Lesson 3: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables

Classwork

Example 1

Students at Rufus King High School were discussing some of the challenges of finding space for athletic teams to practice after school. Part of the problem, according to Kristin, is that the females are more likely to be involved in after-school athletic programs than males. However, the athletic director assigns the available facilities as if males are more likely to be involved. Before suggesting changes to the assignments, the students decided to investigate.

Suppose the following information is known about Rufus King High School: 40% of students are involved in one or more of the after-school athletic programs offered at the school. It is also known that 58% of the school's students are female. The students decide to construct a **hypothetical 1000 two-way table**, like Table 1, to organize the data.

Table 1

Participation in after-school athletic program (Yes or No) by gender

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Females	Cell 1	Cell 2	Cell 3
Males	Cell 4	Cell 5	Cell 6
Total	Cell 7	Cell 8	Cell 9

Exercises 1–6

1. What cell in Table 1 represents a hypothetical group of $1{,}000$ students at Rufus King High School?

2. What cells in Table 1 can be filled based on the information given about the student population? Place these values in the appropriate cells of the table based on this information.

3. Based only on the cells you completed in Exercise 2, which of the following probabilities can be calculated, and which cannot be calculated? Calculate the probability if it can be calculated. If it cannot be calculated, indicate why.
   a. The probability that a randomly selected student is female.

   b. The probability that a randomly selected student participates in after school athletics programs.

   c. The probability that a randomly selected student who does not participate in the after school athletics program is male.

   d. The probability that a randomly selected male student participates in the after school athletics program.

4. The athletic director indicated that 23.2% of the students at Rufus King are females and participate in after school athletics programs. Based on this information, complete Table 1.

5. Consider the cells 1, 2, 4, and 5 of Table 1. Identify which of these cells represent students who are female or who participate in after-school athletic programs.

6. What cells of the two-way table represent students who are males who do not participate in after-school athletic programs?

Example 2

The completed hypothetical 1000 table organizes information in a way that makes it possible to answer various questions. For example, you can investigate whether females at the school are more likely to be involved in the after-school athletic programs.

Consider the following events:

- Let "A" represent the event "a randomly selected student is female."
- Let "not A" represent the "complement of A." The complement of A represents the event "a randomly selected student is not female," which is equivalent to the event "a randomly selected student is male."
- Let "B" represent the event "a randomly selected student participates in the after-school athletic program."
- Let "not B" represent the "complement of B." The complement of B represents the event "a randomly selected student does not participate in the after-school athletic program."
- Let "A or B" (described as A union B) represent the event "a randomly selected student is female or participates in the after-school athletic program."
- Let "A and B" (described as A intersect B) represent the event "a randomly selected student is female and participates in the after-school athletic program."

Exercises 7–9

7. Based on the descriptions above, describe the following events in words:
   a. Not A or Not B.

   b. A and Not B.

8. Based on the above descriptions and Table 1, determine the probability of each of the following events:
   a. A

   b. B

   c. Not A

d. Not B

e. A or B

f. A and B

9. Determine the following values:
   a. The probability of A plus the probability of Not A.

   b. The probability of B plus the probability of Not B.

   c. What do you notice about the results of parts (a) and (b)? Explain.

Example 3: Conditional Probability

Another type of probability is called a conditional probability. Pulling apart the two-way table helps to define a conditional probability.

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Females	Cell 1	Cell 2	Cell 3

Suppose that a randomly selected student is female. What is the probability that the selected student participates in the after-school athletic program? This probability is an example of what is called a conditional probability. This probability is calculated as the number of students who are female students and participate in the after-school athletic program (or the students in cell 1) divided by the total number of female students (or the students in cell 3).

Exercises 10–15

10. The following are also examples of conditional probabilities. Answer each question.
    a. What is the probability that if a randomly selected student is female, she participates in the after-school athletic program?

    b. What is the probability that if a randomly selected student is female, she does not participate in after-school athletics?

11. Describe two conditional probabilities that can be determined from the following row in Table 1.

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Males	Cell 4	Cell 5	Cell 6

12. Describe two conditional probabilities that can be determined from the following column in Table 1.

	Yes - Participate in After-School Athletic Program
Females	Cell 1
Males	Cell 4
Total	Cell 7

13. Determine the following conditional probabilities.

    a. A randomly selected student is female. What is the probability she participates in the after-school athletic program? Explain how you determined your answer.

    b. A randomly selected student is male. What is the probability he participates in the after-school athletic program?

    c. A student is selected at random. What is the probability this student participates in the after-school athletic program?

14. Based on the answers to Exercise 13, do you think that female students are more likely to be involved in after-school athletics? Explain your answer.

15. What might explain the concern females expressed in the beginning of this lesson about the problem of assigning space?

Lesson Summary

Data organized in a two-way frequency table can be used to calculate probabilities. The two-way frequency tables can also be used to calculate conditional probabilities.

In certain problems, probabilities that are known can be used to create a hypothetical 1000 two-way table. This hypothetical population of $1,000$ can be used to calculate conditional probabilities.

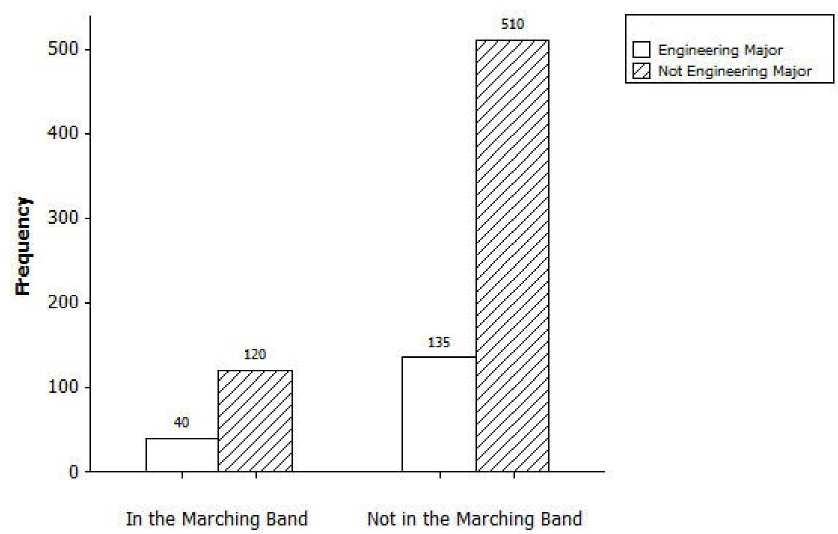
Probabilities are always interpreted by the context of the data.

Problem Set

Oostburg College has a rather large marching band. Engineering majors were heard bragging that students majoring in engineering are more likely to be involved in the marching band than students from other majors.

1. If the above claim is accurate, does that mean that most of the band is engineering students? Explain your answer.

2. The following graph was prepared to investigate the above claim.



Based on the graph, complete the following two-way frequency table:

	In the Marching Band	Not in the Marching Band	Total
Engineering major			
Not an engineering major			
Total			

3. Let M represent the event that a randomly selected student is in the marching band. Let E represent the event that a randomly selected student is an engineering major.
   a. Describe the event represented by the complement of M.
   b. Describe the event represented by the complement of E.
   c. Describe the event A and B (A intersect B).
   d. Describe the event A or B (A union B).

4. Based on the completed two-way frequency table, determine the following and explain how you got your answer.
   a. The probability that a randomly selected student is in the marching band.
   b. The probability that a randomly selected student is an engineering major.
   c. The probability that a randomly selected student is in the marching band and an engineering major.
   d. The probability that a randomly selected student is in the marching band and not an engineering major.

5. Indicate if the following conditional probabilities would be calculated using the rows or the columns of the two-way frequency table.
   a. A randomly selected student is majoring in engineering. What is the probability this student is in the marching band?
   b. A randomly selected student is not in the marching band. What is the probability that this student is majoring in engineering?

6. Based on the two-way frequency table, determine the following conditional probabilities.
   a. A randomly selected student is majoring in engineering. What is the probability that this student is in the marching band?
   b. A randomly selected student is not majoring in engineering. What is the probability that this student is in the marching band?

7. The claim that started this investigation was that students majoring in engineering are more likely to be in the marching band than students from other majors. Describe the conditional probabilities that would be used to determine if this claim is accurate.

8. Based on the two-way frequency table, calculate the conditional probabilities identified in Problem 7.

9. Do you think the claim that students majoring in engineering are more likely to be in the marching band than students for other majors is accurate? Explain your answer.

10. There are 40 students at Oostburg College majoring in computer science. Computer science is not considered an engineering major. Calculate an estimate of the number of computer science majors you think are in the marching band. Explain how you calculated your estimate.