Lesson 31: Credit Cards

Classwork

Mathematical Modeling Exercise

You have charged $\$1{,}500$ for the down payment on your car to a credit card that charges 19.99% annual interest, and you plan to pay a fixed amount toward this debt each month until it is paid off. We will denote the balance owed after the nth payment has been made as b_n.

a. What is the monthly interest rate, i? Approximate i to 5 decimal places.

b. You have been assigned to either the 50-team, the 100-team, or the 150-team, where the number indicates the size of the monthly payment R you will make toward your debt. What is your value of R?

c. Remember that you can make any size payment toward a credit card debt, as long as it is at least the minimum payment specified by the lender. Your lender calculates the minimum payment as the sum of 1% of the outstanding balance and the total interest that has accrued over the month or $\$25$, whichever is greater. Under these stipulations, what is the minimum payment? Is your monthly payment R at least as large as the minimum payment?

d. Complete the following table to show 6 months of payments.

Month, n	Interest Due	Payment, R	Paid to Principal	Balance, b_n
0				1500.00
1				
2				
3				
4				
5				
6				

e. Write a recursive formula for the balance b_n in month n in terms of the balance b_{n-1}.

f. Write an explicit formula for the balance b_n in month n, leaving the expression $1 + i$ in symbolic form.

g. Rewrite your formula in part (f) using r to represent the quantity $(1 + i)$.

h. What can you say about your formula in (g)? What term do we use to describe r in this formula?

i. Write your formula from part (g) in summation notation using Σ.

j. Apply the appropriate formula from Lesson 29 to rewrite your formula from part (g).

k. Find the month when your balance is paid off.

l. Calculate the total amount paid over the life of the debt. How much was paid solely to interest?

Problem Set

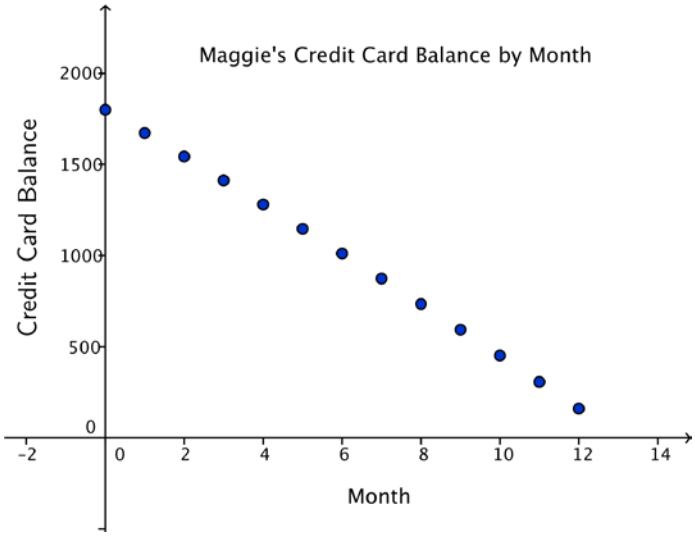
1. Suppose that you have a $2,000 balance on a credit card with a 29.99% annual interest rate, compounded monthly, and you can afford to pay $150 per month toward this debt.
   a. Find the amount of time it will take to pay off this debt. Give your answer in months and years.
   b. Calculate the total amount paid over the life of the debt.
   c. How much money was paid entirely to the interest on this debt?

2. Suppose that you have a $2,000 balance on a credit card with a 14.99% annual interest rate, and you can afford to pay $150 per month toward this debt.
   a. Find the amount of time it will take to pay off this debt. Give your answer in months and years.
   b. Calculate the total amount paid over the life of the debt.
   c. How much money was paid entirely to the interest on this debt?

3. Suppose that you have a $2,000 balance on a credit card with a 7.99% annual interest rate, and you can afford to pay $150 per month toward this debt.
   a. Find the amount of time it will take to pay off this debt. Give your answer in months and years.
   b. Calculate the total amount paid over the life of the debt.
   c. How much money was paid entirely to the interest on this debt?

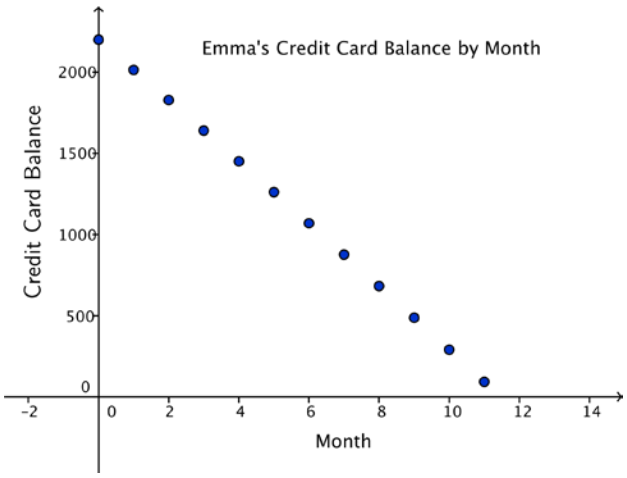
4. Summarize the results of Problems 1, 2, and 3.

5. Brendan owes $1,500 on a credit card with an interest rate of 12%. He is making payments of $100 every month to pay this debt off. Maggie is also making regular payments to a debt owed on a credit card, and she created the following graph of her projected balance over the next 12 months.
   a. Who has the higher initial balance? Explain how you know.
   b. Who will pay their debt off first? Explain how you know.



6. Alan and Emma are both making \$200 monthly payments toward balances on credit cards. Alan has prepared a table to represent his projected balances, and Emma has prepared a graph.

Alan's Credit Card Balance			
Month, n	Interest	Payment	Balance, b_n
0	——	——	2000.00
1	41.65	200	1841.65
2	38.35	200	1680.00
3	34.99	200	1514.99
4	31.55	200	1346.54
5	28.04	200	1174.58
6	24.46	200	999.04
7	20.81	200	819.85
8	17.07	200	636.92
9	13.26	200	450.18
10	9.37	200	259.55
11	5.41	200	64.96



a. What is the annual interest rate on Alan's debt? Explain how you know.

b. Who has the higher initial balance? Explain how you know.

c. Who will pay their debt off first? Explain how you know.

d. What do your answers to parts (a), (b), and (c) tell you about the interest rate for Emma's debt?

7. Both Gary and Helena are paying regular monthly payments to a credit card balance. The balance on Gary's credit card debt can be modeled by the recursive formula $g_n = g_{n-1}(1.01666) - 200$ with $g_0 = 2500$, and the balance on Helena's credit card debt can be modeled by the explicit formula $h_n = 2000(1.01666)^n - 250\left(\frac{1.01666^n - 1}{0.01666}\right)$ for $n \geq 0$.

a. Who has the higher initial balance? Explain how you know.

b. Who has the higher monthly payment? Explain how you know.

c. Who will pay their debt off first? Explain how you know.

8. In the next lesson, we will apply the mathematics we have learned to the purchase of a house. In preparation for that task, you need to come to class prepared with an idea of the type of house you would like to buy.

a. Research the median housing price in the county where you live or where you wish to relocate.

b. Find the range of prices that are within 25% of the median price from part (a). That is, if the price from part (a) was P, then your range is $0.75P$ to $1.25P$.

c. Look at online real estate websites, and find a house located in your selected county that falls into the price range specified in part (b). You will be modeling the purchase of this house in Lesson 32, so bring a printout of the real estate listing to class with you.

9. Select a career that interests you from the following list of careers. If the career you are interested in is not on this list, check with your teacher to obtain permission to perform some independent research. Once it has been selected, you will use the career to answer questions in Lesson 32 and Lesson 33.

Occupation	Median Starting Salary	Education Required
Entry-level full-time (waitstaff, office clerk, lawn care worker, etc.)	$18,000	High school diploma or GED
Accountant	$54,630	4-year college degree
Athletic Trainer	$36,560	4-year college degree
Chemical Engineer	$78,860	4-year college degree
Computer Scientist	$93,950	4-year college degree or more
Database Administrator	$64,600	4-year college degree
Dentist	$136,960	Graduate degree
Desktop Publisher	$34,130	4-year college degree
Electrical Engineer	$75,930	4-year college degree
Graphic Designer	$39,900	2- or 4-year college degree
HR Employment Specialist	$42,420	4-year college degree
HR Compensation Manager	$66,530	4-year college degree
Industrial Designer	$54,560	4-year college degree or more
Industrial Engineer	$68,620	4-year college degree
Landscape Architect	$55,140	4-year college degree
Lawyer	$102,470	Law degree
Occupational Therapist	$60,470	Master's degree
Optometrist	$91,040	Master's degree
Physical Therapist	$66,200	Master's degree
Physician—Anesthesiology	$259,948	Medical degree
Physician—Family Practice	$137,119	Medical degree
Physician's Assistant	$74,980	2 years college plus 2-year program
Radiology Technician	$47,170	2-year degree
Registered Nurse	$57,280	2- or 4-year college degree plus
Social Worker—Hospital	$48,420	Master's degree
Teacher—Special Education	$47,650	Master's degree
Veterinarian	$71,990	Veterinary degree