Lesson 15: Why Were Logarithms Developed?

Classwork

Exercises

1. Solve the following equations. Remember to check for extraneous solutions because logarithms are only defined for positive real numbers.

   a. $\log(x^2) = \log(49)$

   b. $\log(x+1) + \log(x-2) = \log(7x-17)$

   c. $\log(x^2+1) = \log\big(x(x-2)\big)$

d. $\log(x+4) + \log(x-1) = \log(3x)$

e. $\log(x^2 - x) - \log(x-2) = \log(x-3)$

f. $\log(x) + \log(x-1) + \log(x+1) = 3\log(x)$

g. $\log(x-4) = -\log(x-2)$

2. How do you know if you need to use the definition of logarithm to solve an equation involving logarithms as we did in Lesson 15 or if you can use the methods of this lesson?

Lesson Summary

A table of base 10 logarithms can be used to simplify multiplication of multi-digit numbers:

1. To compute $A \times B$ for positive real numbers A and B, look up the values $\log(A)$ and $\log(B)$ in the logarithm table.
2. Add $\log(A)$ and $\log(B)$. The sum can be written as $k + d$, where k is an integer and $0 \le d < 1$ is the decimal part.
3. Look back at the table and find the entry closest to the decimal part, d.
4. The product of that entry and 10^k is an approximation to $A \times B$.

A similar process simplifies division of multi-digit numbers:

1. To compute $A \div B$ for positive real numbers A and B, look up the values $\log(A)$ and $\log(B)$ in the logarithm table.
2. Calculate $\log(A) - \log(B)$. The difference can be written as $k + d$, where k is an integer and $0 \le d < 1$ is the decimal part.
3. Look back at the table to find the entry closest to the decimal part, d.
4. The product of that entry and 10^k is an approximation to $A \div B$.

For any positive values X and Y, if $\log_b(X) = \log_b(Y)$, we can conclude that $X = Y$. This property is the essence of how a logarithm table works, and it allows us to solve equations with logarithmic expressions on both sides of the equation.

Problem Set

1. Use the table of logarithms to approximate solutions to the following logarithmic equations.
   a. $\log(x) = 0.5044$
   b. $\log(x) = -0.5044$ [Hint: Begin by writing -0.5044 as $[(-0.5044) + 1] - 1$.]
   c. $\log(x) = 35.5044$
   d. $\log(x) = 4.9201$

2. Use logarithms and the logarithm table to evaluate each expression.
   a. $\sqrt{2.33}$
   b. $13{,}500 \cdot 3{,}600$
   c. $\dfrac{7.2 \times 10^9}{1.3 \times 10^5}$

3. Solve for x: $\log(3) + 2\log(x) = \log(27)$.

4. Solve for x: $\log(3x) + \log(x+4) = \log(15)$.

5. Solve for x.
   a. $\log(x) = \log(y+z) + \log(y-z)$
   b. $\log(x) = \big(\log(y) + \log(z)\big) + \big(\log(y) - \log(z)\big)$

6. If x and y are positive real numbers, and $\log(y) = 1 + \log(x)$, express y in terms of x.

7. If x, y, and z are positive real numbers, and $\log(x) - \log(y) = \log(y) - \log(z)$, express y in terms of x and z.

8. If x and y are positive real numbers, and $\log(x) = y\big(\log(y+1) - \log(y)\big)$, express x in terms of y.

9. If x and y are positive real numbers, and $\log(y) = 3 + 2\log(x)$, express y in terms of x.

10. If x, y, and z are positive real numbers, and $\log(z) = \log(y) + 2\log(x) - 1$, express z in terms of x and y.

11. Solve the following equations.
    a. $\ln(10) - \ln(7-x) = \ln(x)$
    b. $\ln(x+2) + \ln(x-2) = \ln(9x-24)$
    c. $\ln(x+2) + \ln(x-2) = \ln(-2x-1)$

12. Suppose the formula $P = P_0(1+r)^t$ gives the population of a city P growing at an annual percent rate r, where P_0 is the population t years ago.
    a. Find the time t it takes this population to double.
    b. Use the structure of the expression to explain why populations with lower growth rates take a longer time to double.
    c. Use the structure of the expression to explain why the only way to double the population in one year is if there is a 100 percent growth rate.

13. If $x > 0$, $a + b > 0$, $a > b$, and $\log(x) = \log(a+b) + \log(a-b)$, find x in terms of a and b.

14. Jenn claims that because $\log(1) + \log(2) + \log(3) = \log(6)$, then $\log(2) + \log(3) + \log(4) = \log(9)$.
    a. Is she correct? Explain how you know.
    b. If $\log(a) + \log(b) + \log(c) = \log(a+b+c)$, express c in terms of a and b. Explain how this result relates to your answer to part (a).
    c. Find other values of a, b, and c so that $\log(a) + \log(b) + \log(c) = \log(a+b+c)$.

15. In Problem 7 of the Lesson 12 Problem Set, you showed that for $x \geq 1$, $\log\left(x + \sqrt{x^2 - 1}\right) + \log\left(x - \sqrt{x^2 - 1}\right) = 0$. It follows that $\log\left(x + \sqrt{x^2 - 1}\right) = -\log\left(x - \sqrt{x^2 - 1}\right)$. What does this tell us about the relationship between the expressions $x + \sqrt{x^2 - 1}$ and $x - \sqrt{x^2 - 1}$?

16. Use the change of base formula to solve the following equations.

    a. $\log(x) = \log_{100}(x^2 - 2x + 6)$
    b. $\log(x - 2) = \log_{100}(14 - x)$
    c. $\log_2(x + 1) = \log_4(x^2 + 3x + 4)$
    d. $\log_2(x - 1) = \log_8(x^3 - 2x^2 - 2x + 5)$

17. Solve the following equation:

$$\log(9x) = \frac{2\ln(3) + \ln(x)}{\ln(10)}.$$