Lesson 16: Margin of Error when Estimating a Population Proportion

Classwork
Exercises 1-4

In this lesson you will use data from a random sample drawn from a mystery bag to estimate a population proportion and learn how to find and interpret a margin of error for your estimate.

1. Write down your estimate for the proportion of red chips in the mystery bag based on the random sample of 30 chips drawn in class.

2. Tanya and Raoul had a paper bag that contained red and black chips. The bag was marked 40% red chips. They drew random samples of 30 chips, with replacement, from the bag. (They were careful to shake the bag after they replaced a chip.) They had nine red chips in their sample. They drew another random sample of 30 chips from the bag, and this time they had 12 red chips. They repeated this sampling process 50 times and made a plot of the number of red chips in each sample. A plot of their sampling distribution is shown below.
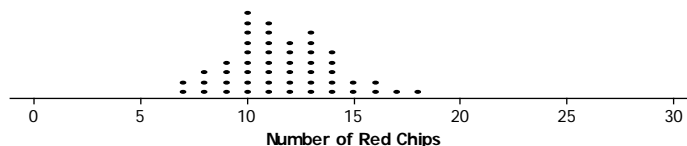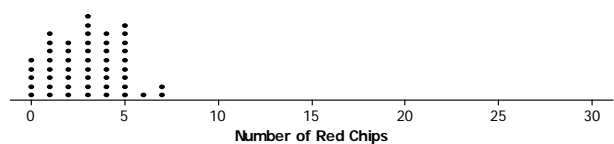

Number of Red Chips

a. What was the most common number of red chips in the 50 samples? Does this seem reasonable? Why or why not?

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b. What number of red chips, if any, never occurred in any of the samples?

c. Give an interval that contains the "likely" number of red chips in samples of size 30 based on the simulated sampling distribution.

d. Do you think the number of red chips in the mystery bag could have come from a sample drawn from a bag that had 40% red chips? Why or why not?

Nine different bags of chips were distributed to small teams of students in the class. Each bag had a different proportion of red chips. Each team simulated drawing 50 different random samples of size 30 from their bag and recorded the number of red chips for each sample. The graphs of their simulated sampling distributions are shown below.
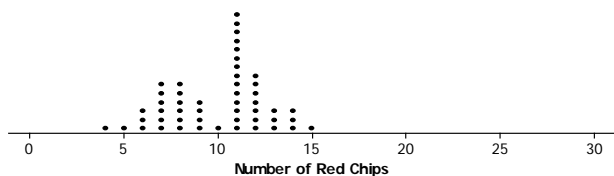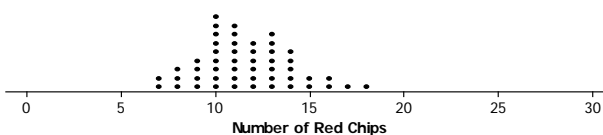
Population 10% red chips

Number of Red Chips

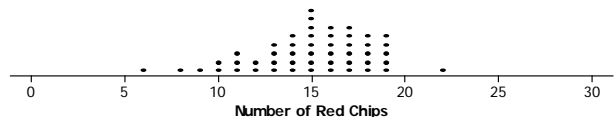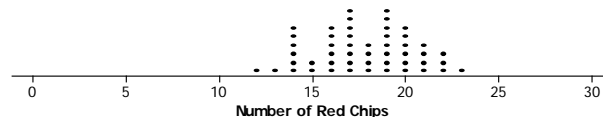Population 20% red chips
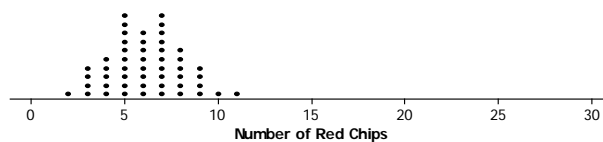
Number of Red Chips

Population 30% red chips

Number of Red Chips

Population 40% red chips

Number of Red Chips

Population 50% red chips

Number of Red Chips

Population 60% red chips

Number of Red Chips

Population 70% red chips
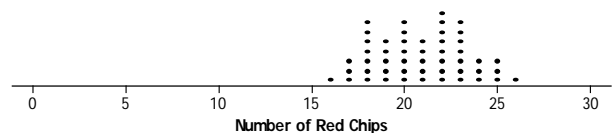
Number of Red Chips

Population 80% red chips
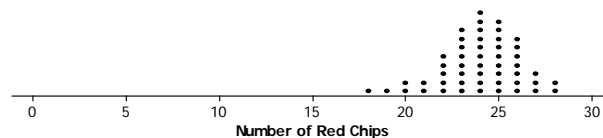
Number of Red Chips

Population 90% red chips
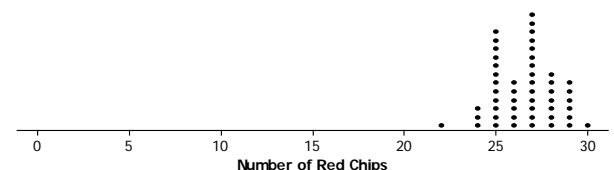
Number of Red Chips

3. Think about the number of red chips in the random sample of size 30 that was drawn from the mystery bag.

   a. Based on the simulated sampling distributions, do you think that the mystery bag might have had 10% red chips? Explain your reasoning.

   b. Based on the simulated sampling distributions, which of the percentages 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90% might reasonably be the percentage of red chips in the mystery bag?

   c. Let p represent the proportion of red chips in the mystery bag. (For example, $p = 0.40$ if there are 40% red chips in the bag.) Based on your answer to part (b), write an inequality that describes plausible values for p. Interpret the inequality in terms of the mystery bag population.

4. If the inequality like the one you described in part (c) of Exercise 3 went from 0.30 to 0.60, it is sometimes written as 0.45 ± 0.15. The value 0.15 is called a "margin of error." The margin of error represents an interval from the expected proportion that would not contain any proportions or very few proportions based on the simulated sampling distribution. Proportions in this interval are not expected to occur when taking a sample from the mystery bag.

   a. Write the inequality you found in Exercise 3 part (c) using this notation. What is the margin of error?

   b. Suppose Sol said, "So this means that the actual proportion of red chips in the mystery bag was 60%." Tonya argued that the actual proportion of red chips in the mystery bag was 20%. What would you say?

Exploratory Challenge 2: Samples of Size 50/Exercises 5–7

5. Do you think the "margin of error" would be different in Exercise 4 if you had sampled 50 chips instead of 30? Try to convince a partner that your conjecture is correct.

6. Below are simulated sampling distributions of the number of red chips for samples of size 50 from populations with various percentages of red chips.

Population with **10**% red chips

Number of Red Chips
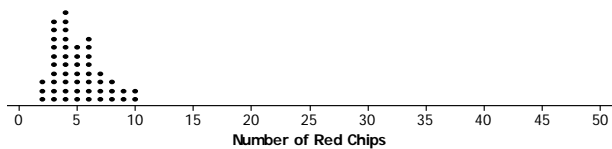
Population with **20**% red chips

Number of Red Chips
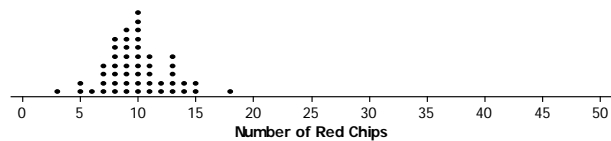
Population with **30**% red chips

Number of Red Chips
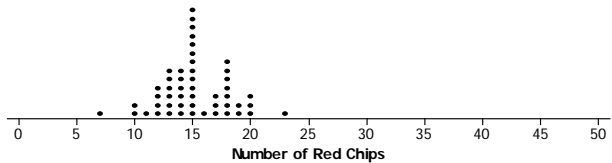
Population with **40**% red chips

Number of Red Chips
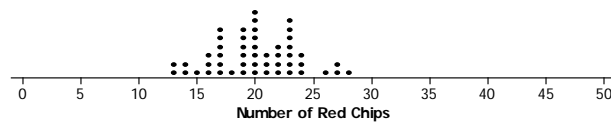
Population with **50**% red chips

Number of Red Chips
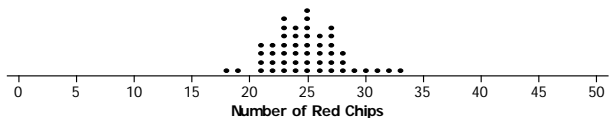
Population with **60**% red chips

Number of Red Chips
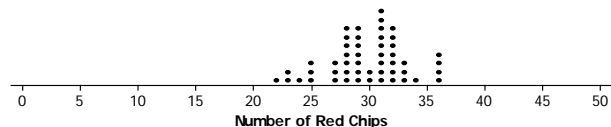
Population with **70**% red chips
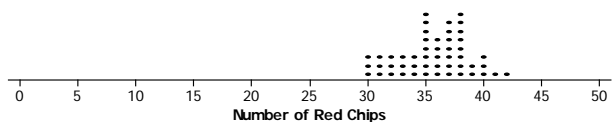
Population with **80**% red chips
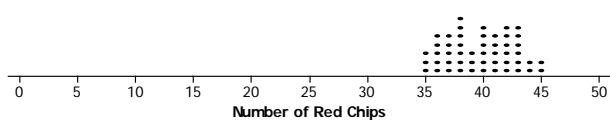
Population with **90**% red chips
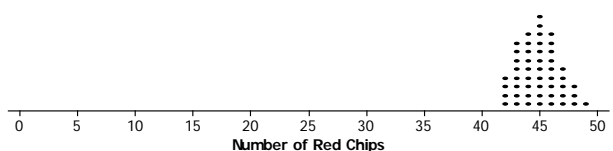
a. Suppose you drew 30 red chips in a random sample of 50 from the mystery bag. What are plausible values for the proportion of red chips in the mystery bag? Explain your reasoning.

b. Write an expression that contains the margin of error based on your answer to part (a).

7. Remember your conjecture from Exercise 5, and compare the margin of error you found for a sample of size 30 (from Exercise 3) to the margin of error you found for a sample of size 50.

a. Was your reasoning in Exercise 5 correct? Why or why not?

b. Explain why the change in the margin of error makes sense.

> **Lesson Summary**
>
> In this lesson, you investigated how to make an inference about an unknown population proportion based on a random sample from that population.
>
> - You learned how random samples from populations with known proportions of successes behave by simulating sampling distributions for samples drawn from those populations.
> - Comparing an observed proportion of successes from a random sample drawn from a population with an unknown proportion of successes to these sampling distributions gives you some information about what populations might produce a random sample like the one you observed.
> - These plausible population proportions can be described as $p \pm M$. The value of M is called a margin of error.

Problem Set

1. Tanya simulated drawing a sample of size 30 from a population of chips and got the following simulated sampling distribution for the number of red chips:

Number of Red Chips
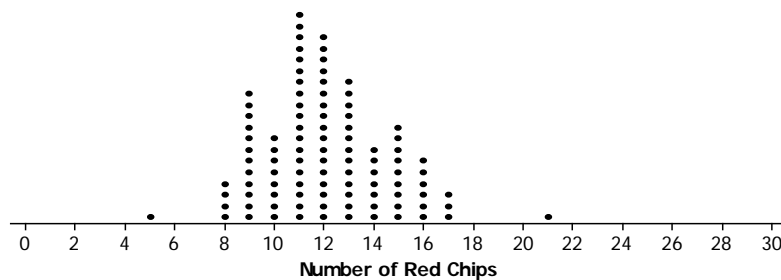
Which of the following results seem like they might have come from this population? Explain your reasoning.

  I.   8 red chips in a random sample of size 30.

  II.   12 red chips in a random sample of size 30.

  III.   24 red chips in a random sample of size 30.

2. 64% percent of the students in a random sample of 100 high school students intended to go onto college. The graphs below show the result of simulating random samples of size 100 from several different populations where the success percentage was known and recording the percentage of successes in the sample.

(i)   Population with **40**% successes

Number of Successes

(ii)   Population with **50**% successes

Number of Successes

(iii)   Population with **60**% successes

Number of Successes

(iv)   Population with **70**% successes

Number of Successes
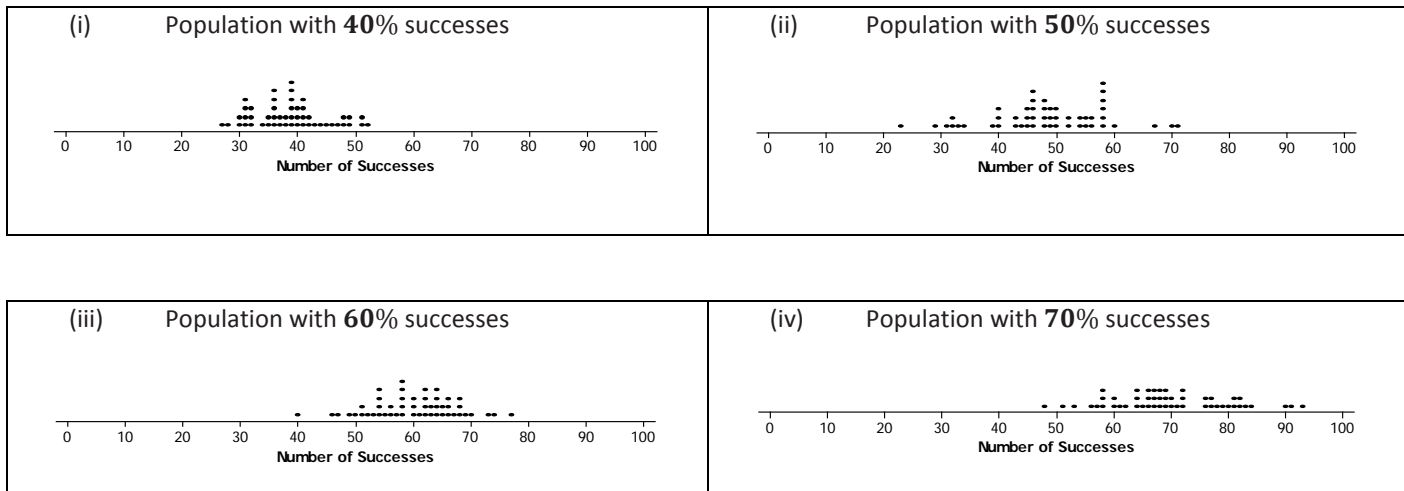
a. Based on these graphs, which of the following are plausible values for the percentage of successes in the population from which the sample was selected: 40%, 50%, 60%, or 70%? Explain your thinking.

b. Would you need more information to determine plausible values for the actual proportion of the population of high school students who intend to go to some postsecondary school? Why or why not?

3. Suppose the mystery bag had resulted in the following number of red chips. Using the simulated sampling distributions found earlier in this lesson, find a margin of error in each case.

a. The number of red chips in a random sample of size 30 was 10.

b. The number of red chips in a random sample of size 30 was 21.

c. The number of red chips in a random sample of size 50 was 22.

4. The following intervals were plausible population proportions for a given sample. Find the margin of error in each case.

a. from 0.35 to 0.65

b. from 0.72 to 0.78

c. from 0.84 to 0.95

d. from 0.47 to 0.57

5. Decide if each of the following statements is true or false. Explain your reasoning in each case.

   a. The smaller the sample size, the smaller the margin of error.

   b. If the margin of error is 0.05 and the observed proportion of red chips is 0.35, then the true population proportion is likely to be between 0.40 and 0.50.

6. Extension: The margin of error for a sample of size 30 is 0.20; for a sample of 50, is 0.10. If you increase the sample size to 70, do you think the margin of error for the percent of successes will be 0.05? Why or why not?