Lesson 8: Parallel and Perpendicular Lines

Classwork

Exercise 1

1.   a.   Write an equation of the line that passes through the origin that intersects the line $2x + 5y = 7$ to form a right angle.

    b.   Determine whether the lines given by the equations $2x + 3y = 6$ and $y = \frac{3}{2}x + 4$ are perpendicular. Support your answer.

    c.   Two lines having the same y-intercept are perpendicular. If the equation of one of these lines is $y = -\frac{4}{5}x + 6$, what is the equation of the second line?

Example 2

    a.   What is the relationship between two coplanar lines that are perpendicular to the same line?

b. Given two lines, l_1 and l_2, with equal slopes and a line k that is perpendicular to one of these two parallel lines, l_1:

   i. What is the relationship between line k and the other line, l_2?

   ii. What is the relationship between l_1 and l_2?

Exercises 2–7

2. Given a point $(-3, 6)$ and a line $y = 2x - 8$:

   a. What is the slope of the line?

   b. What is the slope of any line parallel to the given line?

   c. Write an equation of a line through the point and parallel to the line.

   d. What is the slope of any line perpendicular to the given line? Explain.

3. Find an equation of a line through $(0, -7)$ and parallel to the line $y = \frac{1}{2}x + 5$.

    a. What is the slope of any line parallel to the given line? Explain your answer.

    b. Write an equation of a line through the point and parallel to the line.

    c. If a line is perpendicular to $y = \frac{1}{2}x + 5$, will it be perpendicular to $x - 2y = 14$? Explain.

4. Find an equation of a line through $\left(\sqrt{3}, \frac{1}{2}\right)$ parallel to the line:

    a. $x = -9$

    b. $y = -\sqrt{7}$

    c. What can you conclude about your answer in parts (a) and (b)?

5. Find an equation of a line through $\left(-\sqrt{2}, \pi\right)$ parallel to the line $x - 7y = \sqrt{5}$.

6. Recall that our search robot is moving along the line $y = 3x - 600$ and wishes to make a right turn at the point $(400, 600)$. Find an equation for the perpendicular line on which the robot is to move. Verify that your line intersects the x-axis at $(2200, 0)$.

7. A robot, always moving at a constant speed of 2 feet per second, starts at position $(20, 50)$ on the coordinate plane and heads in a south-east direction along the line $3x + 4y = 260$. After 15 seconds, it turns left 90° and travels in a straight line in this new direction.

   a. What are the coordinates of the point at which the robot made the turn? What might be a relatively straightforward way of determining this point?

   b. Find an equation for the second line on which the robot traveled.

   c. If, after turning, the robot travels for 20 seconds along this line and then stops, how far will it be from its starting position?

   d. What is the equation of the line the robot needs to travel along in order to now return to its starting position? How long will it take for the robot to get there?

Problem Set

1. Write the equation of the line through $(-5, 3)$ and:

   a. Parallel to $x = -1$.

   b. Perpendicular to $x = -1$.

   c. Parallel to $y = \frac{3}{5}x + 2$.

   d. Perpendicular to $y = \frac{3}{5}x + 2$.

2. Write the equation of the line through $\left(\sqrt{3}, \frac{5}{4}\right)$ and:

   a. Parallel to $y = 7$.

   b. Perpendicular to $y = 7$.

   c. Parallel to $\frac{1}{2}x - \frac{3}{4}y = 10$.

   d. Perpendicular to $\frac{1}{2}x - \frac{3}{4}y = 10$.

3. A vacuum robot is in a room and charging at position $(0, 5)$. Once charged, it begins moving on a northeast path at a constant sped of $\frac{1}{2}$ foot per second along the line $4x - 3y = -15$. After 60 seconds, it turns right 90° and travels in the new direction.

   a. What are the coordinates of the point at which the robot made the turn?

   b. Find an equation for the second line on which the robot traveled.

   c. If after turning, the robot travels 80 seconds along this line, how far has it traveled from its starting position?

   d. What is the equation of the line the robot needs to travel along in order to return and recharge? How long will it take the robot to get there?

4. Given the statement $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{DE}$, construct an argument for or against this statement using the two triangles shown.
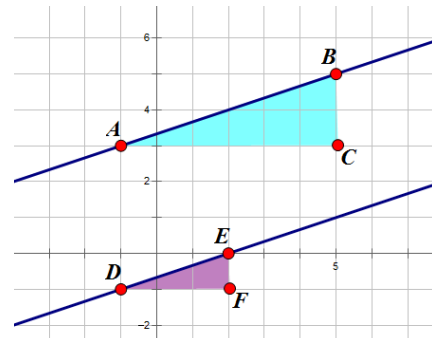
5. Recall the proof we did in Example 1: Let l_1 and l_2 be two non-vertical lines in the Cartesian plane. l_1 and l_2 are perpendicular if and only if their slopes are negative reciprocals of each other. In class, we looked at the case where both y-intercepts were not zero. In Lesson 5, we looked at the case where both y-intercepts were equal to zero, when the vertex of the right angle was at the origin. Reconstruct the proof for the case where one line has a y-intercept of zero, and the other line has a non-zero y-intercept.

6. Challenge: Reconstruct the proof we did in Example 1 if one line has a slope of zero.