Lesson 15: What Is a Trigonometric Identity?

Classwork

Exercises 1–3

1. Recall the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, where θ is any real number.

   a. Find $\sin(x)$, given $\cos(x) = \frac{3}{5}$, for $-\frac{\pi}{2} < x < 0$.

   b. Find $\tan(y)$, given $\cos(y) = -\frac{5}{13}$, for $\frac{\pi}{2} < y < \pi$.

   c. Write $\tan(z)$ in terms of $\cos(z)$, for $\pi < z < \frac{3\pi}{2}$.

2. Use the Pythagorean identity to do the following:

   a. Rewrite the expression $\cos(\theta)\sin^2(\theta) - \cos(\theta)$ in terms of a single trigonometric function.

   b. Rewrite the expression $(1 - \cos^2(\theta))\csc(\theta)$ in terms of a single trigonometric function.

   c. Find all the solutions of the equation $2\sin^2(\theta) = 2 + \cos(\theta)$ in the interval $[0,2\pi)$. Draw a unit circle that shows the solutions.

3. Which of the following equations are identities? For those equations that are identities, which ones are defined for all real numbers and which are not? For the latter, for which values of x are they not defined?

   a. $\sin(x + 2\pi) = \sin(x)$ where the functions on both sides are defined.

   b. $\sec(x) = 1$ where the functions on both sides are defined.

   c. $\sin(-x) = \sin(x)$ where the functions on both sides are defined.

   d. $1 + \tan^2(x) = \sec^2(x)$ where the functions on both sides are defined.

   e. $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ where the functions on both sides are defined.

   f. $\sin^2(x) = \tan^2(x)$ for all real x.

Lesson Summary

The Pythagorean identity: $\sin^2(\theta) + \cos^2(\theta) = 1$ for all real numbers θ.

Problem Set

1. Which of the following are trigonometric identities? Graph the functions on each side of the equation.
   a. $\tan(x) = \frac{\sin(x)}{\cos(x)}$ where the functions on both sides are defined.
   b. $\cos^2(x) = 1 + \sin(x)$ where the functions on both sides are defined.
   c. $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$ where the functions on both sides are defined.

2. Determine the domain of the following trigonometric identities:
   a. $\cot(x) = \frac{\cos(x)}{\sin(x)}$ where the functions on both sides are defined.
   b. $\cos(-u) = \cos(u)$ where the functions on both sides are defined.
   c. $\sec(y) = \frac{1}{\cos(y)}$ where the functions on both sides are defined.

3. Rewrite $\sin(x)\cos^2(x) - \sin(x)$ as an expression containing a single term.

4. Suppose $0 < \theta < \frac{\pi}{2}$, and $\sin(\theta) = \frac{1}{\sqrt{3}}$. What is the value of $\cos(\theta)$?

5. If $\cos(\theta) = -\frac{1}{\sqrt{5}}$, what are possible values of $\sin(\theta)$?

6. Use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, where θ is any real number, to find the following:
   a. $\cos(\theta)$, given $\sin(\theta) = \frac{5}{13}$, for $\frac{\pi}{2} < \theta < \pi$.
   b. $\tan(x)$, given $\cos(x) = -\frac{1}{\sqrt{2}}$, for $\pi < x < \frac{3\pi}{2}$.

7. The three identities below are all called Pythagorean identities. The second and third follow from the first, as you saw in Example 1 and the Exit Ticket.

   a. For which values of θ are each of these identities defined?

      i. $\sin^2(\theta) + \cos^2(\theta) = 1$, where the functions on both sides are defined.

      ii. $\tan^2(\theta) + 1 = \sec^2(\theta)$, where the functions on both sides are defined.

      iii. $1 + \cot^2(\theta) = \csc^2(\theta)$, where the functions on both sides are defined.

   b. For which of the three identities is 0 in the domain of validity?

   c. For which of the three identities is $\frac{\pi}{2}$ in the domain of validity?

   d. For which of the three identities is $-\frac{\pi}{4}$ in the domain of validity?