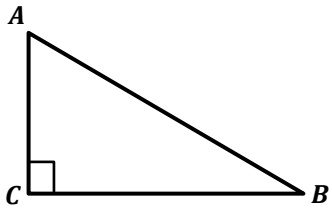
Lesson 5: Criterion for Perpendicularity

Classwork

Opening Exercise

In right triangle ABC, find the missing side.



a. If $AC = 9$ and $CB = 12$, what is AB? Explain how you know.

b. If $AC = 5$ and $AB = 13$, what is CB?

c. If $AC = CB$ and $AB = 2$, what is AC (and CB)?

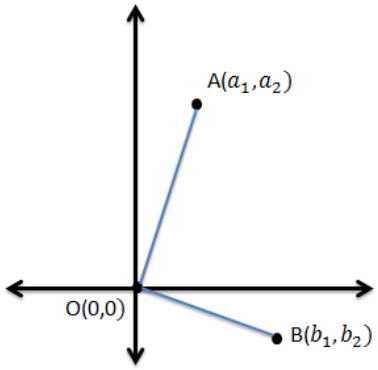
Exercise 1

1. Use the grid at the right.

a. Plot points $O(0,0)$, $P(3,-1)$, and $Q(2,3)$ on the coordinate plane.

b. Determine whether $\overline{OP}$ and $\overline{OQ}$ are perpendicular. Support your findings.

Example 2



Exercises 2–4

2. Given points $A(6,4)$, $B(24,-6)$, $C(1,4)$, $P(2,-3)$, $S(-18,-12)$, $T(-3,-12)$, $U(-8,2)$, and $W(-6,9)$, find all pairs of segments from the list below that are perpendicular. Support your answer.
   $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OP}$, $\overline{OS}$, $\overline{OT}$, $\overline{OU}$, and $\overline{OW}$

3. The points $O(0,0)$, $A(-4,1)$, $B(-3,5)$, and $C(1,4)$ are the vertices of parallelogram $OABC$. Is this parallelogram a rectangle? Support you answer.

Problem Set

1. Prove using the Pythagorean theorem that $\overline{AC}$ is perpendicular to $\overline{AB}$ given $A(-2,-2), B(5,-2),$ and $C(-2,22)$.

2. Using the general formula for perpendicularity of segments through the origin and $(90,0)$, determine if segments $\overline{OA}$ and $\overline{OB}$ are perpendicular.
   a. $A(-3,-4), B(4,3)$
   b. $A(8,9), B(18,-16)$

3. Given points $O(0,0)$, $S(2,7)$, and $T(7,-2)$, where $\overline{OS}$ is perpendicular to $\overline{OT}$, will the images of the segments be perpendicular if the three points O, S, and T are translated four units to the right and eight units up? Explain your answer.

4. In Example 1, we saw that $\overline{OA}$ was perpendicular to $\overline{OB}$ for $O(0,0)$, $A(6,4)$, and $B(-2,3)$. Suppose $P(5,5)$, $Q(11,9)$, and $R(3,8)$. Are segments $\overline{PQ}$ and $\overline{PR}$ perpendicular? Explain without using triangles or the Pythagorean theorem.

5. Challenge: Using what we learned in Exercise 2, if $C(c_1,c_2)$, $A(a_1,a_2)$, and $B(b_1,b_2)$, what is the general condition of a_1, a_2, b_1, b_2, c_1, and c_2 that ensures segments $\overline{CA}$ and $\overline{CB}$ are perpendicular?

6. A robot that picks up tennis balls is on a straight path from $(8,6)$ towards a ball at $(-10,-5)$. The robot picks up a ball at $(-10,-5)$, then turns $90°$ right. What are the coordinates of a point that the robot can move towards to pick up the last ball?

7. Gerry thinks that the points $(4,2)$ and $(-1,4)$ form a line perpendicular to a line with slope 4. Do you agree? Why or why not?