

Lesson 26: Percent Rate of Change

Classwork

Exercise

Answer the following questions.

The youth group from Example 1 is given the option of investing their money at 2.976% interest per year, compounded monthly.

- After two years, how much would be in each account with an initial deposit of \$800?
- Compare the total amount from part (a) to how much they would have made using the interest rate of 3% compounded yearly for two years. Which account would you recommend the youth group invest its money in? Why?

Lesson Summary

- For application problems involving a percent rate of change represented by the unit rate r , we can write $F(t) = P(1 + r)^t$, where F is the future value (or ending amount), P is the present amount, and t is the number of time units. When the percent rate of change is negative, r is negative and the quantity decreases with time.
- The nominal APR is the percent rate of change per compounding period times the number of compounding periods per year. If the nominal APR is given by the unit rate r and is compounded n times a year, then function $F(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ describes the future value at time t of an account given that nominal APR and an initial value of P .
- For continuous compounding, we can write $F = Pe^{rt}$, where e is Euler's number and r is the unit rate associated to the percent rate of change.

Problem Set

- Write each recursive sequence in explicit form. Identify each sequence as arithmetic, geometric, or neither.
 - $a_1 = 3, a_{n+1} = a_n + 5$
 - $a_1 = -1, a_{n+1} = -2a_n$
 - $a_1 = 30, a_{n+1} = a_n - 3$
 - $a_1 = \sqrt{2}, a_{n+1} = \frac{a_n}{\sqrt{2}}$
 - $a_1 = 1, a_{n+1} = \cos(\pi a_n)$
- Write each sequence in recursive form. Assume the first term is when $n = 1$.
 - $a_n = \frac{3}{2}n + 3$
 - $a_n = 3\left(\frac{3}{2}\right)^n$
 - $a_n = n^2$
 - $a_n = \cos(2\pi n)$

3. Consider two bank accounts. Bank A gives simple interest on an initial investment in savings accounts at a rate of 3% per year. Bank B gives compound interest on savings accounts at a rate of 2.5% per year. Fill out the following table.

Number of Years, n	Bank A Balance, a_n	Bank B Balance, b_n
0	\$1000.00	\$1000.00
1		
2		
3		
4		
5		

- What type of sequence do the Bank A balances represent?
 - Give both a recursive and an explicit formula for the Bank A balances.
 - What type of sequence do the Bank B balances represent?
 - Give both a recursive and an explicit formula for the Bank B balances.
 - Which bank account balance is increasing faster in the first five years?
 - If you were to recommend a bank account for a long-term investment, which would you recommend?
 - At what point is the balance in Bank B larger than the balance in Bank A?
4. You decide to invest your money in a bank that uses continuous compounding at 5.5% interest per year. You have \$500.
- Ja'mie decides to invest \$1,000 in the same bank for one year. She predicts she will have double the amount in her account than you will have. Is this prediction correct? Explain.
 - Jonas decides to invest \$500 in the same bank as well, but for two years. He predicts that after two years he will have double the amount of cash that you will after one year. Is this prediction correct? Explain.
5. Use the properties of exponents to identify the percent rate of change of the functions below, and classify them as representing exponential growth or decay. (The first two problems are done for you.)
- $f(t) = (1.02)^t$
 - $(t) = (1.01)^{12t}$
 - $f(t) = (0.97)^t$
 - $f(t) = 1000(1.2)^t$
 - $f(t) = \frac{(1.07)^t}{1000}$
 - $f(t) = 100 \cdot 3^t$
 - $f(t) = 1.05 \cdot \left(\frac{1}{2}\right)^t$
 - $f(t) = 80 \cdot \left(\frac{49}{64}\right)^{\frac{1}{2}t}$
 - $f(t) = 1.02 \cdot (1.13)^{\pi t}$

6. The effective rate of an investment is the percent rate of change per year associated to the nominal APR. The effective rate is very useful in comparing accounts with different interest rates and compounding periods. In general, the effective rate can be found with the following formula: $r_E = \left(1 + \frac{r}{k}\right)^k - 1$. The effective rate presented here is the interest rate needed for annual compounding to be equal to compounding n times per year.
- For investing, which account is better: an account earning a nominal APR of 7% compounded monthly or an account earning a nominal APR of 6.875% compounded daily? Why?
 - The effective rate formula for an account compounded continuously is $r_E = e^r - 1$. Would an account earning 6.875% interest compounded continuously be better than the accounts in part (a)?
7. Radioactive decay is the process in which radioactive elements decay into more stable elements. A half-life is the time it takes for half of an amount of an element to decay into a more stable element. For instance, the half-life for half of an amount of uranium-235 to transform into lead-207 is 704 million years. Thus, after 704 million years, only half of any sample of uranium-235 will remain, and the rest will have changed into lead-207. We will assume that radioactive decay is modeled by exponential decay with a constant decay rate.
- Suppose we have a sample of A g of uranium-235. Write an exponential formula that gives the amount of uranium-235 remaining after m half-lives.
 - Does the formula that you wrote in part (a) work for any radioactive element? Why?
 - Suppose we have a sample of A g of uranium-235. What is the decay rate per million years? Write an exponential formula that gives the amount of uranium-235 remaining after t million years.
 - How would you calculate the number of years it takes to get to a specific percentage of the original amount of material? For example, how many years will it take for there to be 80% of the original amount of uranium-235 remaining?
 - How many millions of years would it take 2.35 kg of uranium-235 to decay to 1 kg of uranium?
8. Doug drank a cup of tea with 130 mg of caffeine. Each hour, the caffeine in Doug's body diminishes by about 12%. (This rate varies between 6% and 14% depending on the person.)
- Write a formula to model the amount of caffeine remaining in Doug's system after each hour.
 - About how long will it take for the level of caffeine in Doug's system to drop below 30 mg?
 - The time it takes for the body to metabolize half of a substance is called a half-life. To the nearest 5 minutes, how long is the half-life for Doug to metabolize caffeine?
 - Write a formula to model the amount of caffeine remaining in Doug's system after m half-lives.
9. A study done from 1950 through 2000 estimated that the world population increased on average by 1.77% each year. In 1950, the world population was 2.519 billion.
- Write a function p for the world population t years after 1950.
 - If this trend continued, when should the world population have reached 7 billion?
 - The world population reached 7 billion October 31, 2011, according to the United Nations. Is the model reasonably accurate?
 - According to the model, when will the world population be greater than 12 billion people?

10. A particular mutual fund offers 4.5% nominal APR compounded monthly. Trevor wishes to deposit \$1,000.
- What is the percent rate of change per month for this account?
 - Write a formula for the amount Trevor will have in the account after m months.
 - Doubling time* is the amount of time it takes for an investment to double. What is the doubling time of Trevor's investment?
11. When paying off loans, the monthly payment first goes to any interest owed before being applied to the remaining balance. Accountants and bankers use tables to help organize their work.
- Consider the situation that Fred is paying off a loan of \$125,000 with an interest rate of 6% per year compounded monthly. Fred pays \$749.44 every month. Complete the following table:

Payment	Interest Paid	Principal Paid	Remaining Principal
\$749.44			
\$749.44			
\$749.44			

- Fred's loan is supposed to last for 30 years. How much will Fred end up paying if he pays \$749.44 every month for 30 years? How much of this is interest if his loan was originally for \$125,000?