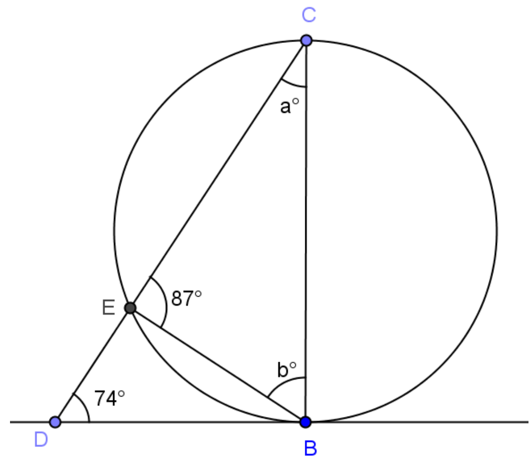
Lesson 14: Secant Lines; Secant Lines That Meet Inside a Circle

Classwork

Opening Exercise

$\overleftrightarrow{DB}$ is tangent to the circle as shown.

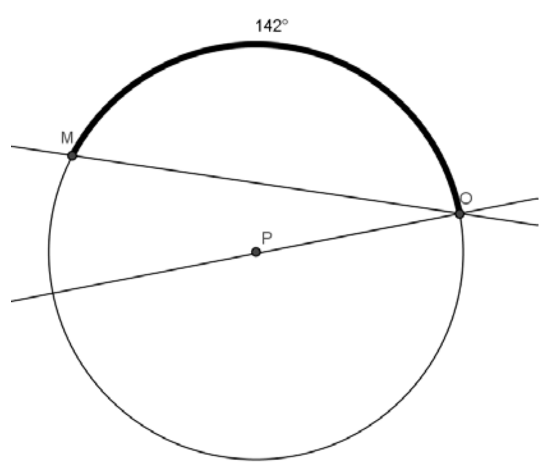
a. Find the values of a and b.



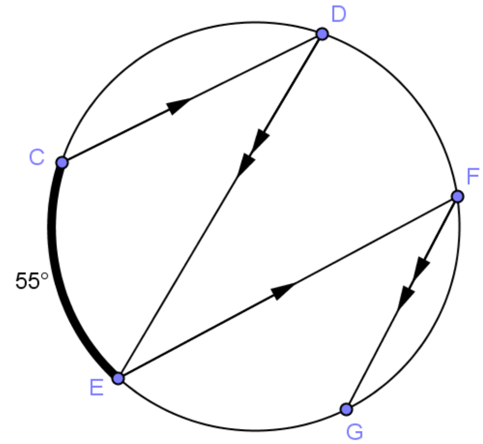
b. Is $\overline{CB}$ a diameter of the circle? Explain.

Exercises 1–2

1. In circle P, $\overline{PO}$ is a radius, and $m\widehat{MO} = 14°$. Find $m\angle MOP$, and explain how you know.

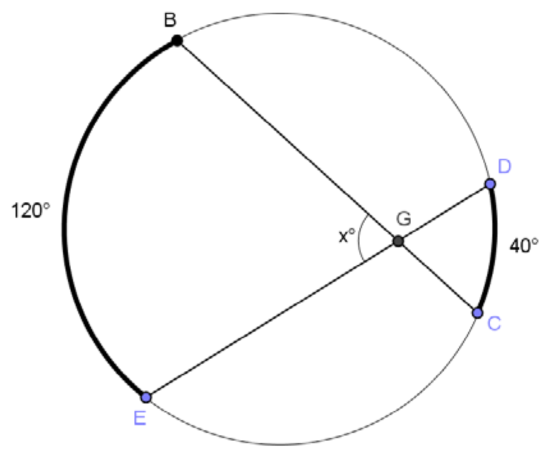


2. In the circle shown, $m\widehat{\mathrm{CE}} = 55°$. Find $m\angle\mathrm{DEF}$ and $m\widehat{\mathrm{EG}}$.
Explain your answer.

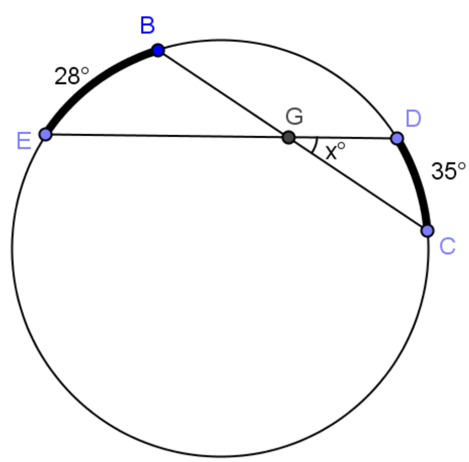


Example 1

a. Find x. Justify your answer.



b. Find x.



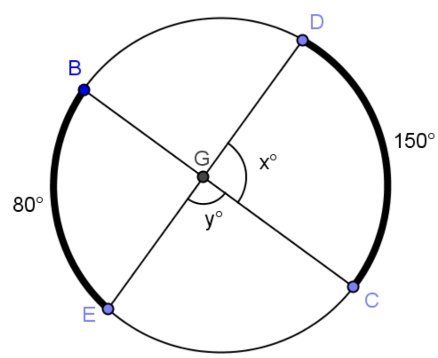
We can state the results of part (b) of this example as the following theorem:

SECANT ANGLE THEOREM: INTERIOR CASE: The measure of an angle whose vertex lies in the interior of a circle is equal to half the sum of the angle measures of the arcs intercepted by it and its vertical angle.

Exercises 3–7

In Exercises 3–5, find x and y.

3.



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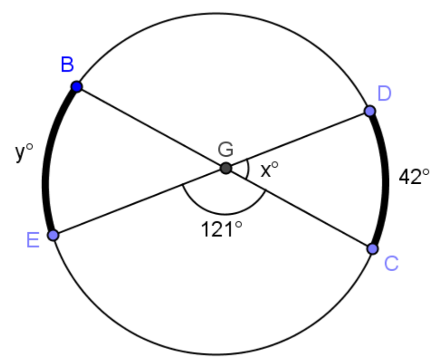

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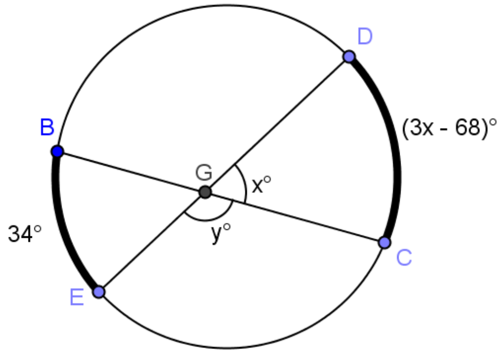
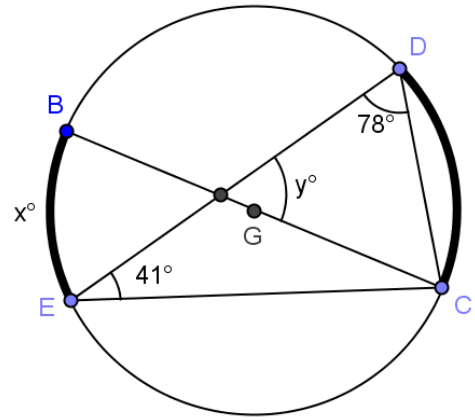
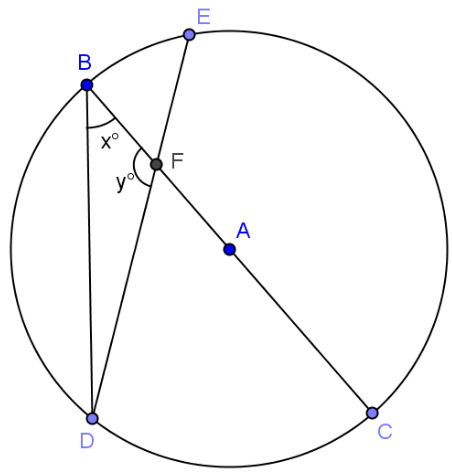


6. In circle, $\overline{BC}$ is a diameter. Find x and y.



7. In the circle shown, $\overline{BC}$ is a diameter. $DC: BE = 2:1$. Prove $y = 180 - \frac{3}{2}x$ using a two-column proof.



Lesson Summary

THEOREMS:

- **SECANT ANGLE THEOREM: INTERIOR CASE.** The measure of an angle whose vertex lies in the interior of a circle is equal to half the sum of the angle measures of the arcs intercepted by it and its vertical angle.

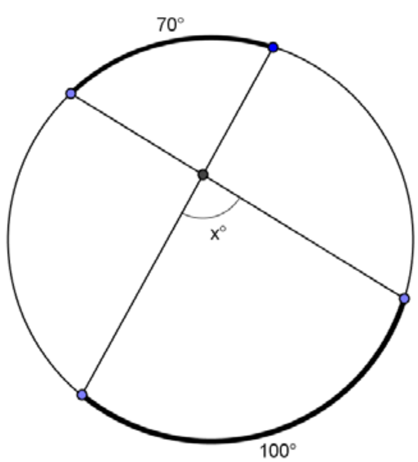
Relevant Vocabulary

- **TANGENT TO A CIRCLE:** A *tangent line to a circle* is a line in the same plane that intersects the circle in one and only one point. This point is called the *point of tangency*.
- **TANGENT SEGMENT/RAY:** A segment is a *tangent segment to a circle* if the line that contains it is tangent to the circle and one of the end points of the segment is a point of tangency. A ray is called a *tangent ray to a circle* if the line that contains it is tangent to the circle and the vertex of the ray is the point of tangency.
- **SECANT TO A CIRCLE:** A *secant line to a circle* is a line that intersects a circle in exactly two points.

Problem Set

In Problems 1–4, find x.

1.



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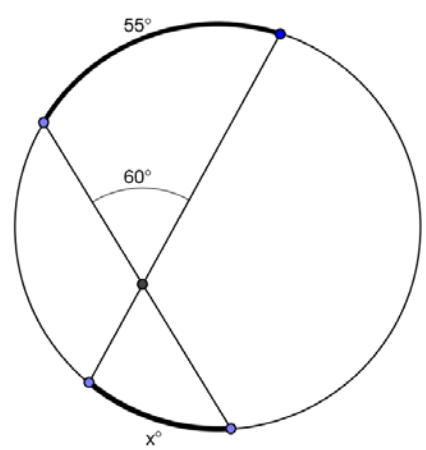

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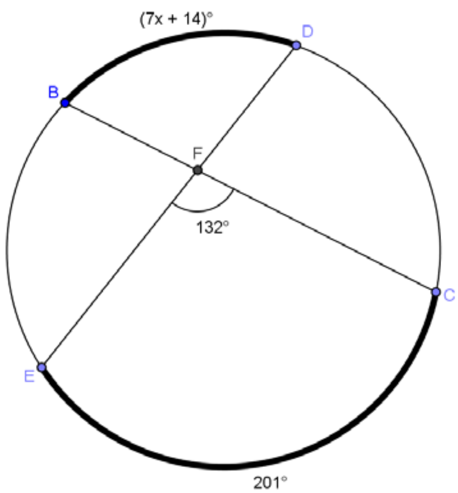

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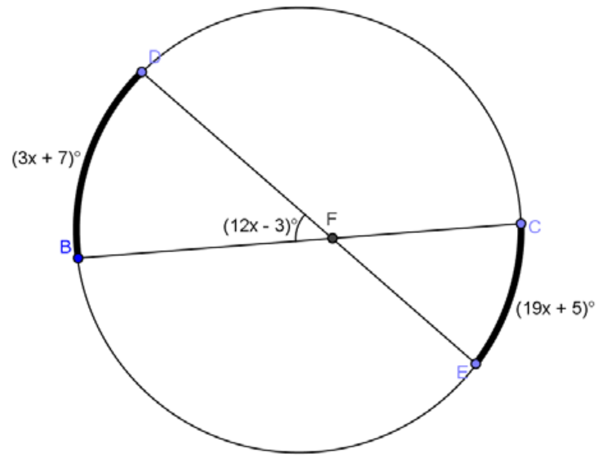
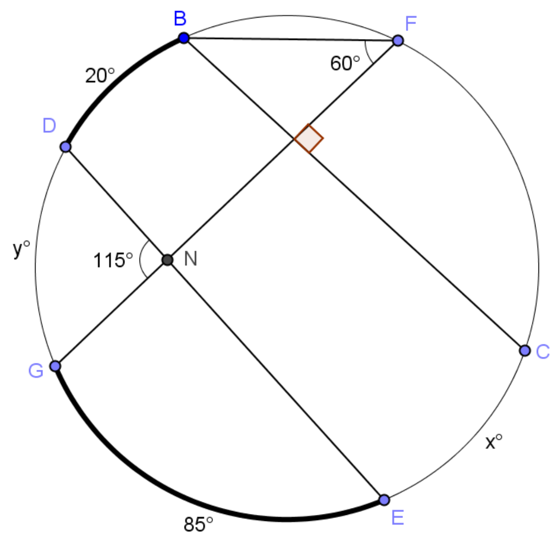
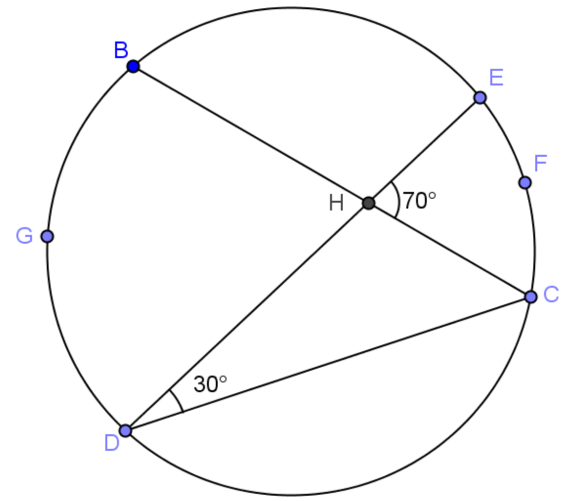


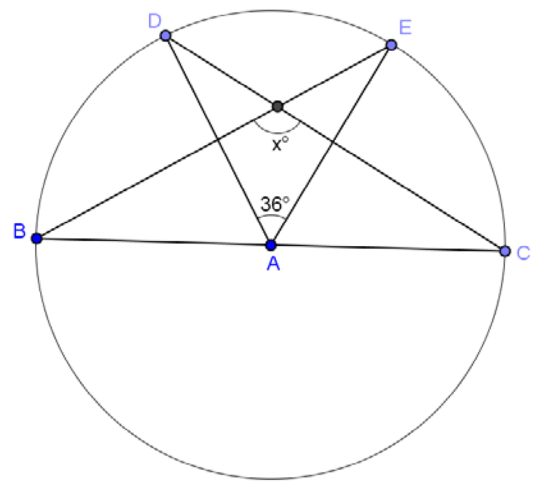
5. Find x ($m\widehat{CE}$) and y ($m\widehat{DG}$).



6. Find the ratio of $m\widehat{EFC}:m\widehat{DGB}$.



7. $\overline{BC}$ is a diameter of circle A. Find x.



8. Show that the general formula we discovered in Example 1 also works for central angles. (Hint: Extend the radii to form 2 diameters, and use relationships between central angles and arc measure.)