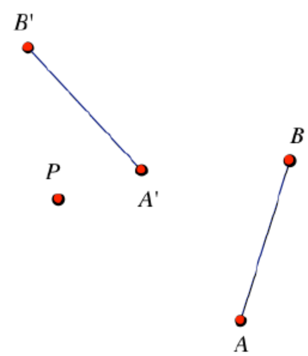
Lesson 6: Dilations as Transformations of the Plane

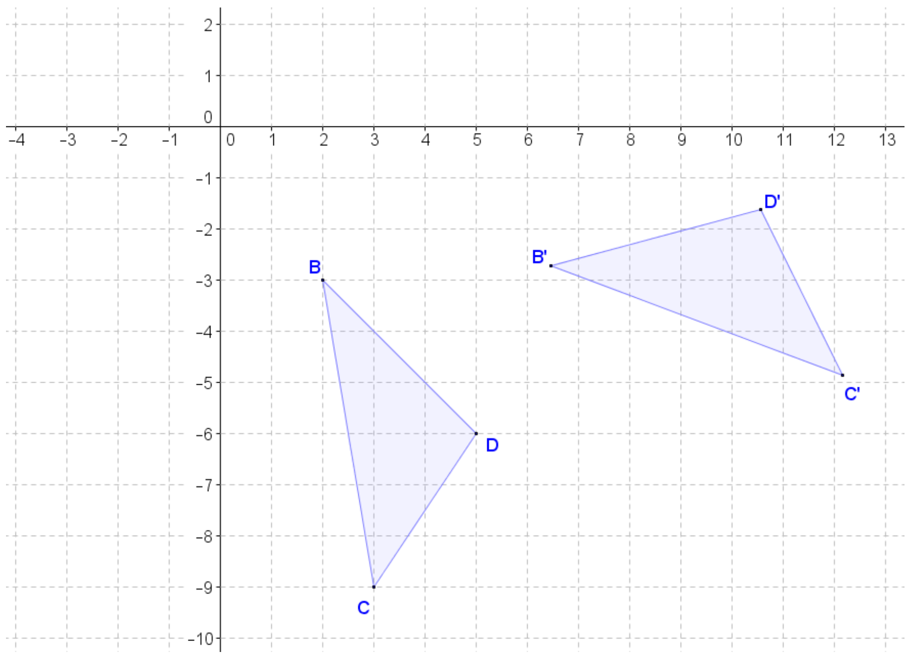
Classwork

Exercises 1–6

1. Find the center and the angle of the rotation that takes AB to $A'B'$. Find the image P' of point P under this rotation.

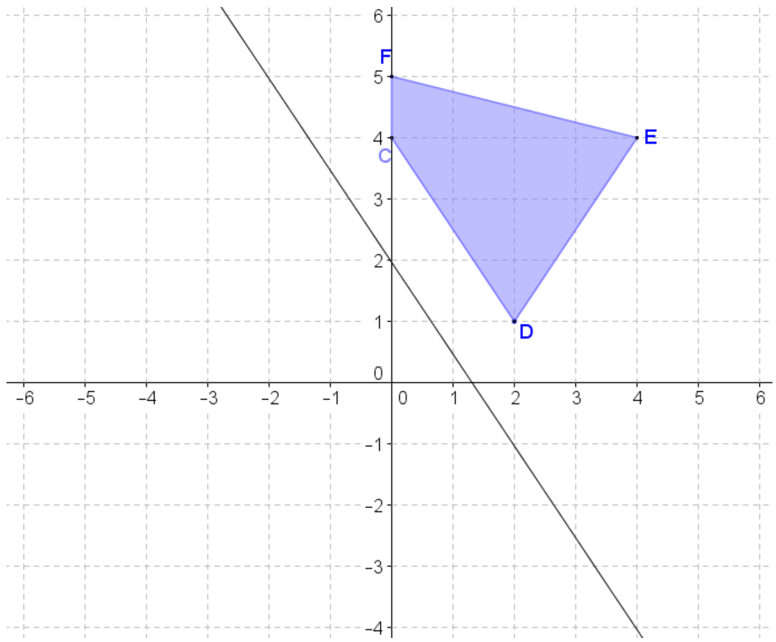


2. In the diagram below, $\triangle B'C'D'$ is the image of $\triangle BCD$ after a rotation about a point A. What are the coordinates of A, and what is the degree measure of the rotation?



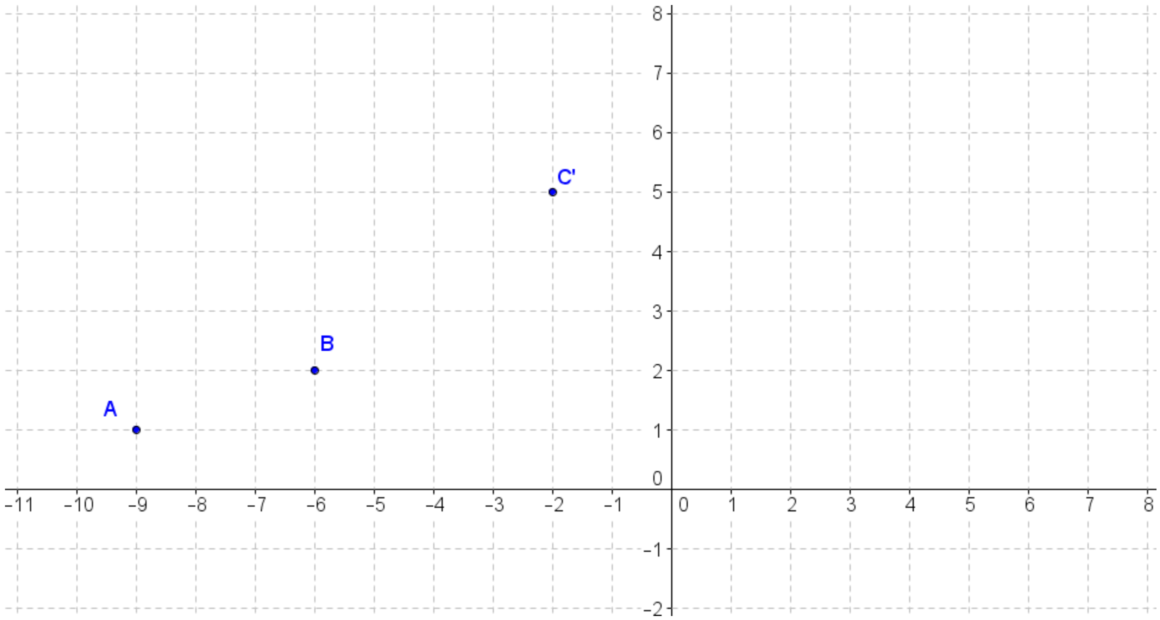
3. Find the line of reflection for the reflection that takes point A to point A'. Find the image P' under this reflection.

4. Quinn tells you that the vertices of the image of quadrilateral $CDEF$ reflected over the line representing the equation $y = -\frac{3}{2}x + 2$ are the following: $C'(-2,3)$, $D'(0,0)$, $E'(-3,-3)$, and $F'(-3,3)$. Do you agree or disagree with Quinn? Explain.



5. A translation takes A to A'. Find the image P' and pre-image P'' of point P under this translation. Find a vector that describes the translation.

6. The point C' is the image of point C under a translation of the plane along a vector.

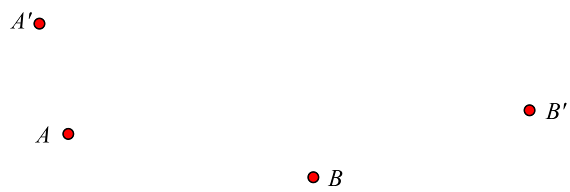


a. Find the coordinates of C if the vector used for the translation is $\overrightarrow{BA}$.

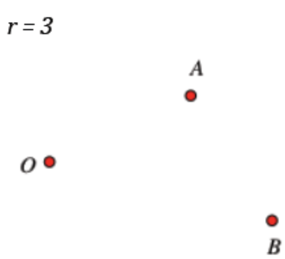
b. Find the coordinates of C if the vector used for the translation is $\overrightarrow{AB}$.

Exercises 7–9

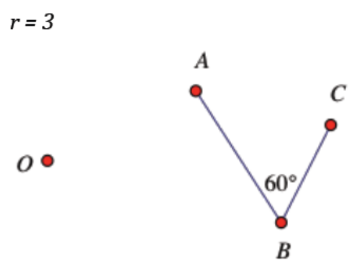
7. A dilation with center O and scale factor r takes A to A' and B to B'. Find the center O and estimate the scale factor r.



8. Given a center O, scale factor r, and points A and B, find the points $A' = D_{O,r}(A)$ and $B' = D_{O,r}(B)$. Compare length AB with length $A'B'$ by division; in other words, find $\frac{A'B'}{AB}$. How does this number compare to r?



9. Given a center O, scale factor r, and points A, B, and C, find the points $A' = D_{O,r}(A)$, $B' = D_{O,r}(B)$, and $C' = D_{O,r}(C)$. Compare $\mathrm{m}\angle ABC$ with $\angle A'B'C'$. What do you find?



Lesson Summary

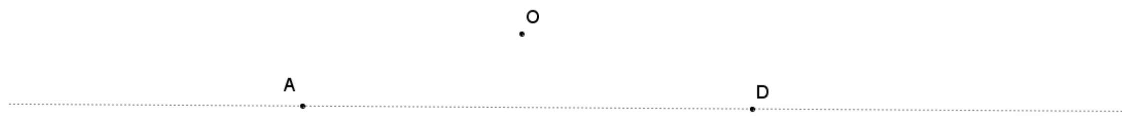
- There are two major classes of transformations; those that are distance-preserving (translations, reflections, rotations) and those that are not (dilations).
- Like rigid motions, dilations involve a rule assignment for each point in the plane and also have inverse functions that return each dilated point back to itself.

Problem Set

1. In the diagram below, A' is the image of A under a single transformation of the plane. Use the given diagram to show your solutions to parts (a)–(d).

   a. Describe the translation that maps $A \to A'$, and then use the translation to locate P', the image of P.
   b. Describe the reflection that maps $A \to A'$, and then use the reflection to locate P', the image of P.
   c. Describe a rotation that maps $A \to A'$, and then use your rotation to locate P', the image of P.
   d. Describe a dilation that maps $A \to A'$, and then use your dilation to locate P', the image of P.

2. On the diagram below, O is a center of dilation and $\overleftrightarrow{AD}$ is a line not through O. Choose two points B and C on $\overleftrightarrow{AD}$ between A and D.



a. Dilate A, B, C, and D from O using scale factor $r = \frac{1}{2}$. Label the images A', B', C', and D', respectively.

b. Dilate A, B, C, and D from O using scale factor $r = 2$. Label the images A'', B'', C'', and D'', respectively.

c. Dilate A, B, C, and D from O using scale factor $r = 3$. Label the images A''', B''', C''', and D''', respectively.

d. Draw a conclusion about the effect of a dilation on a line segment based on the diagram that you drew. Explain.

3. Write the inverse transformation for each of the following so that the composition of the transformation with its inverse will map a point to itself on the plane.

a. $T_{\overrightarrow{AB}}$

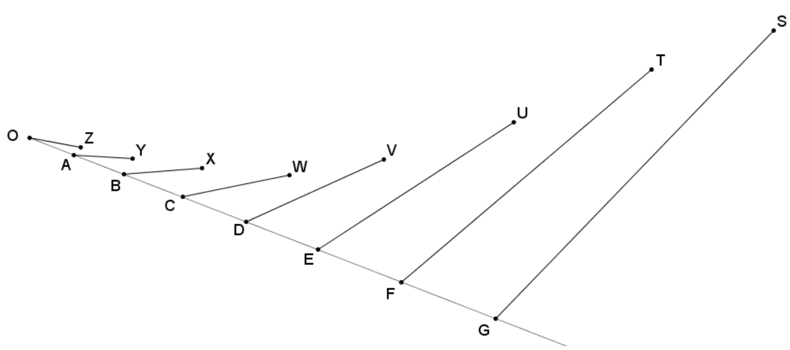
b. $r_{\overleftrightarrow{AB}}$

c. $R_{C,45}$

d. $D_{O,r}$

4. Given $U(1,3)$, $V(-4,-4)$, and $W(-3,6)$ on the coordinate plane, perform a dilation of $\triangle UVW$ from center $O(0,0)$ with a scale factor of $\frac{3}{2}$. Determine the coordinates of images of points U, V, and W, and describe how the coordinates of the image points are related to the coordinates of the pre-image points.

5. Points B, C, D, E, F, and G are dilated images of A from center O with scale factors 2, 3, 4, 5, 6, and 7, respectively. Are points Y, X, W, V, U, T, and S all dilated images of Z under the same respective scale factors? Explain why or why not.



6. Find the center and scale factor that takes A to A' and B to B', if a dilation exists.

7. Find the center and scale factor that takes A to A' and B to B', if a dilation exists.