Lesson 9: Logarithms—How Many Digits Do You Need?

Classwork

Opening Exercise

a. Evaluate $\text{WhatPower}_2(8)$. State your answer as a logarithm and evaluate it.

b. Evaluate $\text{WhatPower}_5(625)$. State your answer as a logarithm and evaluate it.

Exploratory Challenge

Autumn is starting a new club with eight members including herself. She wants everyone to have a secret identification code made up of only A's and B's. For example, using two characters, her ID code could be AB, which also happens to be her initials.

a. Using A's and B's, can Autumn assign each club member a unique two-character ID using only A's and B's? Justify your answer. Here's what Autumn has so far.

Club Member Name	Secret ID
Autumn	AA
Kris	
Tia	
Jimmy	

Club Member Name	Secret ID
Robert	
Jillian	
Benjamin	
Scott	

b. Using A's and B's, how many characters would be needed to assign each club member a unique ID code? Justify your answer by showing the IDs you would assign to each club member by completing the table above (adjust Autumn's ID if needed).

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When the club grew to 16 members, Autumn started noticing a pattern.

Using A's and B's:

     i.   Two people could be given a secret ID with 1 letter: A and B.

     ii.   Four people could be given a secret ID with 2 letters: AA, BA, AB, BB.

     iii.   Eight people could be given a secret ID with 3 letters: AAA, BAA, ABA, BBA, AAB, BAB, ABB, BBB.

    c.   Complete the following statement and list the secret IDs for the 16 people.

       16 people could be given a secret ID with _____ letters using A's and B's.

Club Member Name	Secret ID
Autumn	
Kris	
Tia	
Jimmy	
Robert	
Jillian	
Benjamin	
Scott	

Club Member Name	Secret ID
Gwen	
Jerrod	
Mykel	
Janette	
Nellie	
Serena	
Ricky	
Mia	

    d.   Describe the pattern in words. What type of function could be used to model this pattern?

Exercises 1–2

In the previous problems, the letters A and B were like the digits in a number. A 4-digit ID for Autumn's club could be any 4-letter arrangement of A's and B's because in her ID system, the only digits are the letters A and B.

1. When Autumn's club grows to include more than 16 people, she will need 5 digits to assign a unique ID to each club member. What is the maximum number of people that could be in the club before she needs to switch to a 6-digit ID? Explain your reasoning.

2. If Autumn has 256 members in her club, how many digits would she need to assign each club member a unique ID using only A's and B's? Show how you got your answers.

Example 1

A thousand people are given unique identifiers made up of the digits 0, 1, 2, …, 9. How many digits would be needed for each ID number?

Exercises 3–4

3. There are approximately 317 million people in the United States. Compute and use $\log(100,000,000)$ and $\log(1,000,000,000)$ to explain why Social Security numbers are 9 digits long.

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4. There are many more telephones than the number of people in the United States because of people having home phones, cell phones, business phones, fax numbers, etc. Assuming we need at most 10 billion phone numbers in the United States, how many digits would be needed so that each phone number is unique? Is this reasonable? Explain.

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Problem Set

1. The student body president needs to assign each officially sanctioned club on campus a unique ID number for purposes of tracking expenses and activities. She decides to use the letters A, B, and C to create a unique three-character code for each club.

    a. How many clubs can be assigned a unique ID according to this proposal?

    b. There are actually over 500 clubs on campus. Assuming the student body president still wants to use the letters A, B, and C, how many characters would be needed to generate a unique ID for each club?

2. Can you use the numbers 1, 2, 3, and 4 in a combination of 4 digits to assign a unique ID to each of 500 people? Explain your reasoning.

3. Automobile license plates typically have a combination of letters (26) and numbers (10). Over time, the state of New York has used different criteria to assign vehicle license plate numbers.

    a. From 1973 to 1986, the state used a 3-letter and 4-number code where the three letters indicated the county where the vehicle was registered. Essex County had 13 different 3-letter codes in use. How many cars could be registered to this county?

    b. Since 2001, the state has used a 3-letter and 4-number code but no longer assigns letters by county. Is this coding scheme enough to register 10 million vehicles?

4. The Richter scale uses common (base 10) logarithms to assign a magnitude to an earthquake that is based on the amount of force released at the earthquake's source as measured by seismographs in various locations.

    a. Explain the difference between an earthquake that is assigned a magnitude of 5 versus one assigned a magnitude of 7.

    b. A magnitude 2 earthquake can usually be felt by multiple people who are located near the earthquake's origin. The largest recorded earthquake was magnitude 9.5 in Chile in 1960. How many times greater force than a magnitude 2 earthquake was the largest recorded earthquake?

    c. What would be the magnitude of an earthquake whose force was 1000 times greater than a magnitude 4.3 quake?

5. Sound pressure level is measured in decibels (dB) according to the formula $L = 10 \log\left(\frac{I}{I_0}\right)$, where I is the intensity of the sound and I_0 is a reference intensity that corresponds to a barely perceptible sound.

    a. Explain why this formula would assign 0 decibels to a barely perceptible sound.

    b. Decibel levels above 120 dB can be painful to humans. What would be the intensity that corresponds to this level?

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