Lesson 15: Sampling Variability in the Sample Proportion

Classwork

Example 1

A high school principal claims that 50% of the school's students walk to school in the morning. A student attempts to verify the principal's claim by taking a random sample of 40 students and asking them if they walk to school in the morning. Sixteen of the sampled students say they usually walk to school in the morning, giving a sample proportion of $\frac{16}{40} = 0.40$, which seem to dispel the principal's claim of 50%. But could the principal be correct that the proportion of all students who walk to school is 50%?

a. Make a conjecture about the answer.

b. Develop a plan for how to respond.

Help the student make a decision on the principal's claim by investigating what kind of sample proportions you would expect to see if the principal's claim of **50**% is true. You will do this by using technology to simulate the flipping of a coin **40** times.

Exploratory Challenge 1/Exercises 1-9

In Exercises 1–9, students should assume that the principal is correct that 50% of the population of students walk to school. Designate heads to represent a student who walks to school.

1. Simulate 40 flips of a fair coin. Record your observations in the space below.

2. What is the sample proportion of heads in your sample of 40? Report this value to your teacher.

3. Repeat Exercises 1 and 2 to obtain a second sample of 40 coin flips.

Your teacher will display a graph of all the students' sample proportions of heads.

4. Describe the shape of the distribution.

5. What was the smallest sample proportion observed?

6. What was the largest sample proportion observed?

7. Estimate the center of the distribution of sample proportions.

Your teacher will report the mean and standard deviation of the sampling distribution created by the class.

8. How does the mean of the sampling distribution compare with the population proportion of 0.50?

9. Recall that a student took a random sample of 40 students and found that the sample proportion of students who walk to school was 0.40. Would this have been a surprising result if the actual population proportion were 0.50 as the principal claims?

Example 2: Sampling Variability

What do you think would happen to the sampling distribution you constructed in the previous exercises had everyone in class taken a random sample of size 80 instead of 40? Justify your answer. This will be investigated in the following exercises.

Exploratory Challenge 2/Exercises 10-22

10. Use technology and simulate 80 coin flips. Calculate the proportion of heads. Record your results in the space below.

11. Repeat flipping a coin 80 times until you have recorded a total of 40 sample proportions.

.

12. Construct a dot plot of the 40 sample proportions.

13. Describe the shape of the distribution.

14. What was the smallest proportion of heads observed?

15. What was the largest proportion of heads observed?

16. Using technology, find the mean and standard deviation of the distribution of sample proportions.

17. Compare your results with the others in your group. Did you have similar means and standard deviations?

18. How does the mean of the sampling distribution based on 40 simulated flips of a coin (Exercise 1) compare to the mean of the sampling distribution based on 80 simulated coin flips?

19. Describe what happened to the sampling variability (standard deviation) of the distribution of sample proportions as the number of simulated coin flips increased from 40 to 80.

20. What do you think would happen to the variability (standard deviation) of the distribution of sample proportions if the sample size for each sample were 200 instead of 80? Explain.

21. Recall that a student took a random sample of 40 students and found that the sample proportion of students who walk to school was 0.40. If the student had taken a random sample of 80 students instead of 40, would this have been a surprising result if the actual population proportion was 0.50 as the principal claims?

22. What do you think would happen to the sampling distribution you constructed in the previous exercises if everyone in class took a random sample of size 80 instead of 40? Justify your answer.

Lesson Summary

The sampling distribution of the sample proportion can be approximated by a graph of the sample proportions for many different random samples. The mean of the sample proportions will be approximately equal to the value of the population proportion.

As the sample size increases, the sampling variability in the sample proportion decreases – the standard deviation of the sample proportions decreases.

Problem Set

1. A student conducted a simulation of 30 coin flips. Below is a dot plot of the sampling distribution of the proportion of heads. This sampling distribution has a mean of 0.51 and a standard deviation of 0.09.

    Sample Proportion (n = 30)
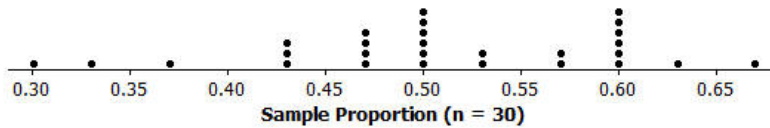
    a. Describe the shape of the distribution.
    b. Describe what would have happened to the mean and the standard deviation of the sampling distribution of the sample proportions if the student had flipped a coin 50 times, calculated the proportion of heads, and then repeated this process for a total of 30 times.

2. What effect does increasing the sample size have on the mean of the sampling distribution?

3. What effect does increasing the sample size have on the standard deviation of the sampling distribution?

4. A student wanted to decide whether or not a particular coin was fair (i.e., the probability of flipping a head is 0.5). She flipped the coin 20 times, calculated the proportion of heads, and repeated this process a total of 40 times. Below is the sampling distribution of sample proportions of heads. The mean and standard deviation of the sampling distribution is 0.379 and 0.091. Do you think this was a fair coin? Why or why not?

    Sample Proportion

---
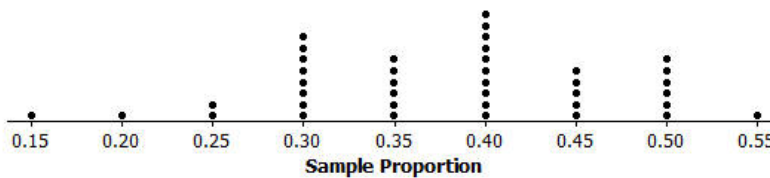
5. The same student flipped the coin 100 times, calculated the proportion of heads, and repeated this process a total of 40 times. Below is the sampling distribution of sample proportions of heads. The mean and standard deviation of the sampling distribution is 0.405 and 0.046. Do you think this was a fair coin? Why or why not?

Sample Proportion