## Lesson 29: Applying Tangents

## Classwork

## Opening Exercise

a. Use a calculator to find the tangent of $\theta$. Give your answer correct to four decimal places.

| $\theta$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin \theta$ | 0 | 0.1736 | 0.3420 | 0.5 | 0.6428 | 0.7660 | 0.8660 | 0.9397 | 0.9848 | 1 |
| $\cos \theta$ | 1 | 0.9848 | 0.9397 | 0.8660 | 0.7660 | 0.6428 | 0.5 | 0.3420 | 0.1736 | 0 |
| $\frac{\sin \theta}{\cos \theta}$ |  |  |  |  |  |  |  |  |  |  |
| $\tan \theta$ |  |  |  |  |  |  |  |  |  |  |

b. The table from Lesson 29 is provided here for you. In the row labeled $\frac{\sin \theta}{\cos \theta}$, divide the sine values by the cosine values. What do you notice?

## Example 1

Scott, whose eye level is 1.5 m above the ground, stands 30 m from a tree. The angle of elevation of a bird at the top of the tree is $36^{\circ}$. How far above ground is the bird?


## Example 2

From an angle of depression of $40^{\circ}$, John watches his friend approach his building while standing on the rooftop. The rooftop is 16 m from the ground, and John's eye level is at about 1.8 m from the rooftop. What is the distance between John's friend and the building?


## Exercise 1

Standing on the gallery of a lighthouse (the deck at the top of a lighthouse), a person spots a ship at an angle of depression of $20^{\circ}$. The lighthouse is 28 m tall and sits on a cliff 45 m tall as measured from sea level. What is the horizontal distance between the lighthouse and the ship? Sketch a diagram to support your answer.


## Exercise 2

A line on the coordinate plane makes an angle of depression of $36^{\circ}$. Find the slope of the line, correct to four decimal places.

## Problem Set

1. A line in the coordinate plane has an angle of elevation of $53^{\circ}$. Find the slope of the line correct to four decimal places.
2. A line in the coordinate plane has an angle of depression of $25^{\circ}$. Find the slope of the line correct to four decimal places.
3. In Problems 1 and 2, why do the lengths of the legs of the right triangles formed not affect the slope of the line?
4. Given the angles of depression below, determine the slope of the line with the indicated angle correct to four decimal places.
a. $35^{\circ}$ angle of depression
b. $49^{\circ}$ angle of depression
c. $80^{\circ}$ angle of depression
d. $87^{\circ}$ angle of depression
e. $89^{\circ}$ angle of depression
f. $89.9^{\circ}$ angle of depression
g. What appears to be happening to the slopes (and tangent values) as the angles of depression get closer to $90^{\circ}$ ?
h. Find the slopes of angles of depression that are even closer to $90^{\circ}$ than $89.9^{\circ}$. Can the value of the tangent of $90^{\circ}$ be defined? Why or why not?
5. For the indicated angle, express the quotient in terms of sine, cosine, or tangent. Then write the quotient in simplest terms.
a. $\frac{4}{2 \sqrt{13}} ; \alpha$
b. $\frac{6}{4} ; \alpha$
c. $\frac{4}{2 \sqrt{13}} ; \beta$
d. $\frac{4}{6} ; \beta$

6. The pitch of a roof on a home is expressed as a ratio of vertical rise: horizontal run where the run has a length of 12 units. If a given roof design includes an angle of elevation of $22.5^{\circ}$, and the roof spans 36 ft . as shown in the diagram, determine the pitch of the roof. Then determine the distance along one of the two sloped surfaces of the roof.

7. An anchor cable supports a vertical utility pole forming a $51^{\circ}$ angle with the ground. The cable is attached to the top of the pole. If the distance from the base of the pole to the base of the cable is 5 meters, how tall is the pole?
8. A winch is a tool that rotates a cylinder, around which a cable is wound. When the winch rotates in one direction, it draws the cable in. Joey is using a winch and a pulley (as shown in the diagram) to raise a heavy box off the floor and onto a cart. The box is 2 ft . tall, and the winch is 14 ft . horizontally from where cable drops down vertically from the pulley. The angle of elevation to the pulley is $42^{\circ}$. What is the approximate length of cable required to connect the winch and the box?

