Lesson 11: Perimeters and Areas of Polygonal Regions Defined by Systems of Inequalities

Classwork
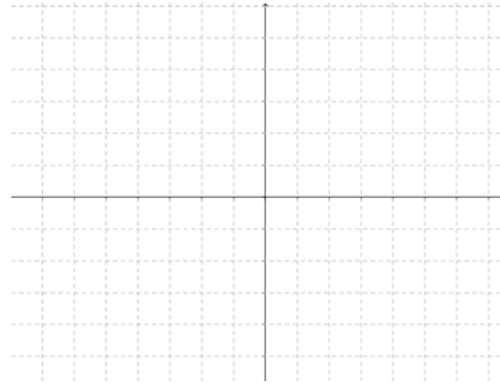
Opening Exercise

Graph the following:

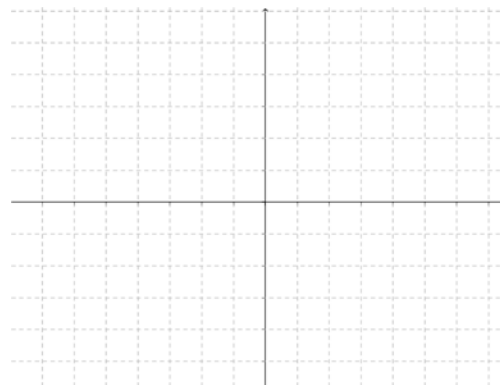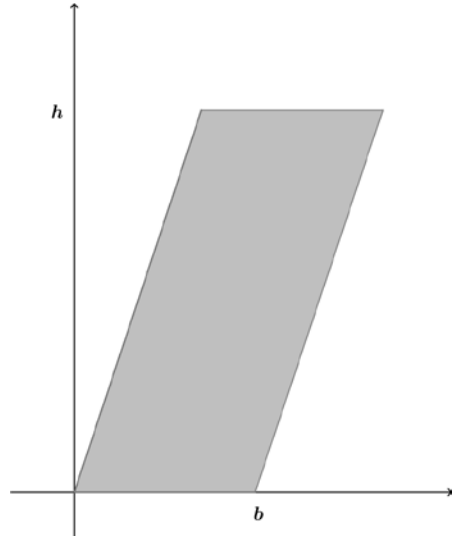a. $y \leq 7$

b. $x > -3$

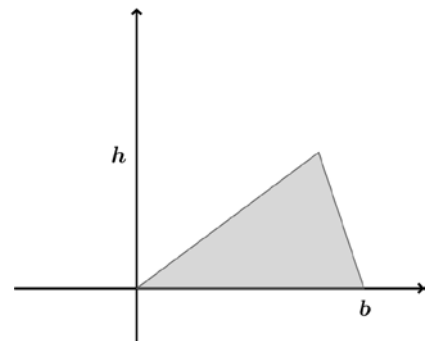c. $y < \frac{1}{2}x - 4$

d. $y \geq -\frac{2}{3}x + 5$

Example 1

A parallelogram with base of length b and height h can be situated in the coordinate plane as shown. Verify that the shoelace formula gives the area of the parallelogram as bh.

Example 2

A triangle with base b and height h can be situated in the coordinate plane as shown. According to Green's theorem, what is the area of the triangle?

Exercises 1–2

1. A quadrilateral region is defined by the system of inequalities below:

    $$y \leq x + 6 \qquad y \leq -2x + 12 \qquad y \geq 2x - 4 \qquad y \geq -x + 2$$
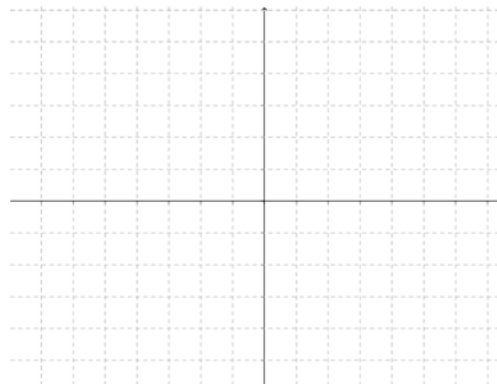
    a. Sketch the region.

    b. Determine the vertices of the quadrilateral.

    c. Find the perimeter of the quadrilateral region.

    d. Find the area of the quadrilateral region.

2. A quadrilateral region is defined by the system of inequalities below:

    $$y \leq x + 5 \qquad y \geq x - 4 \qquad y \leq 4 \qquad y \geq -\frac{5}{4}x - 4$$

    a. Sketch the region.

    b. Determine the vertices of the quadrilateral.

    c. Which quadrilateral is defined by these inequalities? How can you prove your conclusion?

    d. Find the perimeter of the quadrilateral region.

    e. Find the area of the quadrilateral region.

Problem Set

For Problems 1–2 below, identify the system of inequalities that defines the region shown.

1.

2.
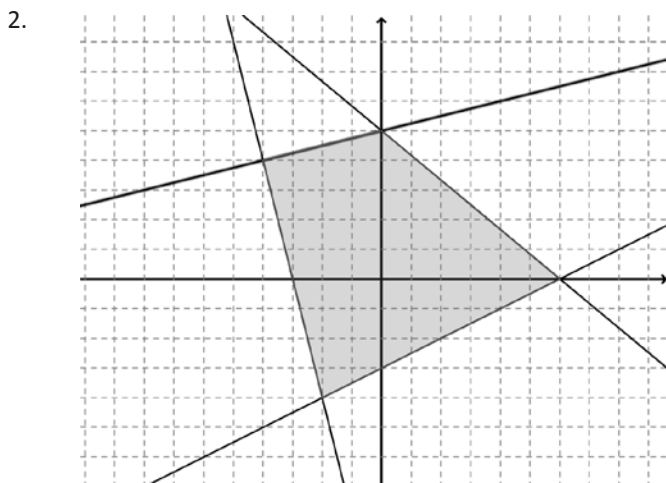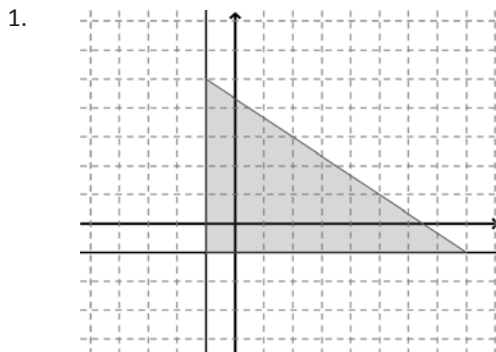
For Problems 3–5 below, a triangular or quadrilateral region is defined by the system of inequalities listed.

   a.   Sketch the region.

   b.   Determine the coordinates of the vertices.

   c.   Find the perimeter of the region rounded to the nearest hundredth if necessary.

   d.   Find the area of the region rounded to the nearest tenth if necessary.

3. $8x - 9y \geq -22$ $\qquad\qquad$ $x + y \leq 10$ $\qquad\qquad$ $5x - 12y \leq -1$

4. $x + 3y \geq 0$ $\qquad\qquad$ $4x - 3y \geq 0$ $\qquad\qquad$ $2x + y \leq 10$

5. $2x - 5y \geq -14$ $\qquad\qquad$ $3x + 2y \leq 17$ $\qquad\qquad$ $2x - y \leq 9$ $\qquad\qquad$ $x + y \geq 0$