

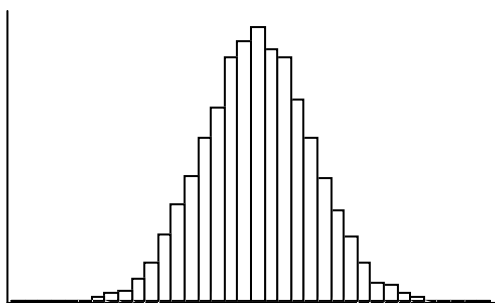
## Lesson 10: Normal Distributions

### Classwork

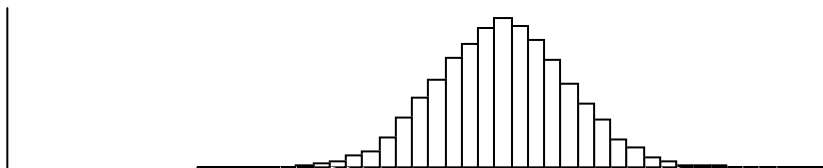
#### Exercise 1

Consider the following data distributions. In the previous lesson, you distinguished between distributions that were approximately normal and those that were not. For each of the following distributions, indicate if it is approximately normal, skewed, or neither, and explain your choice.

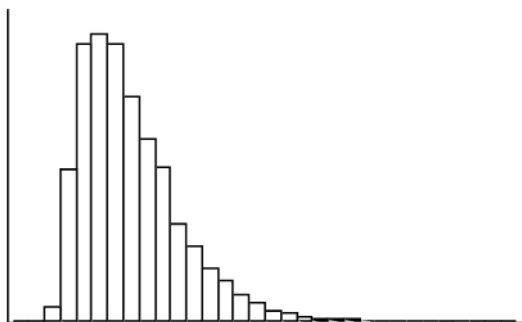
a.



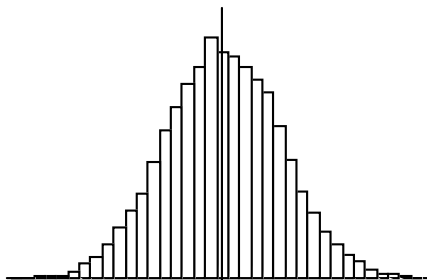
b.



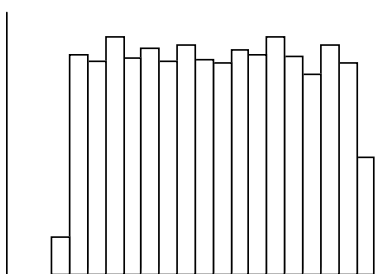
c.



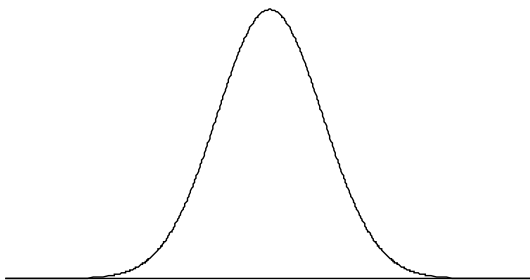
d.



e.



A normal distribution is a distribution that has a particular symmetric mound shape, as shown below.



### Exercise 2

When calculating probabilities associated with normal distributions,  $z$  scores are used. A  $z$  score for a particular value measures the number of standard deviations away from the mean. A positive  $z$  score corresponds to a value that is above the mean, and a negative  $z$  score corresponds to a value that is below the mean. The letter  $z$  was used to represent a variable that has a standard normal distribution where the mean is 0 and standard deviation is 1. This distribution was used to define a  $z$  score. A  $z$  score is calculated by

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

- a. The prices of the printers in a store have a mean of \$240 and a standard deviation of \$50. The printer that you eventually choose costs \$340.
- What is the  $z$  score for the price of your printer?
  - How many standard deviations above the mean was the price of your printer?
- b. Ashish's height is 63 inches. The mean height for boys at his school is 68.1 inches, and the standard deviation of the boys' heights is 2.8 inches.
- What is the  $z$  score for Ashish's height? (Round your answer to the nearest hundredth.)
  - What is the meaning of this value?
- c. Explain how a  $z$  score is useful in describing data.

**Example 1: Use of  $z$  Scores and a Graphing Calculator to find Normal Probabilities**

A swimmer named Amy specializes in the 50 meter backstroke. In competition her mean time for the event is 39.7 seconds, and the standard deviation of her times is 2.3 seconds. Assume that Amy's times are approximately normally distributed.

- a. Estimate the probability that Amy's time is between 37 and 44 seconds.

- b. Using z scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is between 37 and 44 seconds.
- c. Estimate the probability that Amy's time is more than 45 seconds.
- d. Using z scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is more than 45 seconds.
- e. What is the probability that Amy's time would be at least 45 seconds?
- f. Using z scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is less than 36 seconds.

**Exercise 3**

The distribution of lifetimes of a particular brand of car tires has a mean of 51,200 miles and a standard deviation of 8,200 miles.

- a. Assuming that the distribution of lifetimes is approximately normally distributed and rounding your answers to the nearest thousandth, find the probability that a randomly selected tire lasts
  - i. between 55,000 and 65,000 miles.
  - ii. less than 48,000 miles.
  - iii. at least 41,000 miles.
- b. Explain the meaning of the probability that you found in part (a–iii).

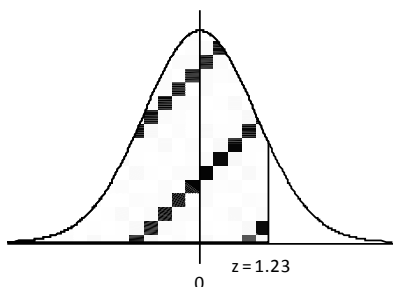
**Exercise 4**

Think again about the brand of tires described in Exercise 3. What is the probability that the lifetime of a randomly selected tire is within 10,000 miles of the mean lifetime for tires of this brand?

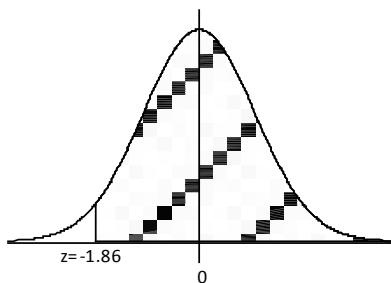
**Example 2: Using Table of Standard Normal Curve Areas**

The standard normal distribution is the normal distribution with a mean of 0 and a standard deviation of 1. The diagrams below show standard normal distribution curves. Use a table of standard normal curve areas to determine the shaded areas.

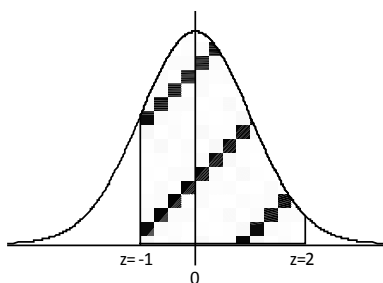
a.



b.



c.



**Lesson Summary**

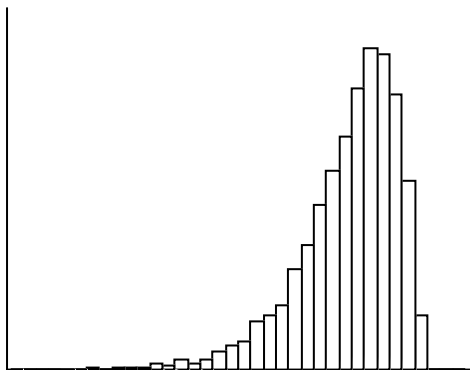
A normal distribution is a continuous distribution that has the particular symmetric mound-shaped curve that is shown at the beginning of the lesson.

Probabilities associated with normal distributions are determined using scores and can be found using a graphing calculator or tables of standard normal curve areas.

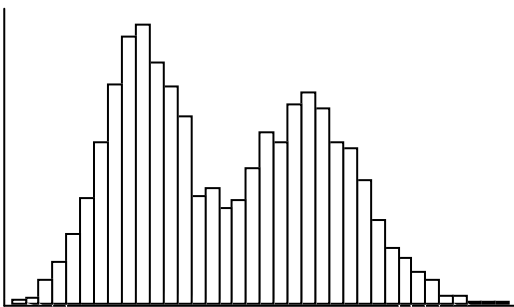
**Problem Set**

1. Which of the following histograms show distributions that are approximately normal?

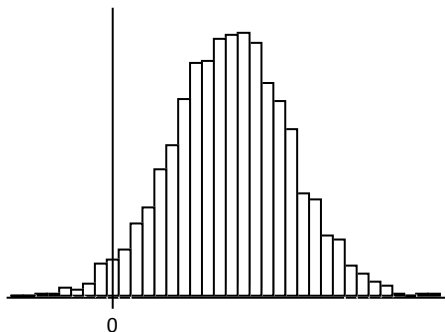
a.



b.



c.



2. Suppose that a particular medical procedure has a cost that is approximately normally distributed with a mean of \$19,800 and a standard deviation of \$2,900. For a randomly selected patient, find the probabilities of the following events. (Round your answers to the nearest thousandth.)
- The procedure costs between \$18,000 and \$22,000.
  - The procedure costs less than \$15,000.
  - The procedure costs more than \$17,250.
3. Consider the medical procedure described in the previous question, and suppose a patient is charged \$24,900 for the procedure. The patient is reported as saying, “I’ve been charged an outrageous amount!” How justified is this comment? Use probability to support your answer.
4. Think again about the medical procedure described in Question 2.
- Rounding your answers to the nearest thousandth, find that probability that, for a randomly selected patient, the cost of the procedure is
    - within two standard deviations of the mean cost.
    - more than one standard deviation from the mean cost.
  - If the mean or the standard deviation were to be changed, would your answers to part (a) be affected? Explain.
5. Use a table of standard normal curve areas to find
- the area to the left of  $z = 0.56$ .
  - the area to the right of  $z = 1.20$ .
  - the area to the left of  $z = -1.47$ .
  - the area to the right of  $z = -0.35$ .
  - the area between  $z = -1.39$  and  $z = 0.80$ .
  - Choose a response from parts (a) through (f), and explain how you determined your answer.