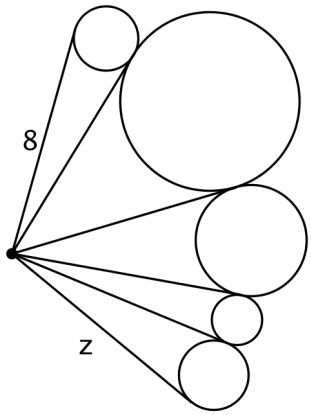
Lesson 12: Tangent Segments

Classwork

Opening Exercise

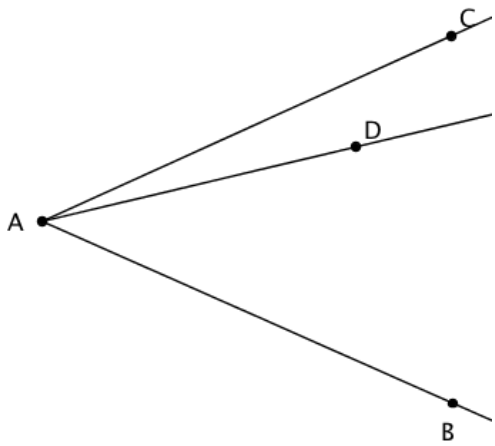
In the diagram, what do you think the length of z could be? How do you know?



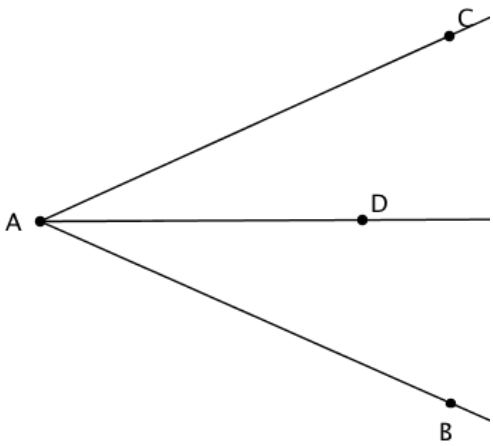
Example 1

In each diagram, try to draw a circle with center D that is tangent to both rays of $\angle BAC$.

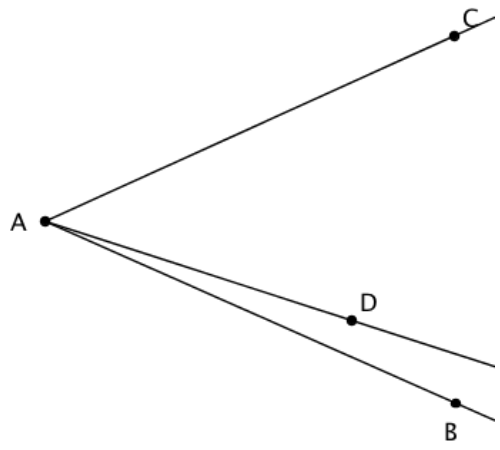
a.



b.



c.



In which diagrams did it seem impossible to draw such a circle? Why did it seem impossible?

What do you conjecture about circles tangent to both rays of an angle? Why do you think that?

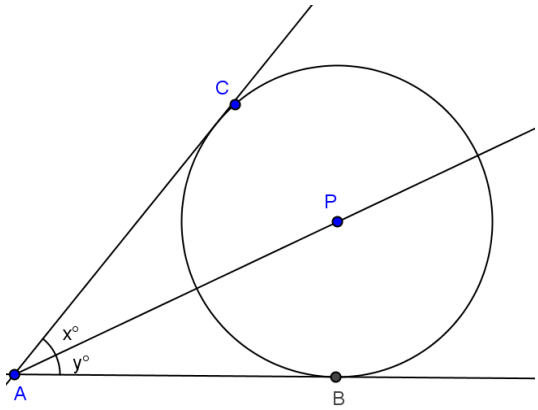
Exercises 1–5

1. You conjectured that *if a circle is tangent to both rays of a circle, then the center lies on the angle bisector*.

   a. Rewrite this conjecture in terms of the notation suggested by the diagram.

   Given:

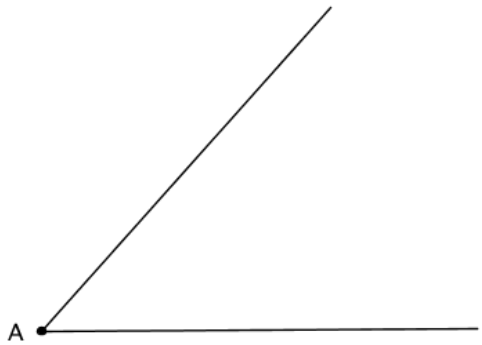
   Need to show:



b. Prove your conjecture using a two-column proof.

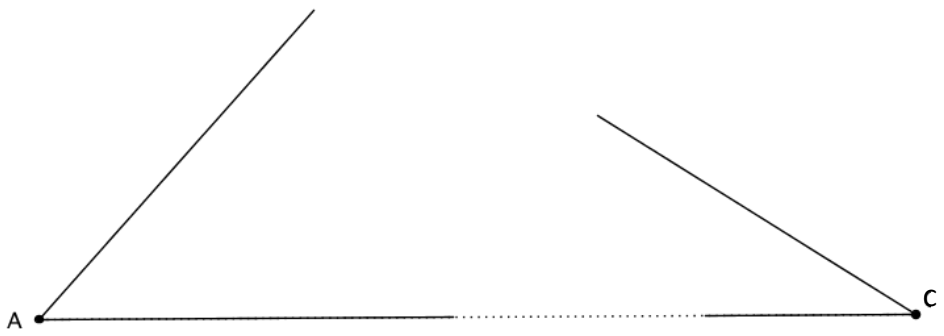
2. An angle is shown below.

a. Draw at least three different circles that are tangent to both rays of the given angle.



b. Label the center of one of your circles with P. How does the distance between P and the rays of the angle compare to the radius of the circle? How do you know?

3. Construct as many circles as you can that are tangent to both the given angles at the same time. You can extend the rays as needed. These two angles share a side.



Explain how many circles you can draw to meet the above conditions and how you know.

4. In a triangle, let P be the location where two angle bisectors meet. Must P be on the third angle bisector as well? Explain your reasoning.

5. Using a straightedge, draw a large triangle ABC.

a. Construct a circle inscribed in the given triangle.

b. Explain why your construction works.

c. Do you know another name for the intersection of the angle bisectors in relation to the triangle?

Lesson Summary

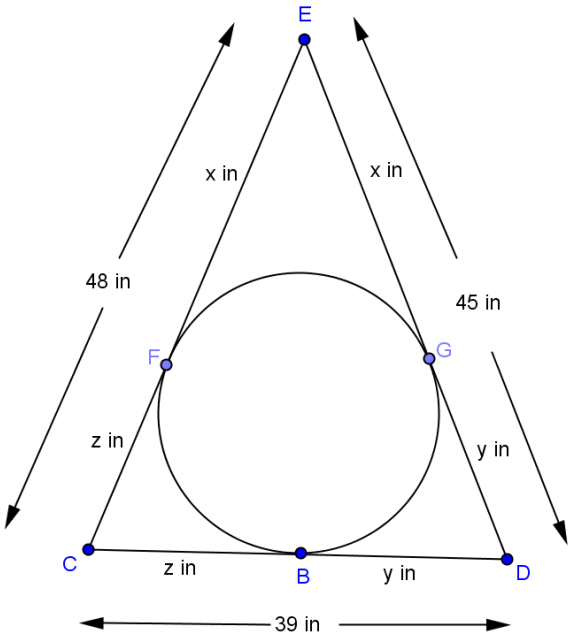
THEOREMS:

- The two tangent segments to a circle from an exterior point are congruent.
- If a circle is tangent to both rays of an angle, then its center lies on the angle bisector.
- Every triangle contains an inscribed circle whose center is the intersection of the triangle's angle bisectors.

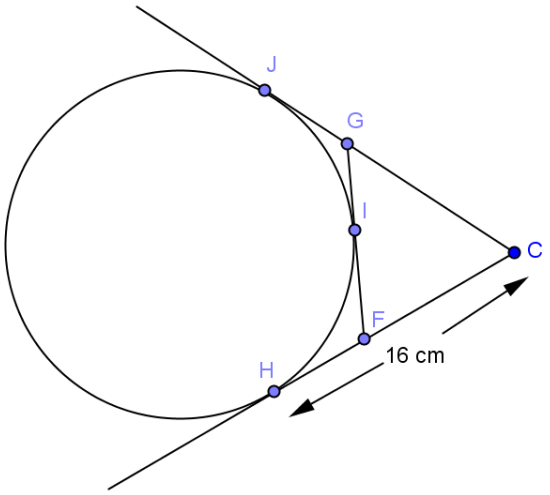
Problem Set

1. On a piece of paper, draw a circle with center A and a point, C, outside of the circle.
   a. How many tangents can you draw from C to the circle?
   b. Draw two tangents from C to the circle, and label the tangency points D and E. Fold your paper along the line AC. What do you notice about the lengths of $\overline{CD}$ and $\overline{CE}$? About the measures of $\angle DCA$ and $\angle ECA$?
   c. $\overline{AC}$ is the _____ of $\angle DCE$.
   d. $\overline{CD}$ and $\overline{CE}$ are tangent to circle A. Find AC.

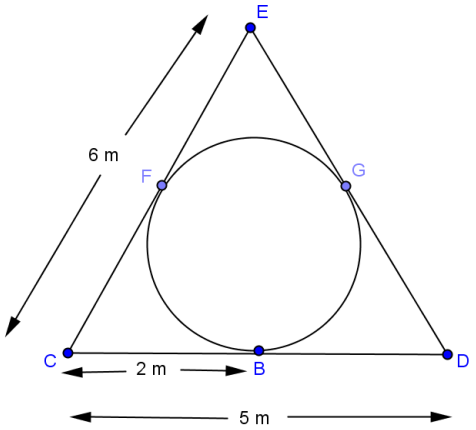
2. In the figure, the three segments are tangent to the circle at points B, F, and G. If $y = \frac{2}{3}x$, find x, y, and z.



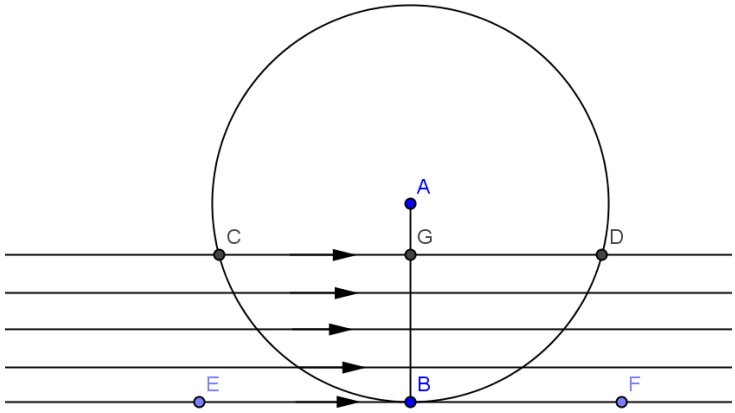
3. In the figure given, the three segments are tangent to the circle at points J, I, and H.
   a. Prove $GF = GJ + HF$.
   b. Find the perimeter of ΔGCF.



4. In the figure given, the three segments are tangent to the circle at points F, B, and G. Find DE.



5. $\overleftrightarrow{EF}$ is tangent to circle A. If points C and D are the intersection points of circle A and any line parallel to $\overleftrightarrow{EF}$, answer the following.



a. Does $CG = GD$ for any line parallel to $\overleftrightarrow{EF}$? Explain.

b. Suppose that $\overleftrightarrow{CD}$ coincides with $\overleftrightarrow{EF}$. Would C, G, and D all coincide with B?

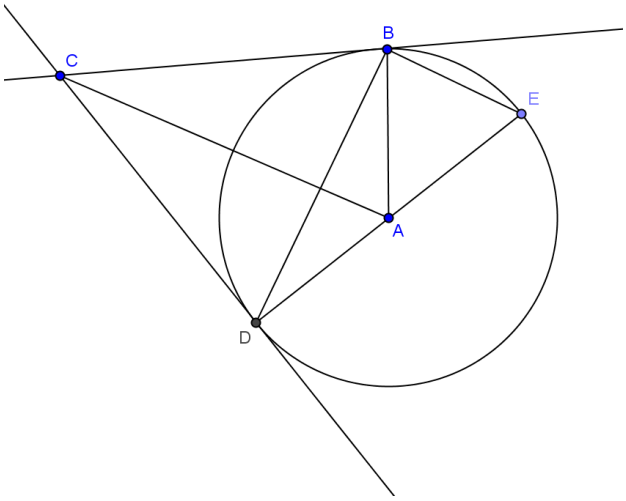
c. Suppose C, G, and D have now reached B, so $\overleftrightarrow{CD}$ is tangent to the circle. What is the angle between $\overleftrightarrow{CD}$ and $\overline{AB}$?

d. Draw another line tangent to the circle from some point, P, in the exterior of the circle. If the point of tangency is point T, what is the measure of $\angle PTA$?

6. The segments are tangent to circle A at points B and D. $\overline{ED}$ is a diameter of the circle.

a. Prove $\overline{BE} \| \overline{CA}$.

b. Prove quadrilateral $ABCD$ is a kite.



7. In the diagram shown, $\overleftrightarrow{BH}$ is tangent to the circle at point B. What is the relationship between $\angle DBH$, the angle between the tangent and a chord, and the arc subtended by that chord and its inscribed angle $\angle DCB$?