

## Lesson 7: Bacteria and Exponential Growth

### Classwork

#### Opening Exercise

Work with your partner or group to solve each of the following equations for  $x$ .

a.  $2^x = 2$

b.  $2^x = 2^3$

c.  $2^x = 16$

d.  $2^x - 64 = 0$

e.  $2^x - 1 = 0$

f.  $2^{3x} = 64$

g.  $2^{x+1} = 32$

**Example**

The *Escherichia coli* bacteria (commonly known as *E. coli*), reproduces once every 30 minutes, meaning that a colony of *E. coli* can double every half hour. *Mycobacterium tuberculosis* has a generation time in the range of 12 to 16 hours. Researchers have found evidence that suggests certain bacteria populations living deep below the surface of the earth may grow at extremely slow rates, reproducing once every several thousand years. With this variation in bacterial growth rates, it is reasonable that we assume a 24-hour reproduction time for a hypothetical bacteria colony in the next example.

Suppose we have a bacteria colony that starts with 1 bacterium, and the population of bacteria doubles every day.

What function  $P$  can we use to model the bacteria population on day  $t$ ?

$t$	$P(t)$

How many days will it take for the bacteria population to reach 8?

How many days will it take for the bacteria population to reach 16?

Roughly how long will it take for the population to reach 10?

We already know from our previous discussion that if  $2^d = 10$ , then  $3 < d < 4$ , and the table confirms that. At this point, we have an underestimate of 3 and an overestimate of 4 for  $d$ . How can we find better under and over estimates for  $d$ ?

$t$	$P(t)$

From our table, we now know another set of under and over estimates for the number  $d$  that we seek. What are they?

Continue this process of “squeezing” the number  $d$  between two numbers until you are confident you know the value of  $d$  to two decimal places.

$t$	$P(t)$

$t$	$P(t)$

What if we had wanted to find  $d$  to 5 decimal places?

To the nearest minute, when does the population of bacteria become 10?

$t$	$P(t)$

$t$	$P(t)$

$t$	$P(t)$

$t$	$P(t)$

**Exercise**

Use the method from the Example to approximate the solution to the equations below to two decimal places.

a.  $2^x = 1000$

b.  $3^x = 1000$

c.  $4^x = 1000$

d.  $5^x = 1000$

e.  $6^x = 1000$

f.  $7^x = 1000$

g.  $8^x = 1000$

h.  $9^x = 1000$

i.  $11^x = 1000$

j.  $12^x = 1000$

k.  $13^x = 1000$

l.  $14^x = 1000$

m.  $15^x = 1000$

n.  $16^x = 1000$

**Problem Set**

1. Solve each of the following equations for  $x$  using the same technique as was used in the Opening Exercise.

- a.  $2^x = 32$
- b.  $2^{x-3} = 2^{2x+5}$
- c.  $2^{x^2-3x} = 2^{-2}$
- d.  $2^x - 2^{4x-3} = 0$
- e.  $2^{3x} \cdot 2^5 = 2^7$
- f.  $2^{x^2-16} = 1$
- g.  $3^{2x} = 27$
- h.  $3^{\frac{2}{x}} = 81$
- i.  $\frac{3^{x^2}}{3^{5x}} = 3^6$

2. Solve the equation  $\frac{2^{2x}}{2^{x+5}} = 1$  algebraically using two different initial steps as directed below.

- a. Write each side as a power of 2.
- b. Multiply both sides by  $2^{x+5}$ .

3. Find consecutive integers that are under and over estimates of the solutions to the following exponential equations.

- a.  $2^x = 20$
- b.  $2^x = 100$
- c.  $3^x = 50$
- d.  $10^x = 432,901$
- e.  $2^{x-2} = 750$
- f.  $2^x = 1.35$

4. Complete the following table to approximate the solution to  $10^x = 34,198$  to two decimal places.

$t$	$P(t)$	$t$	$P(t)$	$t$	$P(t)$	$t$	$P(t)$
1	10	4.1		4.51		4.531	
2	100	4.2		4.52		4.532	
3	1,000	4.3		4.53		4.533	
4	10,000	4.4		4.54		4.534	
5	100,000	4.5				4.535	
		4.6					

5. Complete the following table to approximate the solution to  $2^x = 18$  to two decimal places.

$t$	$P(t)$	$t$	$P(t)$	$t$	$P(t)$	$t$	$P(t)$

6. Approximate the solution to  $5^x = 5555$  to four decimal places.
7. A dangerous bacterial compound forms in a closed environment but is immediately detected. An initial detection reading suggests the concentration of bacteria in the closed environment is one percent of the fatal exposure level. This bacteria is known to double in growth (double in concentration in a closed environment) every hour and can be modeled by the function  $P(t) = 100 \cdot 2^t$ , where  $t$  is measured in hours.
- In the function  $P(t) = 100 \cdot 2^t$ , what does the 100 mean? What does the 2 mean?
  - Doctors and toxicology professionals estimate that exposure to two-thirds of the bacteria’s fatal concentration level will begin to cause sickness. Without consulting a calculator or other technology, offer a rough time limit for the inhabitants of the infected environment to evacuate in order to avoid sickness in the doctors’ estimation. Note that immediate evacuation is not always practical, so offer extra evacuation time if it is affordable.
  - A more conservative approach is to evacuate the infected environment before bacteria concentration levels reach one-third of fatal levels. Without consulting a calculator or other technology, offer a rough time limit for evacuation in this circumstance.

- d. Use the method of the Example to approximate when the infected environment will reach fatal levels (100%) of bacteria concentration, to the nearest minute.

$t$	$2^t$	$t$	$2^t$	$t$	$2^t$	$t$	$2^t$	$t$	$2^t$