Lesson 21: Margin of Error when Estimating a Population Mean

Classwork

This lesson continues to discuss using the sample mean as an estimate of the population mean and judging its accuracy based on the concept of margin of error. In the last lesson, the margin of error was defined as twice the standard deviation of the sampling distribution of the sample mean. In this lesson, a formula will be given for the margin of error that allows you to calculate the margin of error from a single random sample rather than having to create a sampling distribution of sample means.

Example 1: Estimating a Population Mean Using a Random Sample

Provide a one-sentence summary of our findings from the previous lesson.

What were drawbacks of the calculation method?

In practice, you do not have to use that process to find the margin of error. Fortunately, just as was the case with estimating a population proportion, there are some general results that lead to a formula that allows you to estimate the margin of error using a single sample. You can then gauge the accuracy of your estimate of the population mean by calculating the margin of error using the sample standard deviation.

> The standard deviation of the distribution of sample means is approximated by $\frac{s}{\sqrt{n}}$, where s is the standard deviation of the sample, and n is the size of the sample.

Exercises 1–5

1. Suppose a random sample of size ten produced the following ratings in the computer games rating example in the last lesson: $12, 5, 2, 4, 1, 4, 18, 10, 1, 16$. Estimate the population mean rating based on these ten sampled ratings.

2. Calculate the sample standard deviation. Round your answer to three decimal places.

3. Use the formula given above to calculate the approximate standard deviation of the distribution of sample means. Round your answer to three decimal places.

4. Recall that the margin of error is twice the standard deviation of the distribution of sample means. What is the value of the margin of error based on this sample? Write a sentence interpreting the value of the margin of error in the context of this problem on computer game ratings.

5. Based on the sample mean and the value of the margin of error, what is an interval of plausible values for the population mean?

Exercises 6-13

The Gettysburg Address is considered one of history's greatest speeches. Some students noticed that the speech was very short (about 268 words, depending on the version) and wondered if the words were also relatively short. To estimate the mean length of words in the population of words in the Gettysburg Address, work with a partner on the following steps. Your teacher will give you a copy of the Gettysburg Address with words numbered from 001 to 268.

6. Develop and describe a plan for collecting data from the Gettysburg Address and determining the typical length of a word. Then, implement your plan, and report your findings.

7. Use a random-number table or a calculator with a random-number generator to obtain ten different random numbers from 001 to 268.

8. Use the random numbers found in Exercise 7 as identification numbers for the words that will make up your random sample of words from the Gettysburg Address. Make a list of the ten words in your sample.

9. Count the number of letters in each of the ten words in your sample.

10. Calculate the sample mean number of letters for the ten words in your sample.

11. Calculate the sample standard deviation of the number of letters for the ten words in your sample. Round your answer to three decimal places.

12. Use the sample standard deviation from Exercise 11 to calculate the margin of error associated with using your sample mean as an estimate of the population mean.   Round your answer to three decimal places.

13. Write a few sentences describing what you have learned about the mean length of the population of 268 words in the Gettysburg Address. Be sure to include an interpretation of the margin of error.

Lesson Summary

- When using the sample mean to estimate a population mean, it is important to know something about how accurate that estimate might be.
- Accuracy can be described by the margin of error.
- The margin of error can be estimated using data from a single random sample (without the need to create a simulated sampling distribution) by using the formula $2\left(\frac{s}{\sqrt{n}}\right)$, where s is the standard deviation of a single sample, and n is the sample size.

Problem Set

1. A new brand of hot dog claims to have a lower sodium content than the leading brand.
   a. A random sample of ten of these new hot dogs results in the following sodium measurements (mg):

   370 326 322 297 326 289 293 264 327 331.

   Estimate the population mean sodium content of this new brand of hot dog based on the ten sampled measurements.

   b. Calculate the margin of error associated with your estimate of the population mean from part (a). Round your answer to three decimal places.

   c. The mean sodium content of the leading brand of hot dogs is known to be 350 mg. Based on the sample mean and the value of the margin of error for the new brand, is a mean sodium content of 350 mg a plausible value for the mean sodium content of the new brand? Comment on whether you think the new brand of hot dog has a lower sodium content on average than the leading brand.

   d. Another random sample of 40 new brand hot dogs is taken. Should this larger sample of hot dogs produce a more accurate estimate of the population mean sodium content than the sample of size 10? Explain your answer by appealing to the formula for margin of error.

2. It is well known that astronauts increase their height in space missions because of the lack of gravity. A question is whether or not we increase height here on Earth when we are put into a situation where the effect of gravity is minimized. In particular, do people grow taller when confined to a bed? A study was done in which the heights of six men were taken before and after they were confined to bed for three full days.
   a. The before-after differences in height measurements (mm) for the six men were:

   12.6 14.4 14.7 14.5 15.2 13.5.

   Assuming that the men in this study are representative of the population of all men, what is an estimate of the population mean increase in height after three full days in bed?

   b. Calculate the margin of error associated with your estimate of the population mean from part (a). Round your answer to three decimal places.

   c. Based on your sample mean and the margin of error from parts (a) and (b), what are plausible values for the population mean height increase for all men who stay in bed for three full days?