Lesson 8: The "WhatPower" Function

Classwork

Opening Exercise

Evaluate each expression. The first two have been completed for you.

a. $\text{WhatPower}_2(8) = 3$

b. $\text{WhatPower}_3(9) = 2$

c. $\text{WhatPower}_6(36) =$ ______

d. $\text{WhatPower}_2(32) =$ ______

e. $\text{WhatPower}_{10}(1000) =$ ______

f. $\text{WhatPower}_{10}(1{,}000{,}000) =$ ______

g. $\text{WhatPower}_{100}(1{,}000{,}000) =$ ______

h. $\text{WhatPower}_4(64) =$ ______

i. $\text{WhatPower}_2(64) =$ ______

j. $\text{WhatPower}_9(3) =$ ______

k. $\text{WhatPower}_5\left(\sqrt{5}\right) =$ ______

l. $\text{WhatPower}_{\frac{1}{2}}\left(\frac{1}{8}\right) =$ ______

m. $\text{WhatPower}_{42}(1) =$ ______

n. $\text{WhatPower}_{100}(0.01) =$ ______

o. $\text{WhatPower}_2\left(\frac{1}{4}\right) =$ ______

p. $\text{WhatPower}_{\frac{1}{4}}(2) =$ ______

q. With your group members, write a definition for the function WhatPower_b , where b is a number.

Exercises 1–9

Evaluate the following expressions and justify your answers.

2. $\text{WhatPower}_7(49)$

3. $\text{WhatPower}_0(7)$

4. $\text{WhatPower}_5(1)$

5. $\text{WhatPower}_1(5)$

6. $\text{WhatPower}_2(16)$

7. $\text{WhatPower}_{-2}(32)$

8. $\text{WhatPower}_{\frac{1}{3}}(9)$

9. $\text{WhatPower}_{-\frac{1}{3}}(27)$

10. Describe the allowable values of b in the expression $\mathrm{WhatPower}_{\mathrm{b}}(x)$. When can we define a function $f(x) = \mathrm{WhatPower}_b(x)$? Explain how you know.

Examples

1. $\log_2(8) = 3$

2. $\log_3(9) = 2$

3. $\log_6(36) =$ ______

4. $\log_2(32) =$ ______

5. $\log_{10}(1000) =$ ______

6. $\log_{42}(1) =$ ______

7. $\log_{100}(0.01) =$ ______

8. $\log_2\left(\frac{1}{4}\right) =$ ______

Exercise 10

10. Compute the value of each logarithm. Verify your answers using an exponential statement.

    a. $\log_2(32)$

b. $\log_3(81)$

c. $\log_9(81)$

d. $\log_5(625)$

e. $\log_{10}(1{,}000{,}000{,}000)$

f. $\log_{1000}(1{,}000{,}000{,}000)$

g. $\log_{13}(13)$

h. $\log_{13}(1)$

i. $\log_9(27)$

j. $\log_7(\sqrt{7})$

k. $\log_{\sqrt{7}}(7)$

l. $\log_{\sqrt{7}}\left(\frac{1}{49}\right)$

m. $\log_x(x^2)$

Lesson Summary

- If three numbers, L, b, and x are related by $x = b^L$, then L is the *logarithm base* b of x and we write $\log_b(x)$. That is, the value of the expression $L = \log_b(x)$ is the power of b needed to obtain x.
- Valid values of b as a base for a logarithm are $0 < b < 1$ and $b > 1$.

Problem Set

1. Rewrite each of the following in the form $\text{WhatPower}_b(x) = L$.

   a. $3^5 = 243$
   b. $6^{-3} = \frac{1}{216}$
   c. $9^0 = 1$

2. Rewrite each of the following in the form $\log_b(x) = L$.

   a. $16^{\frac{1}{4}} = 2$
   b. $10^3 = 1{,}000$
   c. $b^k = r$

3. Rewrite each of the following in the form $b^L = x$.

   a. $\log_5(625) = 4$
   b. $\log_{10}(0.1) = -1$
   c. $\log_{27}9 = \frac{2}{3}$

4. Consider the logarithms base 2. For each logarithmic expression below, either calculate the value of the expression or explain why the expression does not make sense.

   a. $\log_2(1024)$
   b. $\log_2(128)$
   c. $\log_2(\sqrt{8})$
   d. $\log_2\left(\frac{1}{16}\right)$
   e. $\log_2(0)$
   f. $\log_2\left(-\frac{1}{32}\right)$

5. Consider the logarithms base 3. For each logarithmic expression below, either calculate the value of the expression or explain why the expression does not make sense.

   a. $\log_3(243)$
   b. $\log_3(27)$
   c. $\log_3(1)$
   d. $\log_3\left(\frac{1}{3}\right)$
   e. $\log_3(0)$
   f. $\log_3\left(-\frac{1}{3}\right)$

6. Consider the logarithms base 5. For each logarithmic expression below, either calculate the value of the expression or explain why the expression does not make sense.
   a. $\log_5(3125)$
   b. $\log_5(25)$
   c. $\log_5(1)$
   d. $\log_5\left(\frac{1}{25}\right)$
   e. $\log_5(0)$
   f. $\log_5\left(-\frac{1}{25}\right)$

7. Is there any positive number b so that the expression $\log_b(0)$ makes sense? Explain how you know.

8. Is there any positive number b so that the expression $\log_b(-1)$ makes sense? Explain how you know.

9. Verify each of the following by evaluating the logarithms.
   a. $\log_2(8) + \log_2(4) = \log_2(32)$
   b. $\log_3(9) + \log_3(9) = \log_3(81)$
   c. $\log_4(4) + \log_4(16) = \log_4(64)$
   d. $\log_{10}(10^3) + \log_{10}(10^4) = \log_{10}(10^7)$

10. Looking at the results from Problem 9, do you notice a trend or pattern? Can you make a general statement about the value of $\log_b(x) + \log_b(y)$?

11. To evaluate $\log_2(3)$, Autumn reasoned that since $\log_2(2) = 1$ and $\log_2(4) = 2$, $\log_2(3)$ must be the average of 1 and 2 and therefore $\log_2(3) = 1.5$. Use the definition of logarithm to show that $\log_2(3)$ cannot be 1.5. Why is her thinking not valid?

12. Find the value of each of the following.
   a. If $x = \log_2(8)$ and $y = 2^x$, find the value of y.
   b. If $\log_2(x) = 6$, find the value of x.
   c. If $r = 2^6$ and $s = \log_2(r)$, find the value of s.