jya table to find the apparent height of the space station above the observer at the times listed in the table below.

Time (hour:min:sec)	Angle of elevation θ	Height
12:00:00 a.m.		
12:03:45 a.m.		
12:07:30 a.m.		
12:11:15 a.m.		
12:15:00 a.m.		
12:18:45 a.m.		
12:22:30 a.m.		

   b. When George presents his solution to his classmate Jane, she tells him that his model isn't appropriate for this situation. Is she correct? Explain how you know. (Hint: As we set up our model in the first discussion, we treated our observer as if he was the center of the orbit of the sun around the earth. In part (a) of this problem, we treated our observer as if she was the center of the orbit of the International Space Station around the earth. The radius of the earth is approximately 3963 miles, the space station orbits about 238 miles above the earth's surface, and the distance from the earth to the sun is roughly $93{,}000{,}000$ miles. Draw a picture of the earth and the path of the space station, then compare that to the points with heights and rotation angles from part (a).)