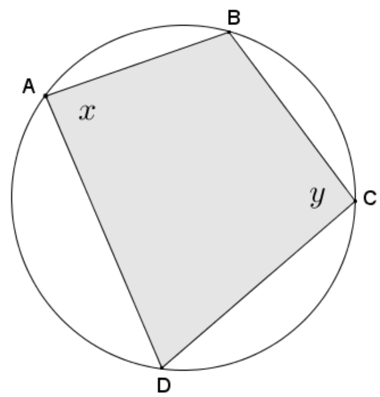
Lesson 20: Cyclic Quadrilaterals

Classwork

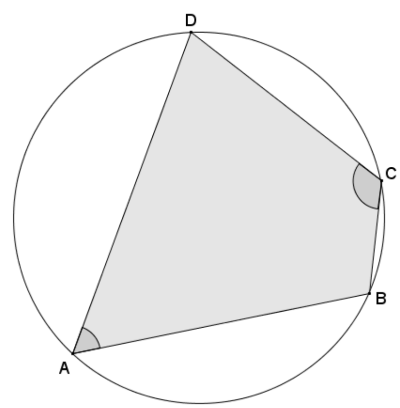
Opening Exercise

Given cyclic quadrilateral $ABCD$ shown in the diagram, prove that $x + y = 180°$.



Example 1:

Given quadrilateral $ABCD$ with $m\angle A + m\angle C = 180°$, prove that quadrilateral $ABCD$ is cyclic; in other words, prove that points A, B, C, and D lie on the same circle.



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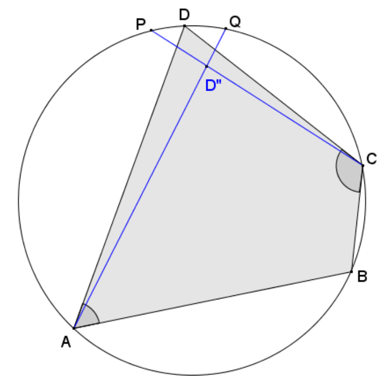
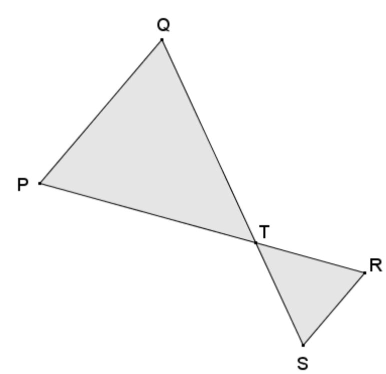
Exercises

1. Assume that vertex D'' lies inside the circle as shown in the diagram. Use a similar argument to Example 1 to show that vertex D'' cannot lie inside the circle.



2. Quadrilateral $PQRS$ is a cyclic quadrilateral. Explain why $\triangle PQT \sim \triangle SRT$.



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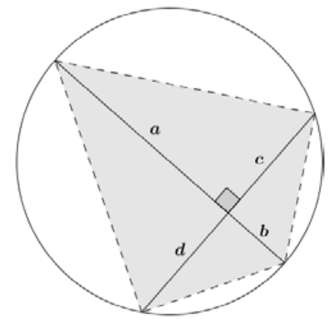
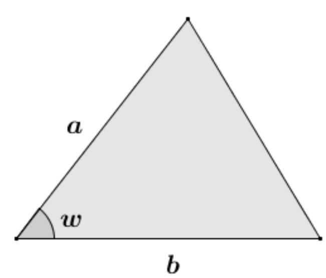
3. A cyclic quadrilateral has perpendicular diagonals. What is the area of the quadrilateral in terms of a, b, c, and d as shown?



4. Show that the triangle in the diagram has area $\frac{1}{2}ab\sin(w)$.



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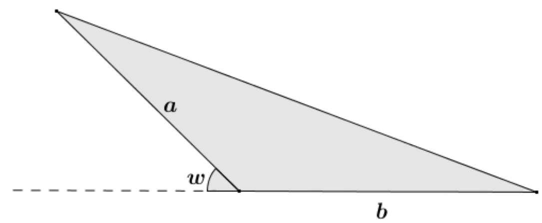
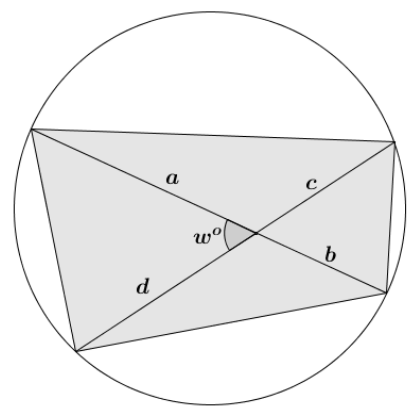
5. Show that the triangle with obtuse angle $(180 - w)°$ has area $\frac{1}{2}ab\sin(w)$.



6. Show that the area of the cyclic quadrilateral shown in the diagram is $Area = \frac{1}{2}(a+b)(c+d)\sin(w)$.



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Lesson Summary

THEOREMS:

Given a convex quadrilateral, the quadrilateral is cyclic if and only if one pair of opposite angles is supplementary.

The area of a triangle with side lengths a and b and acute included angle with degree measure w:

$\text{Area} = \frac{1}{2}ab \cdot \sin(w)$.

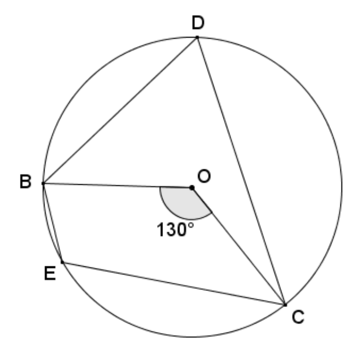
The area of a cyclic quadrilateral $ABCD$ whose diagonals $\overline{AC}$ and $\overline{BD}$ intersect to form an acute or right angle with degree measure w: $\text{Area}(ABCD) = \frac{1}{2} \cdot AC \cdot BD \cdot \sin(w)$.

Relevant Vocabulary

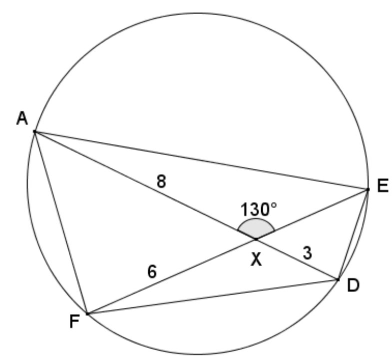
CYCLIC QUADRILATERAL: A quadrilateral inscribed in a circle is called a *cyclic quadrilateral*.

Problem Set

1. Quadrilateral $BDCE$ is cyclic, O is the center of the circle, and $m\angle BOC = 130°$. Find $m\angle BEC$.



2. Quadrilateral $FAED$ is cyclic, $AX = 8$, $FX = 6$, $XD = 3$, and $m\angle AXE = 130°$. Find the area of quadrilateral $FAED$.



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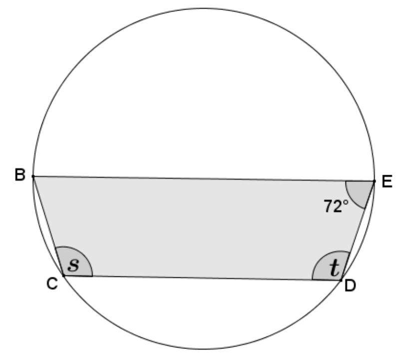
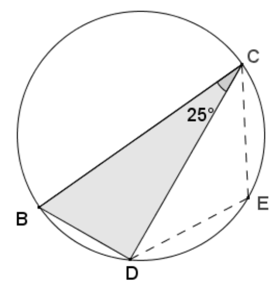
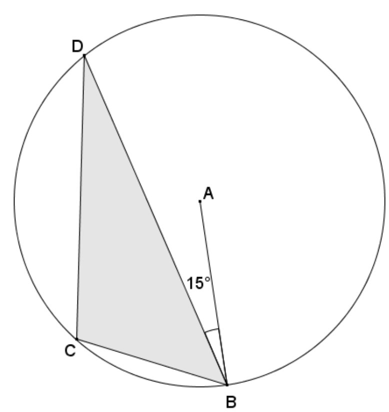
3. In the diagram below, $\overline{BE} \parallel \overline{CD}$, and $m\angle BED = 72°$. Find the value of s and t.



4. In the diagram below, $\overline{BC}$ is the diameter, $m\angle BCD = 25°$, and $\overline{CE} \cong \overline{DE}$. Find $m\angle CED$.



5. In circle A, $m\angle ABD = 15°$. Find $m\angle BCD$.



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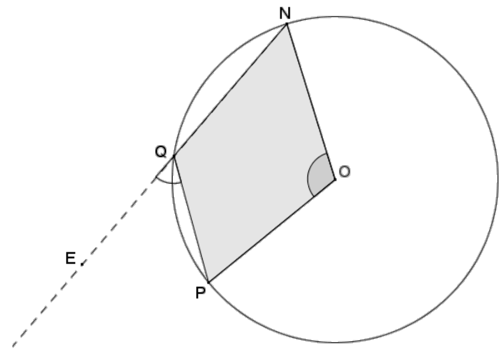
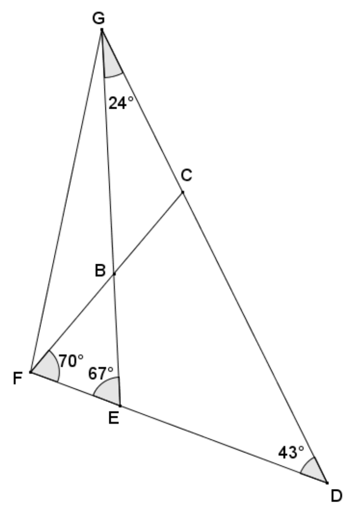
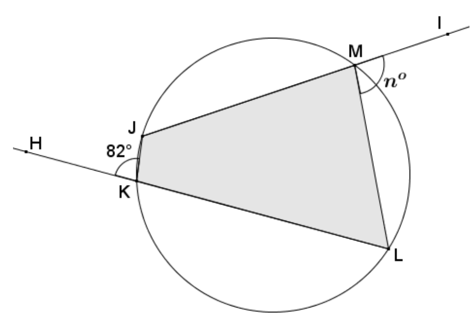
6. Given the diagram below, O is the center of the circle. If $m\angle NOP = 112°$, find $m\angle PQE$.



7. Given the angle measures as indicated in the diagram below, prove that vertices C, B, E, and D lie on a circle.



8. In the diagram below, quadrilateral $JKLM$ is cyclic. Find the value of n.



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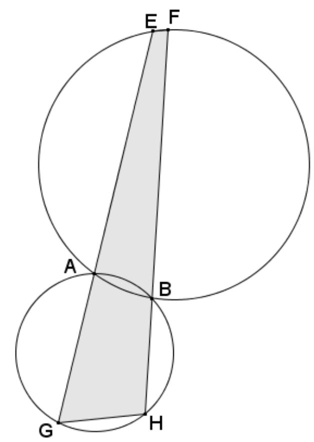
9. Do all four perpendicular bisectors of the sides of a cyclic quadrilateral pass through a common point? Explain.

10. The circles in the diagram below intersect at points A and B. If $m\angle FHG = 100°$ and $m\angle HGE = 70°$, find $m\angle GEF$ and $m\angle EFH$.



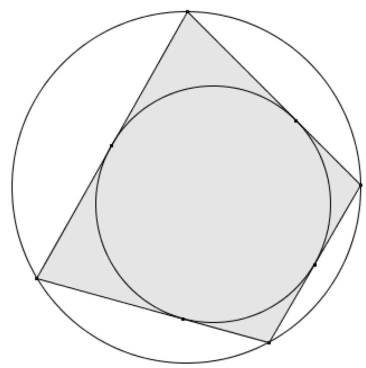
11. A quadrilateral is called *bicentric* if it is both cyclic and possesses an inscribed circle. (See diagram to the right.)

    a. What can be concluded about the opposite angles of a bicentric quadrilateral? Explain.

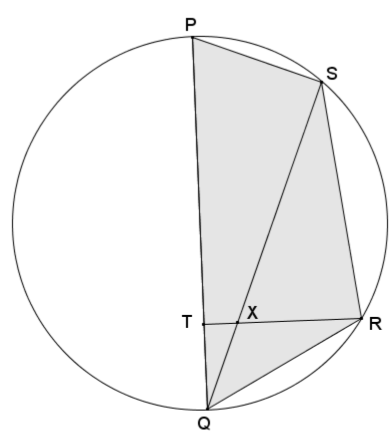
    b. Each side of the quadrilateral is tangent to the inscribed circle. What does this tell us about the segments contained in the sides of the quadrilateral?

    c. Based on the relationships highlighted in part (b), there are four pairs of congruent segments in the diagram. Label segments of equal length with a, b, c, and d.

    d. What do you notice about the opposite sides of the bicentric quadrilateral?



12. Quadrilateral $PSRQ$ is cyclic such that $\overline{PQ}$ is the diameter of the circle. If $\angle QRT \cong \angle QSR$, prove that $\angle PTR$ is a right angle, and show that S, X, T, and P lie on a circle.



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