

Lesson 14: Sampling Variability in the Sample Proportion

Classwork

Example 1: Polls

A recent poll stated that 40% of Americans pay “a great deal” or a “fair amount” of attention to the nutritional information that restaurants provide. This poll was based on a random sample of 2,027 adults living in the U.S.

The 40% corresponds to a proportion of 0.40, and 0.40 is called a sample proportion. It is an estimate of the proportion of all adults who would say they pay “a great deal” or a “fair amount” of attention to the nutritional information that restaurants provide. If you were to take a random sample of 20 Americans, how many would you predict would say that they pay attention to nutritional information? In this lesson, you will investigate this question by generating distributions of sample proportions and investigating patterns in these distributions.

Your teacher will give your group a container of dried beans. Some of the beans in the container are black. With your classmates, you are going to see what happens when you take a sample of beans from the container and use the proportion of black beans in the sample to estimate the proportion of black beans in the container (a population proportion).

Exploratory Challenge 1/Exercises 1-9

1. Each person in the group should randomly select a sample of 20 beans from the container by carefully mixing all the beans and then selecting one bean and recording its color. Replace the bean, mix the bag, and continue to select one bean at a time until 20 beans have been selected. Be sure to replace each bean and mix the bag before selecting the next bean. Count the number of black beans in your sample of 20.
2. What is the proportion of black beans in your sample of 20? (Round your answer to 2 decimal places.) This value is called the sample proportion of black beans.

3. Write your sample proportion on a post-it note, and place the note on the number line that your teacher has drawn on the board. Place your note above the value on the number line that corresponds to your sample proportion.

The graph of all the students' sample proportions is called a distribution of the sampling distribution of sample proportions. This sampling distribution is an approximation of the actual sampling distribution of all possible samples of size 20.

4. Describe the shape of the distribution.
5. What was the smallest sample proportion observed?
6. What was the largest sample proportion observed?
7. What sample proportion occurred most often?
8. Using technology, find the mean and standard deviation of the sample proportions used to construct the sampling distribution created by the class.
9. How does the mean of the sampling distribution compare with the population proportion of 0.40?

Example 2: Sampling Variability

What do you think would happen to the sampling distribution if everyone in class took a random sample of 40 beans from the container? To help answer this question, you will repeat the process described in Example 1, but this time you will draw a random sample of 40 beans instead of 20.

Exploratory Challenge 2/Exercises 10-21

10. Take a random sample with replacement of 40 beans from the container. Count the number of black beans in your sample of 40 beans.
11. What is the proportion of black beans in your sample of 40? (Round your answer to 2 decimal places.)
12. Write your sample proportion on a post-it note, and place it on the number line that your teacher has drawn on the board. Place your note above the value on the number line that corresponds to your sample proportion.
13. Describe the shape of the distribution.
14. What was the smallest sample proportion observed?
15. What was the largest sample proportion observed?
16. What sample proportion occurred most often?
17. Using technology, find the mean and standard deviation of the sample proportions used to construct the sampling distribution created by the class.

18. How does the mean of the sampling distribution compare with the population proportion of 0.40?
19. How does the mean of the sampling distribution based on random samples of size 20 compare to the mean of the sampling distribution based on random samples of size 40?
20. As the sample size increased from 20 to 40 describe what happened to the sampling variability (standard deviation of the distribution of sample proportions)?
21. What do you think would happen to the variability (standard deviation) of the distribution of sample proportions if the sample size for each sample were 80 instead of 40? Explain.

Lesson Summary

The sampling distribution of the sample proportion can be approximated by a graph of the sample proportions for many different random samples. The mean of the sampling distribution of the sample proportions will be approximately equal to the value of the population proportion.

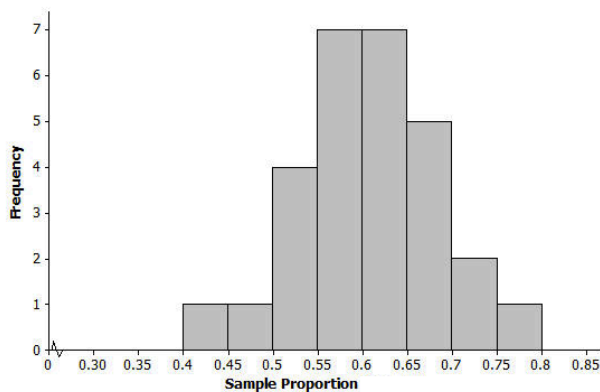
As the sample size increases, the sampling variability in the sample proportion decreases – the standard deviation of the sampling distribution of the sample proportions decreases.

Problem Set

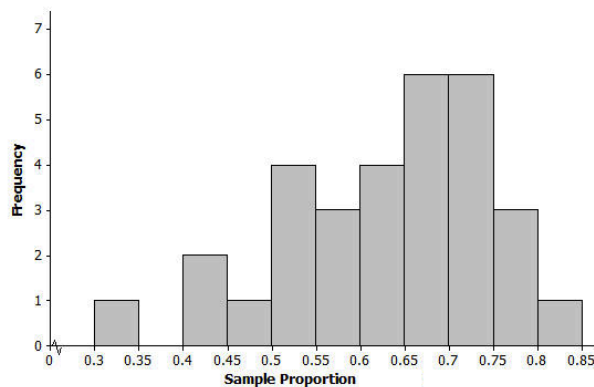
1. A class of 28 eleventh graders wanted to estimate the proportion of all juniors and seniors at their high school with part-time jobs after school. Each eleventh grader took a random sample of 30 juniors and seniors and then calculated the proportion with part-time jobs. Following are the 28 sample proportions.
0.7, 0.8, 0.57, 0.63, 0.7, 0.47, 0.67, 0.67, 0.8, 0.77, 0.4, 0.73, 0.63, 0.67, 0.6, 0.77, 0.77, 0.77, 0.53, 0.57, 0.73, 0.7, 0.67, 0.7, 0.77, 0.57, 0.77, 0.67
 - a. Construct a dot plot of the sample proportions.
 - b. Describe the shape of the distribution.
 - c. Using technology, find the mean and standard deviation of the sample proportions.
 - d. Do you think that the proportion of all juniors and seniors at the school with part-time jobs could be 0.7? Do you think it could be 0.5? Justify your answers based on your dot plot.
 - e. Suppose the eleventh graders had taken random samples of size 60. How would the distribution of sample proportions based on samples of size 60 differ from the distribution for samples of size 30?

2. A group of eleventh graders wanted to estimate the proportion of all students at their high school who suffer from allergies. Each student in one group of eleventh graders took a random sample of 20 students, while another group of eleventh graders each took a random sample of 40 students. Below are the two sampling distributions (shown as histograms) of the sample proportions of high school students who said that they suffer from allergies. Which histogram is based on random samples of size 40? Explain.

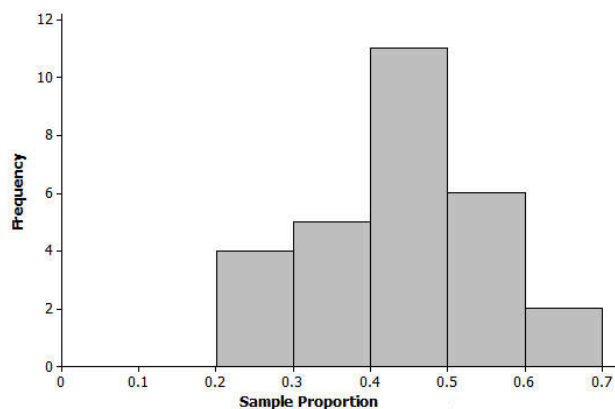
Histogram A



Histogram B



3. The nurse in your school district would like to study the proportion of all high school students in the district who usually get at least eight hours of sleep on school nights. Suppose each student in your class takes a random sample of 20 high school students in the district and each calculates their sample proportion of students who said that they usually get at least eight hours of sleep on school nights. Below is a histogram of the sampling distribution.



- a. Do you think that the proportion of all high school students who usually get at least eight hours of sleep on school nights could have been 0.4? Do you think it could have been 0.55? Could it have been 0.75? Justify your answers based on the histogram.

- b. Suppose students had taken random samples of size 60. How would the distribution of sample proportions based on samples of size 60 differ from those of size 20?