Lesson 14: Solving Logarithmic Equations

Classwork

Opening Exercises

Convert the following logarithmic equations to exponential form:

a. $\log(10{,}000) = 4$

b. $\log(\sqrt{10}) = \frac{1}{2}$

c. $\log_2(256) = 8$

d. $\log_4(256) = 4$

e. $\ln(1) = 0$

f. $\log(x + 2) = 3$

Examples 1–3

Write each of the following equations as an equivalent exponential equation, and solve for x.

1. $\log(3x + 7) = 0$

2. $\log_2(x + 5) = 4$

3. $\log(x + 2) + \log(x + 5) = 1$

Exercises

1. Drew said that the equation $\log_2[(x+1)^4] = 8$ cannot be solved because he expanded $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$ and realized that he cannot solve the equation $x^4 + 4x^3 + 6x^2 + 4x + 1 = 2^8$. Is he correct? Explain how you know.

Solve the equations in Exercises 2–4 for x.

2. $\ln((4x)^5) = 15$

3. $\log((2x+5)^2) = 4$

4. $\log_2((5x+7)^{19}) = 57$

Solve the logarithmic equations in Exercises 5–9, and identify any extraneous solutions.

5. $\log(x^2+7x+12) - \log(x+4) = 0$

6. $\log_2(3x) + \log_2(4) = 4$

7. $2\ln(x+2) - \ln(-x) = 0$

8. $\log(x) = 2 - \log(x)$

9. $\ln(x+2) = \ln(12) - \ln(x+3)$

Problem Set

1. Solve the following logarithmic equations.
   a. $\log(x) = \frac{5}{2}$
   b. $5\log(x+4) = 10$
   c. $\log_2(1-x) = 4$
   d. $\log_2(49x^2) = 4$
   e. $\log_2(9x^2+30x+25) = 8$

2. Solve the following logarithmic equations.
   a. $\ln(x^6) = 36$
   b. $\log[(2x^2+45x-25)^5] = 10$
   c. $\log[(x^2+2x-3)^4] = 0$

3. Solve the following logarithmic equations.
   a. $\log(x) + \log(x-1) = \log(3x+12)$
   b. $\ln(32x^2) - 3\ln(2) = 3$
   c. $\log(x) + \log(-x) = 0$
   d. $\log(x+3) + \log(x+5) = 2$
   e. $\log(10x+5) - 3 = \log(x-5)$
   f. $\log_2(x) + \log_2(2x) + \log_2(3x) + \log_2(36) = 6$

4. Solve the following equations.
   a. $\log_2(x) = 4$
   b. $\log_6(x) = 1$
   c. $\log_3(x) = -4$
   d. $\log_{\sqrt{2}}(x) = 4$
   e. $\log_{\sqrt{5}}(x) = 3$
   f. $\log_3(x^2) = 4$
   g. $\log_2(y^{-3}) = 12$
   h. $\log_3(8x+9) = 4$
   i. $2 = \log_4(3x-2)$
   j. $\log_5(3-2x) = 0$
   k. $\ln(2x) = 3$
   l. $\log_3(x^2-3x+5) = 2$
   m. $\log((x^2+4)^5) = 10$
   n. $\log(x) + \log(x+21) = 2$
   o. $\log_4(x-2) + \log_4(2x) = 2$
   p. $\log(x) - \log(x+3) = -1$
   q. $\log_4(x+3) - \log_4(x-5) = 2$
   r. $\log(x) + 1 = \log(x+9)$
   s. $\log_3(x^2-9) - \log_3(x+3) = 1$
   t. $1 - \log_8(x-3) = \log_8(2x)$
   u. $\log_2(x^2-16) - \log_2(x-4) = 1$
   v. $\log\left(\sqrt{(x+3)^3}\right) = \frac{3}{2}$
   w. $\ln(4x^2-1) = 0$
   x. $\ln(x+1) - \ln(2) = 1$