Lesson 33: The Million Dollar Problem

Classwork

Opening Exercise

In Problem 1 of the Problem Set of Lesson 32, you calculated the monthly payment for a 15-year mortgage at a 5% annual interest rate for the house you chose. You will need that monthly payment for these questions.

a. About how much do you expect your home to be worth in 15 years?

b. For $0 \leq x \leq 15$, plot the graph of the function $f(x) = P(1+r)^x$ where r is the appreciation rate and P is the initial value of your home.

c. Compare the image of the graph you plotted in part (b) with a partner, and write your observations of the differences and similarities. What do you think is causing the differences that you see in the graphs? Share your observations with another group to see if your conclusions are correct.

Your friend Julia bought a home at the same time as you but chose to finance the loan over 30 years. Julia also was able to avoid a down payment and financed the entire value of her home. This allowed her to purchase a more expensive home, but 15 years later she still has not paid off the loan. Consider the following amortization table representing Julia's mortgage, and answer the following questions by comparing the table with your graph.

Payment #	Beginning Balance	Payment on Interest	Payment on Principal
1	$\$145000$	$\$543.75$	$\$190.94$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
178	$\$96,784.14$	$\$362.94$	$\$371.75$
179	$\$96,412.38$	$\$361.55$	$\$373.15$
180	$\$96,039.23$	$\$360.15$	$\$374.55$

d. In Julia's neighborhood, her home has grown in value at around 2.95% per year. Considering how much she still owes the bank, how much of her home does she own after 15 years (the equity in her home)? Express your answer in dollars to the nearest thousand and as a percent of the value of her home.

e. Reasoning from your graph in part (b) and the table above, if both you and Julia sell your homes in 15 years at the homes' appreciated values, who would have more equity?

f. How much more do you need to save over 15 years to have assets over $\$1,000,000$?

Mathematical Modeling Exercises

Assume you can earn 7% interest annually, compounded monthly, in an investment account. Develop a savings plan so that you will have $\$1$ million in assets in 15 years (including the equity in your paid-off house).

1. Use your answer to Opening Exercise, part (g) as the future value of your savings plan.
   a. How much will you have to save every month to save up $\$1$ million in assets?

   b. Recall the monthly payment to pay off your home in 15 years (from Problem 1 of the Problem Set of Lesson 32). How much are the two together? What percentage of your monthly income is this for the profession you chose?

2. Write a report supported by the calculations you did above on how to save $1 million (or more) in your lifetime.

Problem Set

1. Consider the following scenario: You would like to save up $\$50{,}000$ after 10 years and plan to set up a structured savings plan to make monthly payments at 4.125% interest annually, compounded monthly.
   a. What lump sum amount would you need to invest at this interest rate in order to have $\$50{,}000$ after 10 years?
   b. Use an online amortization calculator to find the monthly payment necessary to take a loan for the amount in part (a) at this interest rate and for this time period.
   c. Use $A_f = R\left(\frac{(1+i)^n - 1}{i}\right)$ to solve for R.
   d. Compare your answers to part (b) and part (c). What do you notice? Why did this happen?

2. For structured savings plans, the future value of the savings plan as a function of the number of payments made at that point is an interesting function to examine. Consider a structured savings plan with a recurring payment of $\$450$ made monthly and an annual interest rate of 5.875% compounded monthly.
   a. State the formula for the future value of this structured savings plan as a function of the number of payments made. Use f for the function name.
   b. Graph the function you wrote in part (a) for $0 \leq x \leq 216$.
   c. State any trends that you notice for this function.
   d. What is the approximate value of the function f for $x = 216$?
   e. What is the domain of f? Explain.
   f. If the domain of the function is restricted to natural numbers, is the function a geometric sequence? Why or why not?
   g. Recall that the n^{th} partial sums of a geometric sequence can be represented with S_n. It is true that $f(x) = S_x$ for positive integers x, since it is a geometric sequence; that is, $S_x = \sum_{i=1}^{x} ar^i$. State the geometric sequence whose sums of the first x terms are represented by this function.
   h. April has been following this structured savings plan for 18 years. April says that taking out the money and starting over will not affect the total money earned because the interest rate does not change. Explain why April is incorrect in her reasoning.

3. Henry plans to have $\$195{,}000$ in property in 14 years and would like to save up to $\$1$ million by depositing $\$3{,}068.95$ each month at 6% interest per year, compounded monthly. Tina's structured savings plan over the same time span is described in the following table:
   a. Who has the higher interest rate? Who pays more every month?
   b. At the end of 14 years, who has more money from their structured savings plan? Does this agree with what you expected? Why or why not?
   c. At the end of 40 years, who has more money from their structured savings plan?

Deposit #	Amount Saved
30	\$110,574.77
31	\$114,466.39
32	\$118,371.79
33	\$122,291.02
34	\$126,224.14
⋮	⋮
167	\$795,266.92
168	\$801,583.49

4. Edgar and Paul are two brothers that both get an inheritance of $\$150{,}000$. Both plan to save up over $\$1{,}000{,}000$ in 25 years. Edgar takes his inheritance and deposits the money into an investment account earning 8% interest annually, compounded monthly, payable at the end of 25 years. Paul spends his inheritance but uses a structured savings plan that is represented by the sequence $b_n = 1275 + b_{n-1} \cdot \left(1 + \frac{0.0775}{12}\right)$ with $b_0 = 1275$ in order to save up over $\$1{,}000{,}000$.

   a. Which of the two has more money at the end of 25 years?

   b. What are the pros and cons of both brothers' plans? Which would you rather do? Why?