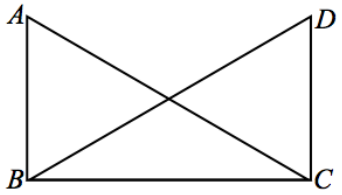
Lesson 23: Base Angles of Isosceles Triangles

Classwork

Opening Exercise

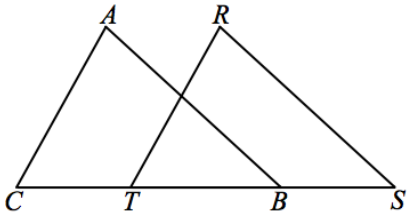
Describe the additional piece of information needed for each pair of triangles to satisfy the SAS triangle congruence criteria.

1. Given: $AB = DC$



Prove: $\triangle ABC \cong \triangle DCB$

2. Given: $AB = RS$
$\overline{AB} \parallel \overline{RS}$



Prove: $\triangle ABC \cong \triangle RST$

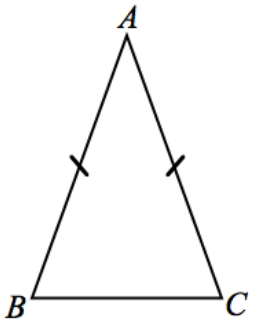
Exploratory Challenge

Today we examine a geometry fact that we already accept to be true. We are going to prove this known fact in two ways: (1) by using transformations and (2) by using SAS triangle congruence criteria.

Here is isosceles triangle ABC. We accept that an isosceles triangle, which has (at least) two congruent sides, also has congruent base angles.

Label the congruent angles in the figure.

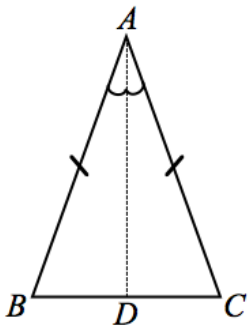
Now we will prove that the base angles of an isosceles triangle are always congruent.



Prove Base Angles of an Isosceles are Congruent: Transformations

Given: Isosceles $\triangle ABC$, with $AB = AC$

Prove: $\text{m}\angle B = \text{m}\angle C$



Construction: Draw the angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{BC}$. We need to show that rigid motions will map point B to point C and point C to point B.

Let r be the reflection through $\overleftrightarrow{AD}$. Through the reflection, we want to demonstrate two pieces of information that map B to point C and vice versa: (1) $\overline{AB}$ maps to $\overline{AC}$, and (2) $AB = AC$.

Since A is on the line of reflection, $\overleftrightarrow{AD}$, $r(A) = A$. Reflections preserve angle measures, so the measure of the reflected angle $r(\angle BAD)$ equals the measure of $\angle CAD$; therefore, $r(\overrightarrow{AB}) = \overrightarrow{AC}$. Reflections also preserve lengths of segments; therefore, the reflection of $\overline{AB}$ will still have the same length as $\overline{AB}$. By hypothesis, $AB = AC$, so the length of the reflection will also be equal to AC. Then $r(B) = C$. Using similar reasoning, we can show that $r(C) = B$.

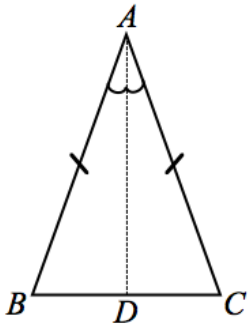
Reflections map rays to rays, so $r(\overrightarrow{BA}) = \overrightarrow{CA}$ and $r(\overrightarrow{BC}) = \overrightarrow{CB}$. Again, since reflections preserve angle measures, the measure of $r(\angle ABC)$ is equal to the measure of $\angle ACB$.

We conclude that $\text{m}\angle B = \text{m}\angle C$. Equivalently, we can state that $\angle B \cong \angle C$. In proofs, we can state that "base angles of an isosceles triangle are equal in measure" or that "base angles of an isosceles triangle are congruent."

Prove Base Angles of an Isosceles are Congruent: SAS

Given: Isosceles $\triangle ABC$, with $AB = AC$

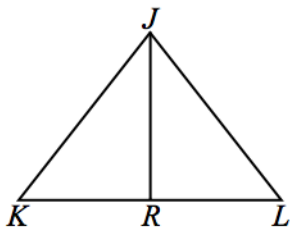
Prove: $\angle B \cong \angle C$



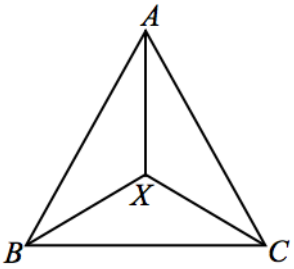
Construction: Draw the angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{BC}$. We are going to use this auxiliary line towards our SAS criteria.

Exercises

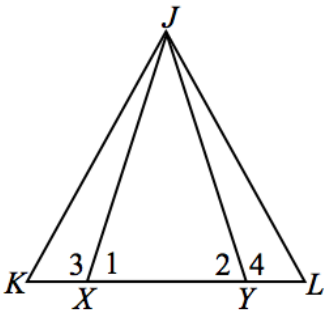
1. Given: $JK = JL; \overline{JR}$ bisects $\overline{KL}$
   Prove: $\overline{JR} \perp \overline{KL}$



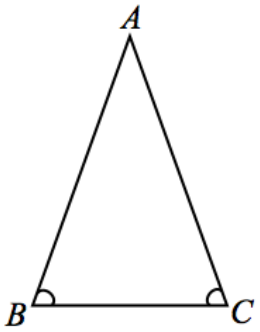
2. Given: $AB = AC,\ XB = XC$
   Prove: $\overline{AX}$ bisects $\angle BAC$



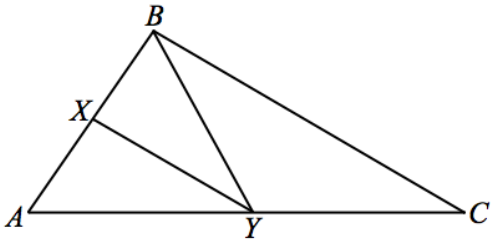
3. Given: $JX = JY,\ KX = LY$
   Prove: $\triangle JKL$ is isosceles



4. Given: $\triangle ABC$, with $\text{m}\angle CBA = \text{m}\angle BCA$
   Prove: $BA = CA$
   (Converse of base angles of isosceles triangle)
   Hint: Use a transformation.

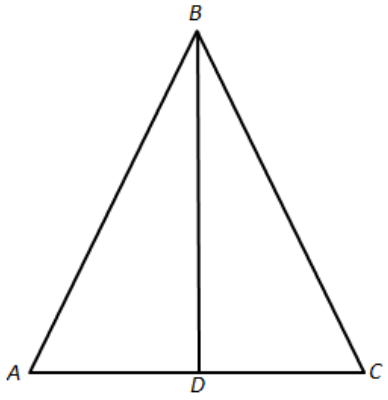


5. Given: $\triangle ABC$, with $\overline{XY}$ is the angle bisector of $\angle BYA$, and $\overline{BC} \parallel \overline{XY}$
   Prove: $YB = YC$

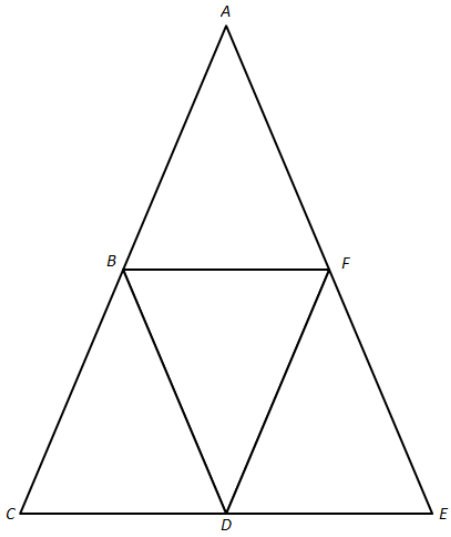


Problem Set

1. Given: $AB = BC, AD = DC$
   Prove: $\triangle ADB$ and $\triangle CDB$ are right triangles



2. Given: $AC = AE$ and $\overline{BF} \parallel \overline{CE}$
   Prove: $AB = AF$



3. In the diagram, $\triangle ABC$ is isosceles with $\overline{AC} \cong \overline{AB}$. In your own words, describe how transformations and the properties of rigid motions can be used to show that $\angle C \cong \angle B$.