Lesson 11: Normal Distributions

Classwork

Example 1: Calculation of Normal Probabilities Using z scores and Tables of Standard Normal Areas

The U.S. Department of Agriculture, in its Official Food Plans (www.cnpp.usda.gov), states that the average cost of food for a 14–18-year-old male (on the "Moderate-cost" plan) is $\$261.50$ per month. Assume that the monthly food cost for a 14–18-year-old male is approximately normally distributed with a mean of $\$261.50$ and a standard deviation of $\$16.25$.

a. Use a table of standard normal curve areas to find the probability that the monthly food cost for a randomly selected 14–18 year old male is

i. less than $\$280$.

ii. more than $\$270$.

iii. more than $\$250$.

iv. between $240 and $275.

b. Explain the meaning of the probability that you found in part (a–iv).

Exercise 1

The USDA document described in Example 1 also states that the average cost of food for a 14–18 year old female (again, on the "Moderate-cost" plan) is $215.20 per month. Assume that the monthly food cost for a 14–18 year old female is approximately normally distributed with mean $215.20 and standard deviation $14.85.

a. Use a table of standard normal curve areas to find the probability that the monthly food cost for a randomly selected 14–18 year old female is

i. less than $225.

ii. less than $200.

iii. more than $\$250$.

iv. between $\$190$ and $\$220$.

b. Explain the meaning of the probability that you found in part (a–iv).

Example 2: Use of a Graphing Calculator to Find Normal Probabilities Directly

Return to the information given in Example 1. Using a graphing calculator, and *without* using z scores, find the probability (rounded to the nearest thousandth) that the monthly food cost for a randomly selected 14–18 year old male is

a. between $\$260$ and $\$265$.

b. at least $\$252$.

c. at most $248.

Exercise 2

Return to the information given in Exercise 1.

a. In Exercise 1, you calculated the probability that that the monthly food cost for a randomly selected 14–18 year old female is between $190 and $220. Would the probability that the monthly food cost for a randomly selected 14–18 year old female is between $195 and $230 be greater than or smaller than the probability for between $190 and $220? Explain your thinking.

b. Do you think that the probability that the monthly food cost for a randomly selected 14–18 year old female is between $195 and $230 is closer to 0.50, 0.75, or 0.90? Explain your thinking.

c. Using a graphing calculator, and without using z scores, find the probability (rounded to the nearest thousandth) that the monthly food cost for a randomly selected 14–18 year old female is between $195 and $230. Is this probability consistent with your answer to part (b)?

d. How does the probability you calculated in part (c) compare to the probability that would have been obtained using the table of normal curve areas?

e. What is one advantage to using a graphing calculator to calculate this probability?

f. In Exercise 1, you calculated the probability that the monthly food cost for a randomly selected 14–18 year old female is at most \$200. Would the probability that the monthly food cost for a randomly selected 14–18 year old female is at most $\$210$ be greater than or less than the probability for at most $\$200$? Explain your thinking.

g. Do you think that the probability that the monthly food cost for a randomly selected 14–18 year old female is at most $\$210$ is closer to 0.10, 0.30, or 0.50? Explain your thinking.

h. Using a graphing calculator, and without using z scores, find the probability (rounded to the nearest thousandth) that the monthly food cost for a randomly selected 14–18 year old female is at most $\$210$.

i. Using a graphing calculator, and without using z scores, find the probability (rounded to the nearest thousandth) that the monthly food cost for a randomly selected 14–18 year old female is at least $\$235$.

Example 3: Using a Spreadsheet to Find Normal Probabilities

Return to the information given in Example 1. The U.S. Department of Agriculture, in its Official Food Plans (www.cnpp.usda.gov), states that the average cost of food for a 14–18-year-old male (on the "moderate-cost" plan) is $\$261.50$ per month. Assume that the monthly food cost for a 14–18-year-old male is approximately normally distributed with a mean of $\$261.50$ and a standard deviation of $\$16.25$. Round your answers to four decimal places.

Use a spreadsheet to find the probability that the monthly food cost for a randomly selected $14-18$ year old male is

a. less than $\$280$.

b. more than $\$270$.

c. more than $\$250$.

d. between $\$240$ and $\$275$.

Exercise 3

The USDA document described in Example 1 also states that the average cost of food for a 14–18 year old female (again, on the "moderate-cost" plan) is $\$215.20$ per month. Assume that the monthly food cost for a 14–18 year old female is approximately normally distributed with a mean of $\$215.20$ and a standard deviation of $\$14.85$. Round your answers to 4 decimal places.

Use a spreadsheet to find the probability that the monthly food cost for a randomly selected 14–18 year old female is

a. less than $\$225$.

b. less than $\$200$.

c. more than $\$250$.

d. between $\$190$ and $\$220$.

Exercise 4

The reaction times of 490 people were measured. The results are shown in the frequency distribution below.

Reaction Time (seconds)	0.1 to < 0.15	0.15 to < 0.2	0.2 to < 0.25	0.25 to < 0.3	0.3 to < 0.35	0.35 to < 0.4
Frequency	9	82	220	138	37	4

a. Construct a histogram that displays these results.

b. Looking at the histogram, do you think a normal distribution would be an appropriate model for this distribution?

c. The mean of the reaction times for these 490 people is 0.2377, and the standard deviation of the reaction times is 0.0457. For a normal distribution with this mean and standard deviation, what is the probability that a randomly selected reaction time is at least 0.25?

d. The actual proportion of these 490 people who had a reaction time that was at least 0.25 is 0.365 (this can be calculated from the frequency distribution). How does this proportion compare to the probability that you calculated in part (c)? Does this confirm that the normal distribution is an appropriate model for the reaction time distribution?

Lesson Summary

Probabilities associated with normal distributions can be found using z scores and tables of standard normal curve areas.

Probabilities associated with normal distributions can be found directly (without using z scores) using a graphing calculator.

When a data distribution has a shape that is approximately normal, a normal distribution can be used as a model for the data distribution. The normal distribution with the same mean and the standard deviation as the data distribution is used.

Problem Set

1. Use a table of standard normal curve areas to find
   a. the area to the left of $z = 1.88$.
   b. the area to the right of $z = 1.42$.
   c. the area to the left of $z = -0.39$.
   d. the area to the right of $z = -0.46$.
   e. the area between $z = -1.22$ and $z = -0.5$.

2. Suppose that the durations of high school baseball games are approximately normally distributed with mean 105 minutes and standard deviation 11 minutes. Use a table of standard normal curve areas to find the probability that a randomly selected high school baseball game lasts
   a. less than 115 minutes.
   b. more than 100 minutes.
   c. between 90 and 110 minutes.

3. Using a graphing calculator, and *without* using z values, check your answers to Problem 2. (Round your answers to the nearest thousandth
   a.
   b.
   c.

4. In Problem 2, you were told that the durations of high school baseball games are approximately normally distributed with mean 105 minutes and standard deviation 11 minutes. Suppose also that the durations of high school softball games are approximately normally distributed with a mean of 95 minutes and the same standard deviation, 11 minutes. Is it more likely that a high school baseball game will last between 100 and 110 minutes or that a high school softball game will last between 100 and 110 minutes? Answer this question without doing any calculations!

5. A farmer has 625 female adult sheep. The sheep have recently been weighed, and the results are shown in the table below.

Weight (pounds)	140 to < 150	150 to < 160	160 to < 170	170 to < 180	180 to < 190	190 to < 200	200 to < 210
Frequency	8	36	173	221	149	33	5

a. Construct a histogram that displays these results.

b. Looking at the histogram, do you think a normal distribution would be an appropriate model for this distribution?

c. The weights of the 625 sheep have mean 174.21 pounds and standard deviation 10.11 pounds. For a normal distribution with this mean and standard deviation, what is the probability that a randomly selected sheep has a weight of at least 190 pounds? (Round your answer to the nearest thousandth.)