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Arithmetic Operations Including Division of Fractions

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[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Topics A through D (assessment 1 day, return 1 day, remediation or further applications 1 day)

Grade 6 • Module 2

Arithmetic Operations Including Division of Fractions

OVERVIEW

In Module 1, students used their existing understanding of multiplication and division as they began their study of ratios and rates. In Module 2, students complete their understanding of the four operations as they study division of whole numbers, division by a fraction, and operations on multi-digit decimals. This expanded understanding serves to complete their study of the four operations with positive rational numbers, thereby preparing students for understanding, locating, and ordering negative rational numbers (Module 3) and algebraic expressions (Module 4).

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. They construct division stories and solve word problems involving division of fractions (**6.NS.A.1**). Through the context of word problems, students understand and use partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask, "How much is one share?" and "What part of the unit is that share?" Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. For example, students understand that $\frac{6 \text{ cm}}{2 \text{ cm}}$ derives a quotient of 3 because 2 divides into 6 three times. They apply this method to quotients of fractions, understanding $\frac{6}{7} \div \frac{2}{7} = \frac{6 \text{ sevenths}}{2 \text{ sevenths}} = 3$ because, again, 2 divides into 6 three times. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Using this relationship, students create equations and formulas to represent and solve problems. Later in the module, students learn the direct correlation of division of fractions to division of decimals along with the application of this concept.

Prior to division of decimals, students will revisit all decimal operations in Topic B. Students have had extensive experience with decimal operations to the hundredths and thousandths (**5.NBT.B.7**), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to the last lesson in Topic A, which focused on mixed numbers. They find that sums and differences of large mixed numbers can sometimes be more efficiently determined by first converting the number to a decimal and then applying the standard algorithms (**6.NS.B.3**). They use estimation to justify their answers.

Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. By gaining fluency in the distributive property throughout this module and the next, students will be proficient in applying the distributive property in Module 4 (**6.EE.A.3**). Estimation and place

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value enable students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students learn to use connections between fraction multiplication and decimal multiplication.

In Grades 4 and 5, students used concrete models, pictorial representations, and properties to divide whole numbers (**4.NBT.B.6**, **5.NBT.B.6**). They became efficient in applying the standard algorithm for long division. Students broke dividends apart into like base-ten units, applying the distributive property to find quotients place by place. In Topic C, students connect estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (**6.NS.B.2**). Students understand that when they "bring down" the next digit in the algorithm, they are essentially distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (**6.NS.B.3**). They make connections to division of fractions and rely on mental math strategies to implement the division algorithm when finding the quotients of decimals.

In the final topic, students think logically about multiplicative arithmetic. In Topic D, students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid's Algorithm is a more efficient way to find the greatest common factor of larger numbers and see that Euclid's Algorithm is based on long division.

The module comprises 19 lessons; six days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.A.1   Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc). How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.B.2   Fluently divide multi-digit numbers using the standard algorithm.

6.NS.B.3   Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

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6.NS.B.4   Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Foundational Standards

Gain familiarity with factors and multiples.

4.OA.B.4   Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Understand the place value system.

5.NBT.A.2   Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.B.6   Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.B.7   Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.4   Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a.   Interpret the product *(a/b) × q* as a parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a × q ÷ b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

5.NF.B.7   Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by fractions.[2]

   a.   Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

   b.   Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

Focus Standards for Mathematical Practice

MP.1   **Make sense of problems and persevere in solving them.** Students use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" For instance, when determining the quotient of fractions, students ask themselves how many sets or groups of the divisor is in the dividend. That quantity is the quotient of the problem. They solve simpler problems to gain insight into the solution. They will confirm, for example, that $10 ÷ 2$ can be found by determining how many groups of two are in ten. They will apply that strategy to the division of fractions. Students may use pictorial representations such as area models, array models, number lines, and drawings to conceptualize and solve problems.

MP.2   **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships in problems. They understand "how many" as it pertains to the divisor in a quotient of fractions problem. They understand and use connections between divisibility and the greatest common factor to apply the distributive property. Students consider units and labels for numbers in contextual problems and consistently refer to what the labels represent to make sense in the problem. Students rely on estimation and properties of operations to justify the reason for their answers when manipulating decimal numbers and their operations. Students reason abstractly when applying place value and fraction sense when determining the placement of a decimal point.

MP.6   **Attend to Precision.** Students use precise language and place value when adding, subtracting, multiplying, and dividing by multi-digit decimal numbers. Students read decimal numbers using place value. For example, 326.31 is read as three hundred twenty-six and thirty-one hundredths. Students calculate sums, differences, products, and quotients of decimal numbers with a degree of precision appropriate to the problem context.

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[2] Students who are able to multiply fractions in general can develop strategies to divide fractions in general by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement in Grade 5.

MP.7 **Look for and make use of structure**. Students find patterns and connections when multiplying and dividing multi-digit decimals. For instance, they use place value to recognize that the quotient of $22.5 \div 0.15$ is the same as the quotient of $2250 \div 15$. In the example $36 + 48 = 12(3 + 4)$, students recognize that when expressing the sum of two whole numbers using the distributive property, the number 12 represents the greatest common factor of 36 and 48 and that 36 and 48 are both multiples of 12. When dividing fractions, students recognize and make use of a related multiplication problem or create a number line and use skip counting to determine the number of times the divisor is added to obtain the dividend. Students use the familiar structure of long division to find the greatest common factor in another way, specifically the Euclidean Algorithm.

MP.8 **Look for and express regularity in repeated reasoning**. Students determine reasonable answers to problems involving operations with decimals. Estimation skills and compatible numbers are used. For instance, when 24.385 is divided by 3.91, students determine that the answer will be close to the quotient of $24 \div 4$, which equals 6. Students discover, relate, and apply strategies when problem-solving, such as the use of the distributive property to solve a multiplication problem involving fractions and/or decimals (e.g., $350 \times 1.8 = 350(1 + 0.8) = 350 + 280 = 630$). When dividing fractions, students may use the following reasoning: Since $2/7 + 2/7 + 2/7 = 6/7$, then $6/7 \div 2/7 = 3$, so I can solve fraction division problems by first getting common denominators and then solving the division problem created by the numerators. Students understand the long-division algorithm and the continual breakdown of the dividend into different place value units. Further, students use those repeated calculations and reasoning to determine the greatest common factor of two numbers using the Euclidean Algorithm.

Terminology

New or Recently Introduced Terms

- **Greatest Common Factor** (The largest positive integer that divides into two or more integers without a remainder; the GCF of 24 and 36 is 12 because when all of the factors of 24 and 36 are listed, the largest factor they share is 12.)
- **Least Common Multiple** (The smallest positive integer that is divisible by two or more given integers without a remainder; the LCM of 4 and 6 is 12 because when the multiples of 4 and 6 are listed, the smallest or first multiple they share is 12.)
- **Multiplicative Inverses** (Two numbers whose product is 1 are multiplicative inverses of one another. For example, $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$. Multiplicative inverses do not always have to be the reciprocal. For example $\frac{1}{5}$ and $\frac{10}{2}$ both have a product of 1, which makes them multiplicative inverses.)

Familiar Terms and Symbols[3]

- Prime Number
- Composite Number
- Factors
- Multiples
- Dividend
- Divisor
- Reciprocal
- Algorithm
- Distributive Property
- Estimate

Suggested Tools and Representations

- Counters
- Fraction Tiles (example shown to the right)
- Tape Diagrams
- Area Models (example shown below)

For example: $\frac{3}{4} \div \frac{1}{2}$

$\frac{2}{5} \div 5$
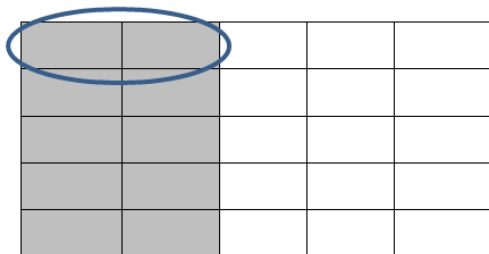
Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.NS.A.1, 6.NS.B.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.NS.A.1, 6.NS.B.2, 6.NS.B.3, 6.NS.B.4

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[3] These are terms and symbols students have seen previously.

Topic A:

Dividing Fractions by Fractions

6.NS.A.1

Focus Standard:	6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc). How much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*
Instructional Days:	8	
Lesson 1:		Interpreting Division of a Fraction by a Whole Number—Visual Models (P)[1]
Lesson 2:		Interpreting Division of a Whole Number by a Fraction—Visual Models (P)
Lessons 3–4:		Interpreting and Computing Division of a Fraction by a Fraction—More Models (P)
Lesson 5:		Creating Division Stories (P)
Lesson 6:		More Division Stories (P)
Lesson 7:		The Relationship Between Visual Fraction Models and Equations (S)
Lesson 8:		Dividing Fractions and Mixed Numbers (P)

In Topic A, students extend their previous understanding of multiplication and division to divide fractions by fractions. Students determine quotients through visual models, such as bar diagrams, tape diagrams, arrays, and number line diagrams. They construct division stories and solve word problems involving division of fractions (**6.NS.A.1**). Students understand and apply partitive division of fractions to determine how much is in each group. They explore real-life situations that require them to ask themselves, "How much is one share?" and "What part of the unit is that share?" Students use measurement to determine quotients of fractions. They are presented conceptual problems where they determine that the quotient represents how many of the divisor is in the dividend. Students look for and uncover patterns while modeling quotients of fractions to ultimately discover the relationship between multiplication and division. Later in the module, students will understand and apply the direct correlation of division of fractions to division of decimals.

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[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Student Outcomes

- Students use visual models, such as fraction bars, number lines, and area models, to show the quotient of whole numbers and fractions and to show the connection between them and the multiplication of fractions.
- Students divide a fraction by a whole number.

Classwork

Opening Exercise (5 minutes)

At the beginning of class, hand each student a fraction card (see page 18). Ask students to do the following Opening Exercise.

> **Opening Exercise**
>
> **Draw a model of the fraction.**
>
>
> **Describe what the fraction means.**

After two minutes, have students share some of their models and descriptions. Emphasize the key point that a fraction shows division of the numerator by the denominator. In other words, a fraction shows a part being divided by a whole. Also, remind students that fractions are numbers; therefore, they can be added, subtracted, multiplied, or divided.

To conclude the Opening Exercise, students can share where their fractions would be located on a number line. A number line can be drawn on a chalkboard or projected onto a board. Then, students can describe how the fractions on the cards would be placed in order on the number line.

Example 1 (7 minutes)

This lesson will focus on fractions divided by whole numbers. Students learned how to divide unit fractions by whole numbers in Grade 5. Teachers can become familiar with what was taught on this topic by reviewing the materials used in the Grade 5, Module 4 lessons and assessments.

Scaffolding:

Each class should have a set of fraction tiles. Students who are struggling may benefit from using the fraction tiles to see the division until they are better at drawing the models.

> **Example 1**
>
> Maria has $\frac{3}{4}$ lb. of trail mix. She needs to share it equally among 6 friends. How much will each friend be given? What is this question asking us to do?
>
> *We are being asked to divide the trail mix into six equal portions. So, we need to divide three-fourths by six.*
>
> **How can this question be modeled?**

- Let's take a look at how to solve this using a number line and a fraction bar.

We will start by creating a number line broken into fourths and a fraction bar broken into fourths.
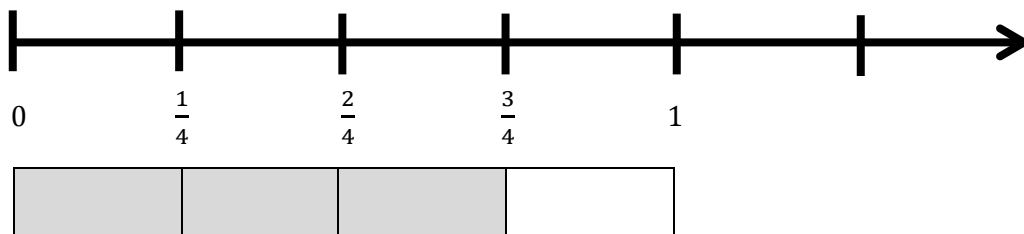
- We are going to give equal amounts of trail mix to each person. How can we show this in the model?
    - *We will divide the shaded portion so that it includes six equal-sized pieces.*
- How will we show this on the number line?
    - *There are three equal sections on the number line that also need to be divided into six equal shares.*
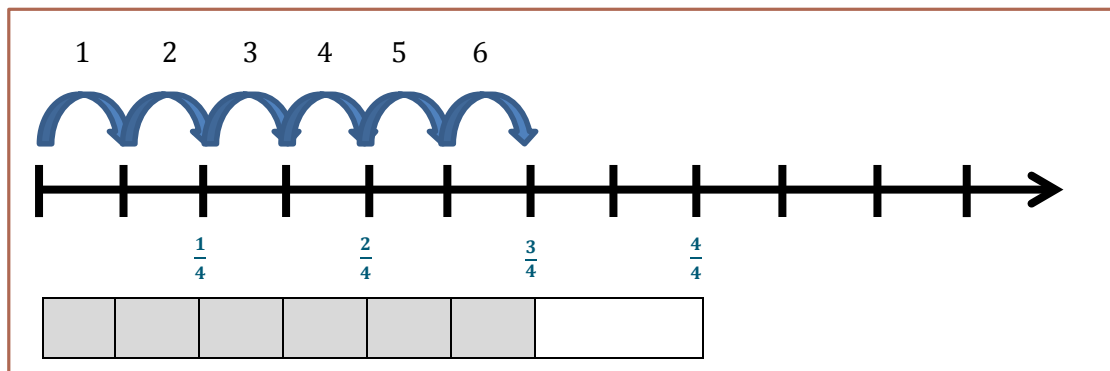
- Next, we need to determine the unit. What did we do to each of the three sections in the fraction bar?
    - *We divided them into two pieces.*
- What should we do to the remaining piece of the fraction bar?
    - *Divide it into two pieces.*
- How many pieces are there total?
    - *8 pieces*

- What does each piece or section represent?

  - $\frac{1}{8}$

Therefore, $\frac{3}{4} \div 6 = \frac{1}{8}$. This visual model also shows that $\frac{1}{6}$ of $\frac{3}{4}$ is $\frac{1}{8}$.
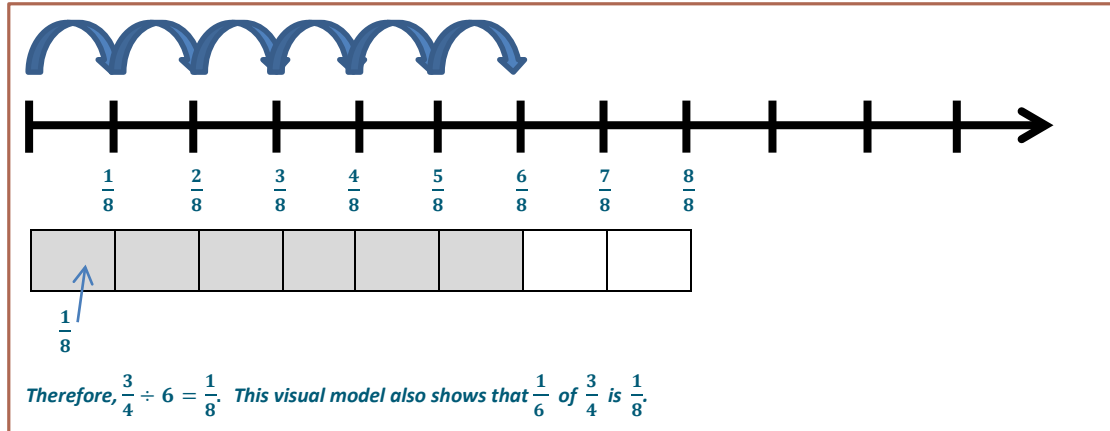
- This is an example of partitive division. You can tell because we were given the original amount of trail mix and how many "parts" of trail mix to make. We needed to determine the size of each part, where the size of each part is less than the original amount.

Example 2 (7 minutes)

MP.1 & MP.2

Example 2

Let's look at a slightly different example. Imagine that you have $\frac{2}{5}$ of a cup of frosting to share equally among three desserts. How would we write this as a division question?

$$\frac{2}{5} \div 3$$

We can start by drawing a model of two-fifths.
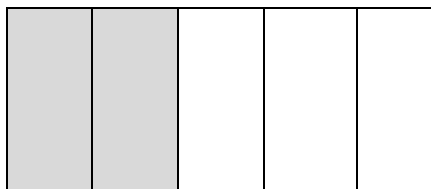
How can we show that we are dividing two-fifths into three equal parts?
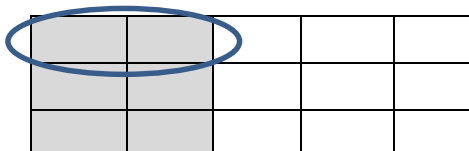
> **What does this part represent?**
>
> *From the visual model, we can determine that $\frac{2}{5} \div 3 = \frac{2}{15}$.*

Exercises 1–5 (16 minutes)

Students will work in pairs to solve the following questions.

Exercises 1–5

For each question below, rewrite the problem as a multiplication question. Then, model the answer.

1. $\frac{1}{2} \div 6 = \frac{1}{12}$

$\frac{1}{6}$ *of* $\frac{1}{2}$ *is* $\frac{1}{12}$.
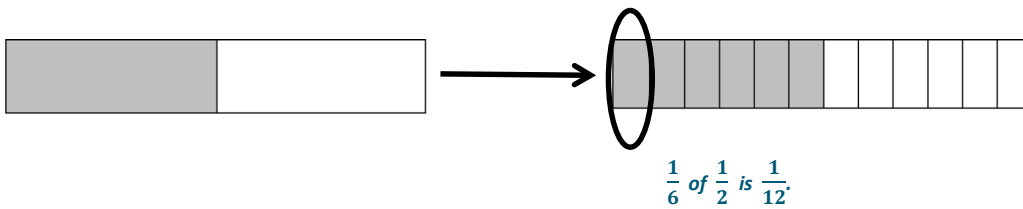
I need to divide $\frac{1}{2}$ into 6 equal sections. Or, I need to rewrite the problem as $\frac{1}{6}$ of $\frac{1}{2}$.

2. $\frac{1}{3} \div 3 = \frac{1}{9}$

$\frac{1}{3}$ *of* $\frac{1}{3}$ *is* $\frac{1}{9}$.
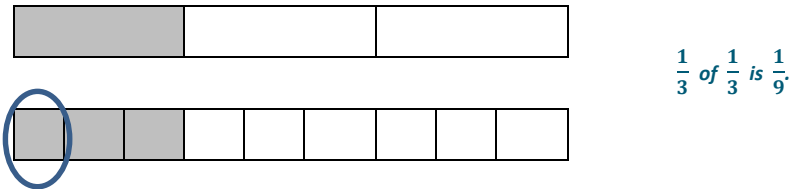
I need to divide $\frac{1}{3}$ into 3 equal sections. Or, I need to rewrite the problem as $\frac{1}{3}$ of $\frac{1}{3}$.

3. $\frac{1}{5} \div 4 = \frac{1}{20}$

$\frac{1}{5}$

$\frac{1}{4}$ *of* $\frac{1}{5}$*, or* $\frac{1}{20}$
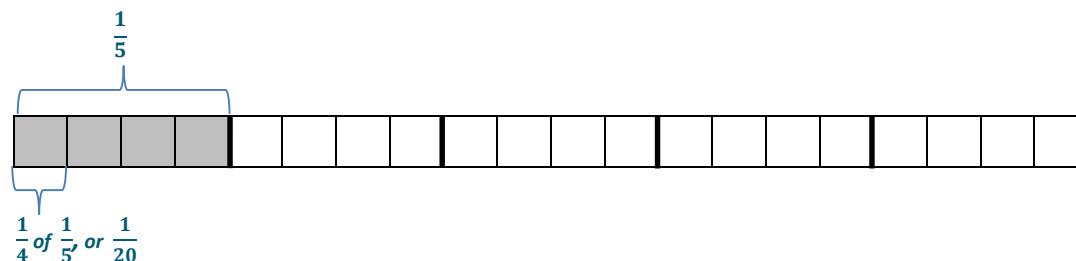
I need to divide $\frac{1}{5}$ into 4 equal sections. Or, I need to rewrite the problem as $\frac{1}{4}$ of $\frac{1}{5}$.

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4. $\dfrac{3}{5} \div 4 = \dfrac{3}{20}$

I need to divide $\dfrac{3}{5}$ into 4 equal sections. Or, I need to rewrite the problem as $\dfrac{1}{4}$ of $\dfrac{3}{5}$.

5. $\dfrac{2}{3} \div 4 = \dfrac{2}{12}$, *or* $\dfrac{1}{6}$

I need to divide $\dfrac{2}{3}$ into 4 equal sections. Or, I need to rewrite the problem as $\dfrac{1}{4}$ of $\dfrac{2}{3}$.

Closing (5 minutes)

- When a fraction is divided by a whole number, how does the answer compare with the dividend (the original fraction)?
  - □ *Students should notice that the quotient is smaller than the original fraction.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 1: Interpreting Division of a Fraction by a Whole Number—Visual Models

Exit Ticket

Find the quotient using a model.

1. $\dfrac{2}{3} \div 3$

2. $\dfrac{5}{6} \div 2$

Exit Ticket Sample Solutions

Solve each division problem using a model.

1. $\dfrac{2}{3} \div 3 = \dfrac{2}{9}$

2. $\dfrac{5}{6} \div 2$

$$\dfrac{5}{12}$$
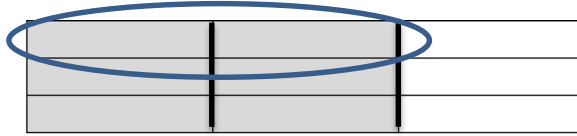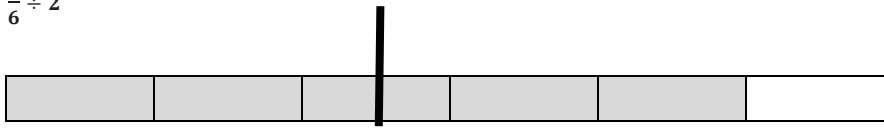
$$\dfrac{5}{6} \div 2 = \dfrac{5}{12}$$

Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model your answer.
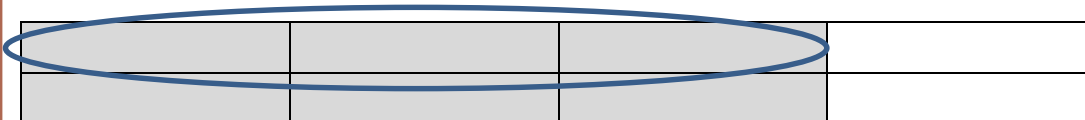
1. $\dfrac{2}{5} \div 5$

I need to find $\dfrac{1}{5}$ of $\dfrac{2}{5}$. I would get $\dfrac{2}{25}$.
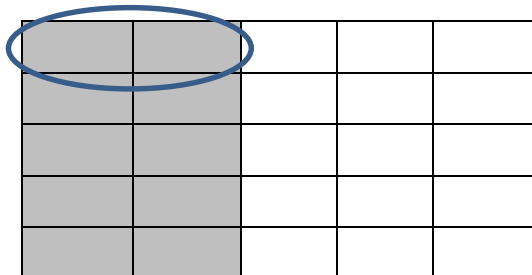
2. $\frac{3}{4} \div 2$

I need to find $\frac{1}{2}$ of $\frac{3}{4}$. I would get $\frac{3}{8}$.

Fraction cards to use at the beginning of class:

$\dfrac{1}{2}$	$\dfrac{3}{4}$	$\dfrac{2}{3}$	$\dfrac{2}{5}$
$\dfrac{3}{5}$	$\dfrac{4}{5}$	$\dfrac{1}{5}$	$\dfrac{2}{6}$
$\dfrac{1}{3}$	$\dfrac{3}{6}$	$\dfrac{4}{6}$	$\dfrac{5}{6}$
$\dfrac{7}{8}$	$\dfrac{5}{8}$	$\dfrac{1}{8}$	$\dfrac{3}{8}$

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Student Outcomes

- Students use fraction bars, number lines, and area models to show the quotient of whole numbers and fractions and to show the connection between those models and the multiplication of fractions.
- Students understand the difference between a whole number being divided by a fraction and a fraction being divided by a whole number.

Classwork

Example 1 (15 minutes)

At the beginning of class, break students into groups. Each group will need to answer the question they have been assigned and draw a model to represent their answer. Multiple groups could have the same question.

Group 1: How many half-miles are in 12 miles? $12 \div \frac{1}{2} = 24$

Group 2: How many quarter hours are in 5 hours? $5 \div \frac{1}{4} = 20$

Group 3: How many one-third cups are in 9 cups? $9 \div \frac{1}{3} = 27$

Group 4: How many one-eighth pizzas are in 4 pizzas? $4 \div \frac{1}{8} = 32$

MP.1 & MP.2

Group 5: How many one-fifths are in 7 wholes? $7 \div \frac{1}{5} = 35$

Models will vary, but could include fraction bars, number lines, or area models (arrays).

Students will draw models on blank paper, construction paper, or chart paper. Hang up only student models, and have students travel around the room answering the following:

1. Write the division question that was answered with each model.
2. What multiplication question could this model also answer?
3. Rewrite the question given to each group as a multiplication question.

Students will be given a table to fill in as they visit each model.

When discussing the opening of this example, ask students how these questions are different from the questions solved in Lesson 1. Students should notice that these questions are dividing whole numbers by fractions, while the questions in Lesson 1 were dividing fractions by whole numbers.

Discuss how the division problem is related to the multiplication problem. Students should recognize that when 12 is divided into halves, it is the same as doubling 12.

Example 1

Question # _____

Write it as a division question. _____

Write it as a multiplication question. _____

Make a rough draft of a model to represent the question:

As you travel to each model, be sure to answer the following questions:

Original Questions	Write the division question that was answered in each model.	What multiplication question could the model also answer?	Write the question given to each group as a multiplication question.
1. How many $\frac{1}{2}$ miles are in 12 miles?	$12 \div \frac{1}{2}$	$12 \times 2 = ?$	*Answers will vary.*
2. How many quarter hours are in 5 hours?	$5 \div \frac{1}{4}$	$5 \times 4 = ?$	
3. How many $\frac{1}{3}$ cups are in 9 cups?	$9 \div \frac{1}{3}$	$9 \times 3 = ?$	
4. How many $\frac{1}{8}$ pizzas are in 4 pizzas?	$4 \div \frac{1}{8}$	$4 \times 8 = ?$	
5. How many one-fifths are in 7 wholes?	$7 \div \frac{1}{5}$	$7 \times 5 = ?$	

Example 2 (5 minutes)

- All of the problems in the first example show what is called *measurement division*. When we know the original amount and the size or measure of one part, we use measurement division to find the number of parts. You can tell when a question is asking for measurement division because it asks, "How many _____ are in _____?"

- Let's take a look at a different example:

Example 2

Molly uses 9 cups of flour to bake bread. If this is $\frac{3}{4}$ of the total amount of flour she started with, what was the original amount of flour?

- How is this question different from the measurement questions?
  - *In this example, we are not trying to figure out how many three-fourths are in* 9*. We know that* 9 *cups is a part of the entire amount of flour needed. Instead, we need to determine three-fourths of what number is* 9*.*

---

a. **Create a model to represent what the question is asking.**

9

?
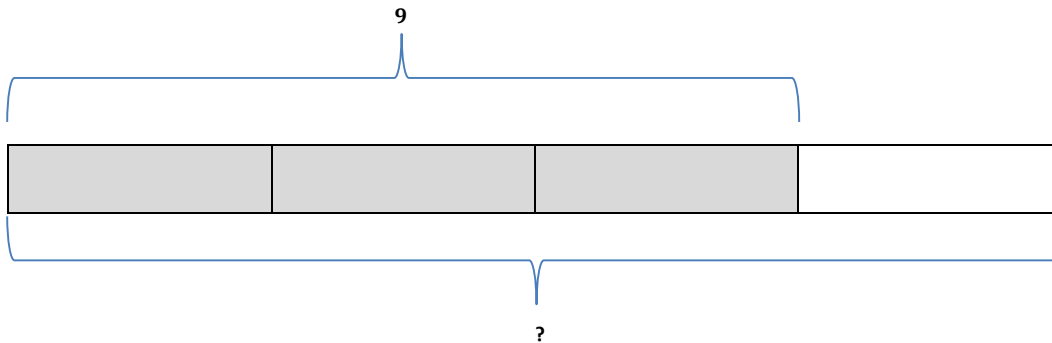
b. **Explain how you would determine the answer using the model.**

To divide 9 *by* $\frac{3}{4}$ *, we divide* 9 *by* 3 *to get the amount for each rectangle; then, we multiply by* 4 *because there are* 4 *rectangles total.*

$9 \div 3 = 3$ $3 \times 4 = 12$. *Now, I can see that there were originally* 12 *cups of flour.*

9

3   3   3   3

12
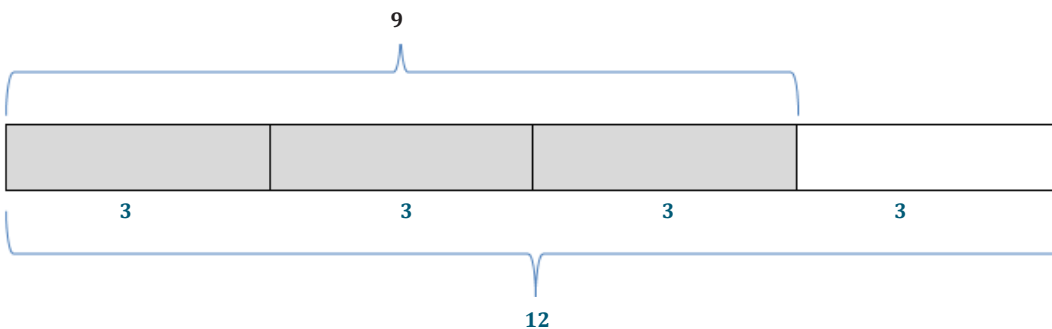
Exercises 1–5 (15 minutes)

Students will work in pairs or on their own to solve the following questions. First, students will write a division expression to represent the situations. Then, students will rewrite each problem as a multiplication question. Finally, they will draw a model to represent the solution.

Allow time for students to share their models. Take time to have students compare the different models that were used to solve each question. For example, allow students to see how a fraction bar and a number line can be used to model Exercise 1.

Exercises 1–5

1. A construction company is setting up signs on 4 miles of the road. If the company places a sign every $\frac{1}{8}$ of a mile, how many signs will it need?

    $4 \div \frac{1}{8}$ ➔ $\frac{1}{8}$ *of what number is 4?*

    | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
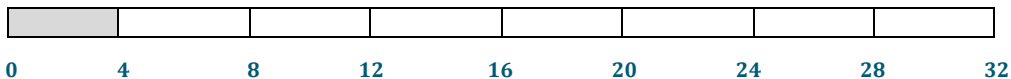
    The company will need 32 *signs.*

2. George bought 12 pizzas for a birthday party. If each person will eat $\frac{3}{8}$ of a pizza, how many people can George feed with 12 pizzas?

    $12 \div \frac{3}{8}$ ➔ $\frac{3}{8}$ *of what number is 12?*

    | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
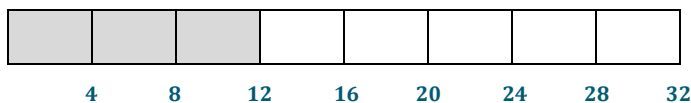
    The pizzas will feed 32 *people.*

3. The Lopez family adopted 6 miles of trail on the Erie Canal. If each family member can clean up $\frac{3}{4}$ of a mile, how many family members are needed to clean the adopted section?

    $6 \div \frac{3}{4}$ ➔ $\frac{3}{4}$ *of what number is 6?*

    | 2 | 4 | 6 | 8 |
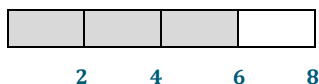
    The Lopez family needs to bring 8 *family members to clean the adopted section.*

---

4. Margo is freezing 8 cups of strawberries. If this is $\frac{2}{3}$ of the total strawberries that were picked, how many cups of strawberries did Margo pick?

$8 \div \frac{2}{3} \rightarrow \frac{2}{3}$ *of what number is 8?*

8

4	4	4

? = 12
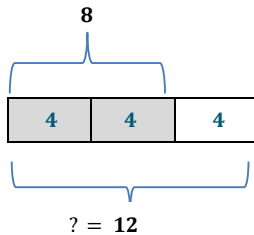
Margo picked 12 cups of strawberries.

5. Regina is chopping up wood. She has chopped 10 logs so far. If the 10 logs represent $\frac{5}{8}$ of all the logs that need to be chopped, how many logs need to be chopped in all?

$10 \div \frac{5}{8} \rightarrow \frac{5}{8}$ *of what number is 10?*

?

2	4	6	8	10	12	14	16

10
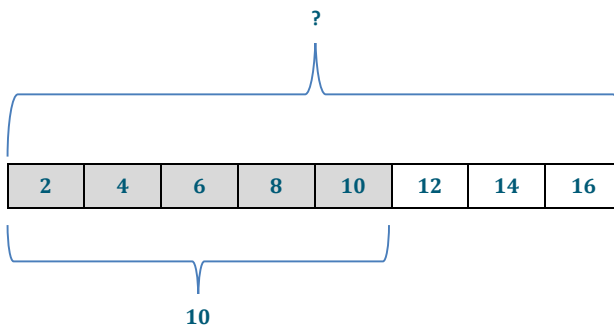
Regina needs to chop 16 logs in all.

Closing (5 minutes)

- What are the key ideas from Lessons 1 and 2?
  - *We can use models to divide a whole number by a fraction and a fraction by a whole number.*
- Over the past two lessons, we have reviewed how to divide a whole number by a fraction and how to divide a fraction by a whole number. The next two lessons will focus on dividing fractions by fractions. Explain how you would use what we have learned about dividing with fractions in the next two lessons.
  - *We can use models to help us divide a fraction by a fraction. We can also use the multiplication problems we wrote as a tool to help us divide fractions by fractions.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 2: Interpreting Division of a Whole Number by a Fraction—Visual Models

Exit Ticket

Solve each division problem using a model.

1.   Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

2.   Rachel completed $\frac{3}{4}$ of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?

Exit Ticket Sample Solutions

Solve each division problem using a model.

1. Henry bought 4 pies which he plans to share with a group of his friends. If there is exactly enough to give each member of the group one-sixth of the pie, how many people are in the group?

$4 \div \frac{1}{6}$ *$\frac{1}{6}$ of what is 4?*

4   8   12   16   20   24
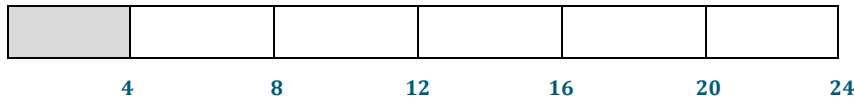
24 people are in the group.

2. Rachel completed $\frac{3}{4}$ of her cleaning in 6 hours. How many total hours will Rachel spend cleaning?

$6 \div \frac{3}{4}$

$\frac{3}{4}$ of what is 6?

?

2	2	2	2

6
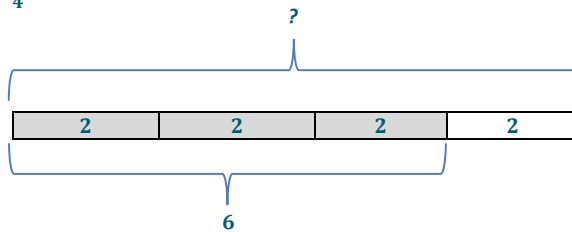
Rachel will spend 8 total hours cleaning.

Problem Set Sample Solutions

Rewrite each problem as a multiplication question. Model your answer.

1. Nicole has used 6 feet of ribbon. This represents $\frac{3}{8}$ of the total amount of ribbon she started with. How much ribbon did Nicole have at the start?

$6 \div \frac{3}{8}$

$\frac{3}{8}$ of what number is 6?

?

2	2	2	2	2	2	2	2

6
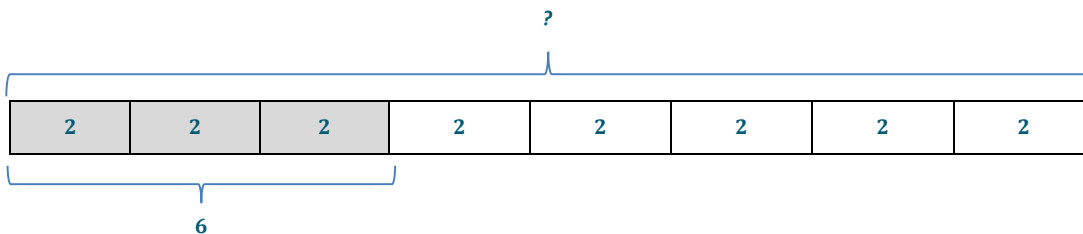
Nicole started with 16 feet of ribbon.

2.   **How many quarter hours are in 5 hours?**

$5 \div \dfrac{1}{4}$

$\dfrac{1}{4}$ of what is 5?

5	5	5	5

There are 20 quarter hours in 5 hours.

Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes

- Students use fraction bars and area models to show the division of fractions by fractions with common denominators.
- Students make connections to the multiplication of fractions. In addition, students understand that to get the quotient when dividing fractions, they must ask, "How many groups of the divisor are in the dividend?"

Classwork

Opening Exercise (5 minutes)

Begin class with a review of how to divide a whole number by a whole number using a model.

Opening Exercise

Draw a model to represent $12 \div 3$.

There are two interpretations:

Measurement Division

12

4

Partitive Division

12

3

COMMON CORE   Lesson 3:   Interpreting and Computing Division of a Fraction by a Fraction—More
                   Models   engage^ny   27
                   Date: 7/3/14
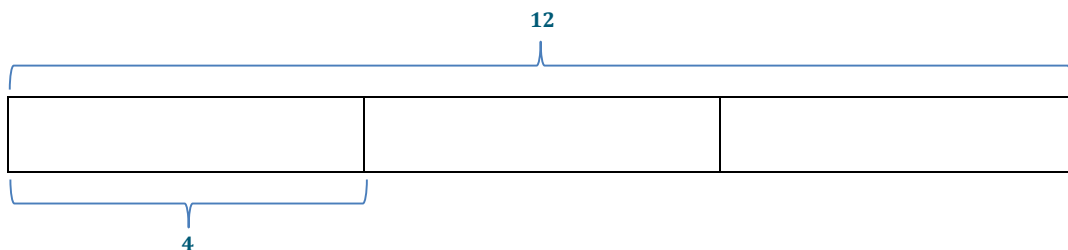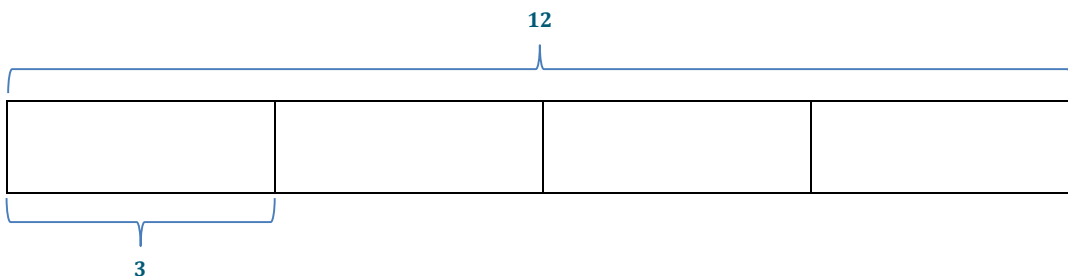
> **How could we reword this question?**
>
> *Answers will vary. Sample Solutions:*
>
> *If we divide* 12 *into three groups of equal size, what is the size of each group?*
>
> *If we divide* 12 *into groups of size 3, how many groups would we have?*
>
> *If I have* 12 *chickens and put* 3 *chickens in each cage, how many cages will I need?*
>
> *If I have* 12 *flowers and place* 3 *flowers in each vase, how many vases will I need?*

Example 1 (5 minutes)

Next, we will introduce an example where students are asked to divide a fraction by a fraction with the same denominator. The whole number examples in the opening are used to give students ideas to build off of when dealing with fractions.

- What is $\frac{8}{9} \div \frac{2}{9}$? Take a moment to use what you know about division to create a model to represent this division problem.

Give students a chance to explore this question and draw models without giving them the answer. After three minutes or so, ask students to share the models that they have created and to discuss what conclusions they have made about dividing fractions with the same denominator.

- *One way to interpret the question is to say how many $\frac{2}{9}$ are in $\frac{8}{9}$. From the model, I can see that there are* 4 *groups of $\frac{2}{9}$ in $\frac{8}{9}$. This would give the same solution as dividing* 8 *by* 2 *to get* 4.

MP.1

Example 1

$$\frac{8}{9} \div \frac{2}{9}$$

Draw a model to show the division problem.
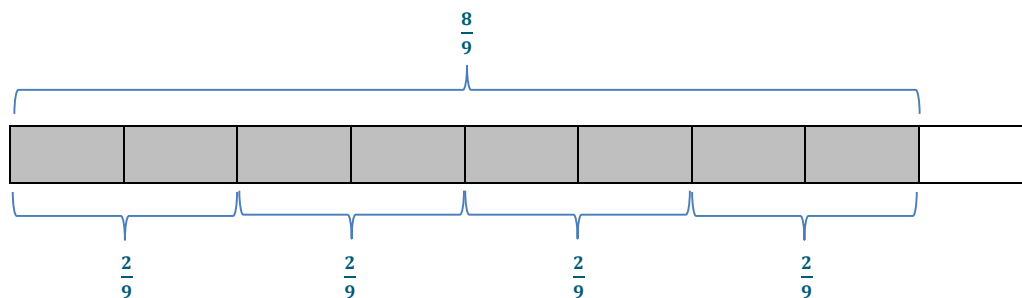
Here we have 4 *groups of $\frac{2}{9}$. Therefore, the answer is* 4.

Example 2 (5 minutes)

Another example of fractions divided by fractions will help students see the connection between the two concepts.

- What is $\frac{9}{12} \div \frac{3}{12}$? Be sure to create a model to support your answer.

    - *One way to interpret this question is by saying how many $\frac{3}{12}$ are in $\frac{9}{12}$. In other words, I need to divide nine twelfths by three twelfths, which is the same as 9 units $\div 3$ units, which is 3.*

MP.1

<div style="border:1px solid">

Example 2

$\frac{9}{12} \div \frac{3}{12}$

Be sure to draw a model to support your answer.

$\frac{9}{12}$

$\frac{3}{12}$ $\frac{3}{12}$ $\frac{3}{12}$
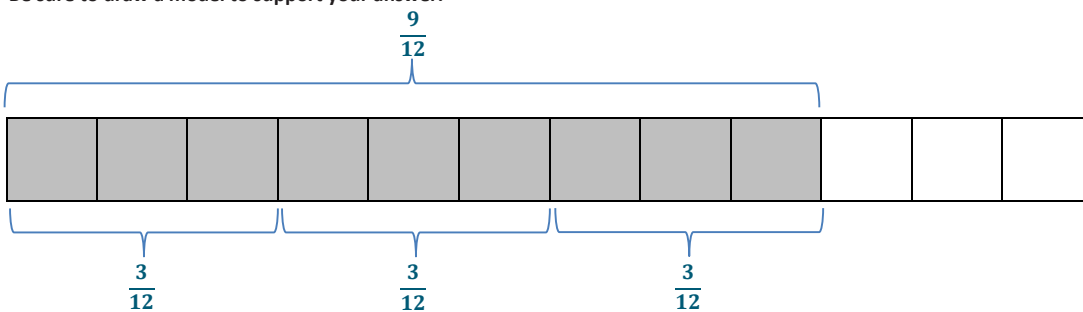
Students might also circle the sections that are equal to $\frac{3}{12}$.
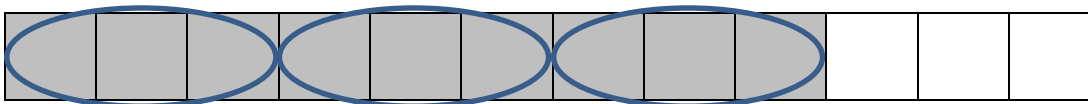
Here we have 3 groups of $\frac{3}{12}$. Therefore, the answer is 3.

</div>

Example 3 (3 minutes)

- What is $\frac{7}{9} \div \frac{3}{9}$? Be sure to create a model to support your answer.

    - *One way to interpret this question is by saying how many $\frac{3}{9}$ are in $\frac{7}{9}$. In other words, I need to divide seven ninths by three ninths, which is the same as 7 units $\div 3$ units, which is $2\frac{1}{3}$.*

- Start by drawing a model in order to divide the two fractions.

    - $\frac{7}{9} \div \frac{3}{9}$

Example 3

$$\frac{7}{9} \div \frac{3}{9}$$

Be sure to create a model to support your answer.

$$\frac{7}{9}$$

MP.1

$$\frac{3}{9} \qquad \frac{3}{9} \qquad \frac{3}{9}$$
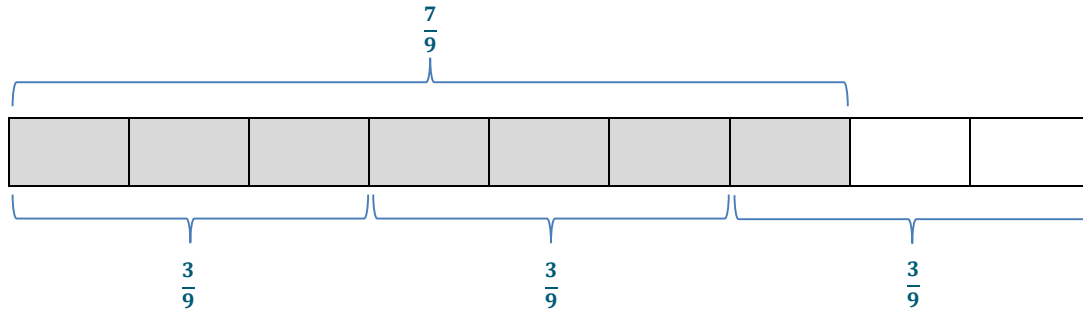
- ▪ How many whole $\frac{3}{9}$ will go into $\frac{7}{9}$?

  - □ *2 wholes and then part of a whole*

- ▪ How do we represent the remainder?

  - □ *There is one out of the three needed pieces to make another whole. So, the remainder is $\frac{1}{3}$.*

  - □ *This means that $\frac{7}{9} \div \frac{3}{9} = 2\frac{1}{3}$.*

  - □ *This is the same as $7 \div 3$.*

Exercises 1–6 (17 minutes)

Students will work in pairs or alone to solve more questions about division with like denominators.

Exercises 1–6

For the following exercises, rewrite the division problem. Then, be sure to draw a model to support your answer.

1. **How many fourths are in three fourths?**

   I need to divide three fourths by one fourth, which is 3.

   $$\frac{3}{4} \div \frac{1}{4}$$

   Draw a model to support your answer.

   $$0 \qquad \frac{1}{4} \qquad \frac{2}{4} \qquad \frac{3}{4}$$
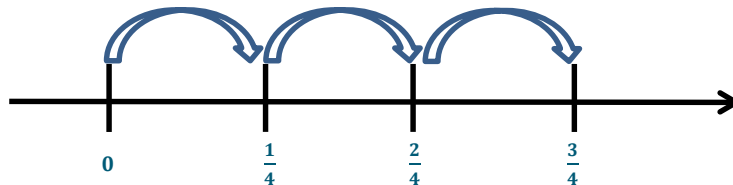
   There are 3 one-fourths in three-fourths.

How are Example 2 and Exercise 1 similar?

Both questions have a quotient of 3.

How are the divisors and dividends related?

$\frac{9}{12}$ *is equivalent to* $\frac{3}{4}$*, and* $\frac{3}{12}$ *is equivalent to* $\frac{1}{4}$*.*

$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$*, and* $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$*.*

What conclusions can you draw from these observations?

When the dividend and divisor are scaled up or scaled down by the same amounts, the quotient stays the same.

2. $\frac{4}{5} \div \frac{2}{5}$

This is really four fifths divided by two fifths, which is 2.
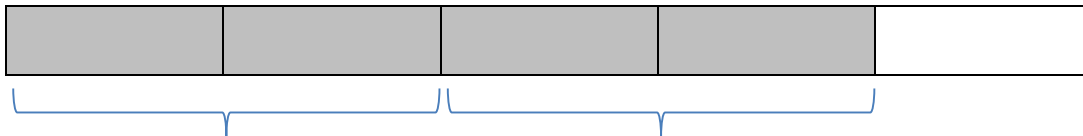
3. $\frac{9}{4} \div \frac{3}{4}$
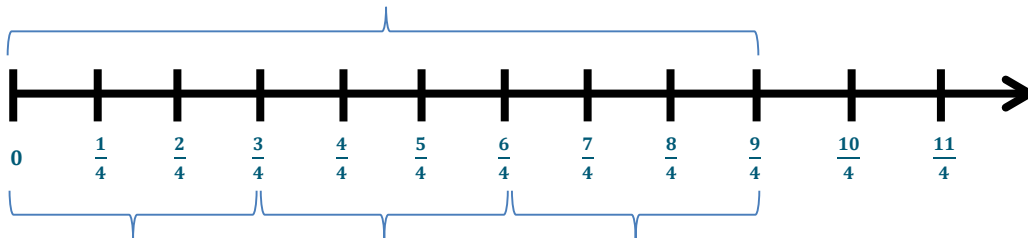
This is really nine fourths divided by three fourths, which is 3.

4. $\frac{7}{8} \div \frac{2}{8}$

This is really seven eighths divided by two eighths, which is $\frac{7}{2}$ *or* $3\frac{1}{2}$*.*

5. $\dfrac{13}{10} \div \dfrac{2}{10}$
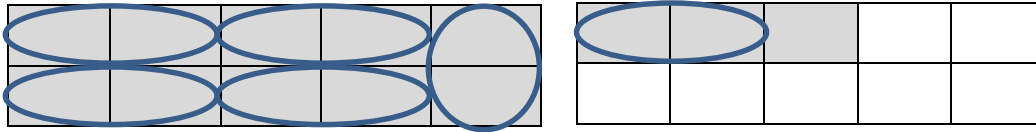
This is really thirteen sixths divided by two sixths, which is $\dfrac{13}{2}$ or $6\dfrac{1}{2}$.

6. $\dfrac{11}{9} \div \dfrac{3}{9}$

| 0 | $\dfrac{1}{9}$ | $\dfrac{2}{9}$ | $\dfrac{3}{9}$ | $\dfrac{4}{9}$ | $\dfrac{5}{9}$ | $\dfrac{6}{9}$ | $\dfrac{7}{9}$ | $\dfrac{8}{9}$ | $\dfrac{9}{9}$ | $\dfrac{10}{9}$ | $\dfrac{11}{9}$ |
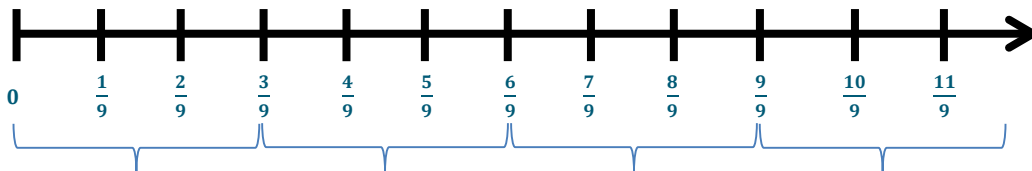
This is really eleven ninths divided by three ninths, which is $\dfrac{11}{3}$ or $3\dfrac{2}{3}$.

Closing (5 minutes)

Depending on how much time you have in the class, you could have each student write to another student an actual note that contains models and a description of the ideas discussed in class. Or, if time is short, this can be a discussion.

- Imagine that your best friend missed today's lesson. What key ideas would you want your friend to know in order to be able to divide fractions by fractions with the same denominator?
  - □ *We can use a variety of models to show that when dividing fractions by fractions with the same denominator, it is equivalent to dividing the numerators.*

> **Lesson Summary**
>
> When dividing a fraction by a fraction with the same denominator, we can use the general rule $\dfrac{a}{c} \div \dfrac{b}{c} = \dfrac{a}{b}$.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 3: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Exit Ticket

Draw a model to support your answer to the division questions.

1. $\dfrac{9}{4} \div \dfrac{3}{4}$

2. $\dfrac{7}{3} \div \dfrac{2}{3}$

Exit Ticket Sample Solutions

Draw a model to support your answer to the division questions.

1. $\dfrac{9}{4} \div \dfrac{3}{4}$

   This is really nine fourths ÷ three fourths = 3.

| 0 | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | $\frac{4}{4}$ | $\frac{5}{4}$ | $\frac{6}{4}$ | $\frac{7}{4}$ | $\frac{8}{4}$ | $\frac{9}{4}$ | $\frac{10}{4}$ | $\frac{11}{4}$ |
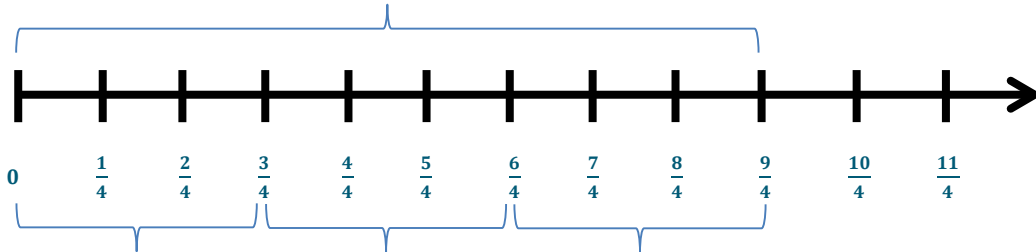
2. $\dfrac{7}{3} \div \dfrac{2}{3}$

   This is really asking seven thirds ÷ two thirds, which is $\dfrac{7}{2}$ or $3\dfrac{1}{2}$.

| 0 | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{3}{3}$ | $\frac{4}{3}$ | $\frac{5}{3}$ | $\frac{6}{3}$ | $\frac{7}{3}$ | $\frac{8}{3}$ | $\frac{9}{3}$ | $\frac{10}{3}$ | $\frac{11}{3}$ |
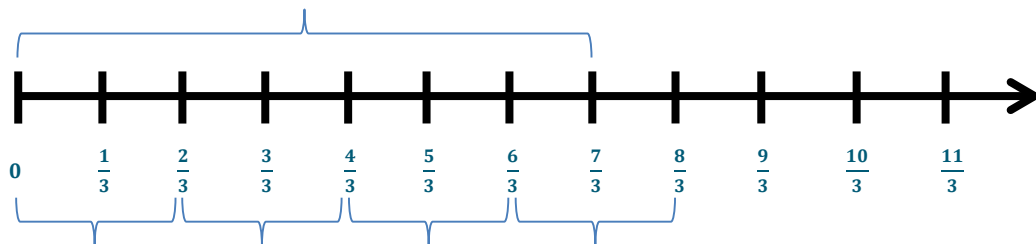
Problem Set Sample Solutions

For the following exercises, rewrite the division problem in words. Then, be sure to draw a model to support your answer.

1. $\dfrac{15}{4} \div \dfrac{3}{4}$

   Fifteen fourths ÷ three fourths = 5.

   $\dfrac{15}{4}$

   $\dfrac{3}{4}$

Date: 7/3/14
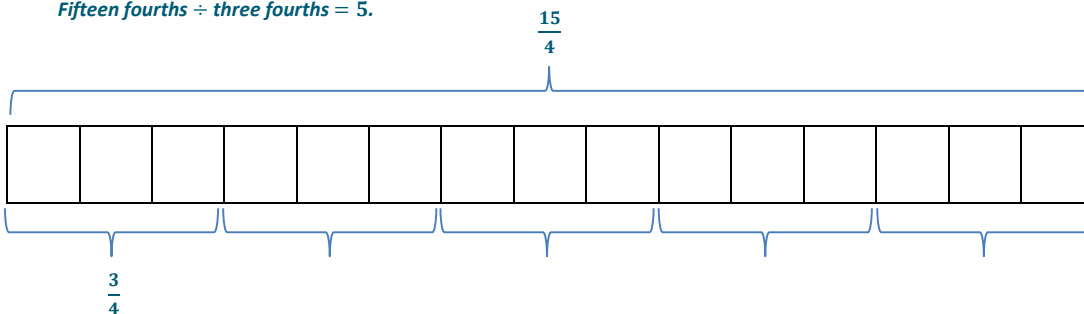
2. $\frac{8}{5} \div \frac{3}{5}$

Eight fifths ÷ three fifths $= \frac{8}{3}$ *or* $2\frac{2}{3}$.
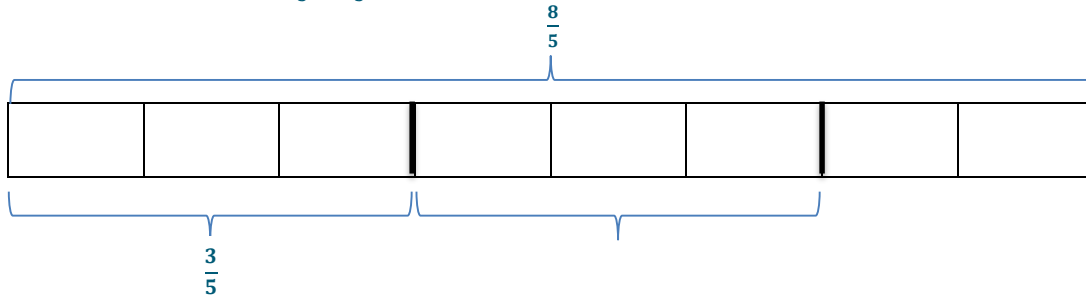
$\frac{8}{5}$

$\frac{3}{5}$

Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Student Outcomes

- Students use fraction bars and area models to divide fractions by fractions with different denominators.
- Students make connections between visual models and multiplication of fractions.

Classwork

Opening Exercise (2 minutes)

Begin class with a review of equivalent fractions. Ask each student for a new example of an equivalent fraction. Students need to share how they know that the new fraction is equivalent to the old fraction.

Opening Exercise

Write at least three equivalent fractions for each fraction below. Be sure to show how the two fractions are related.

a. $\frac{2}{3}$

   Sample solutions include $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}$.

b. $\frac{10}{12}$

   Sample solutions include $\frac{5}{6}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}$.

Example 1 (3 minutes)

For the first example, students will be asked to solve a word problem using the skills they used in Lesson 3 to divide fractions with the same denominator.

MP.1

- Molly purchased $1\frac{3}{8}$ cups of strawberries. This can also be represented as $\frac{11}{8}$. She eats $\frac{2}{8}$ cups per serving. How many servings did Molly purchase?

    □ *This question is really asking me how many $\frac{2}{8}$ are in $\frac{11}{8}$ or, in other words, to divide eleven eighths by two eighths. I can use a model to show that there are $5\frac{1}{2}$ servings in the $\frac{11}{8}$ cups of strawberries.*

Example 1

Molly purchased $\frac{11}{8}$ cups of strawberries. If she eats $\frac{2}{8}$ cups per serving, how many servings does Molly have?

Use a model to prove your answer.

$$1 \quad + \quad 1 \quad + \quad 1 \quad + \quad 1 \quad + \quad 1 \quad + \quad \frac{1}{2} = 5\frac{1}{2}$$

| 0 | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ | $\frac{4}{8}$ | $\frac{5}{8}$ | $\frac{6}{8}$ | $\frac{7}{8}$ | $\frac{8}{8}$ | $\frac{9}{8}$ | $\frac{10}{8}$ | $\frac{11}{8}$ | $\frac{12}{8}$ |
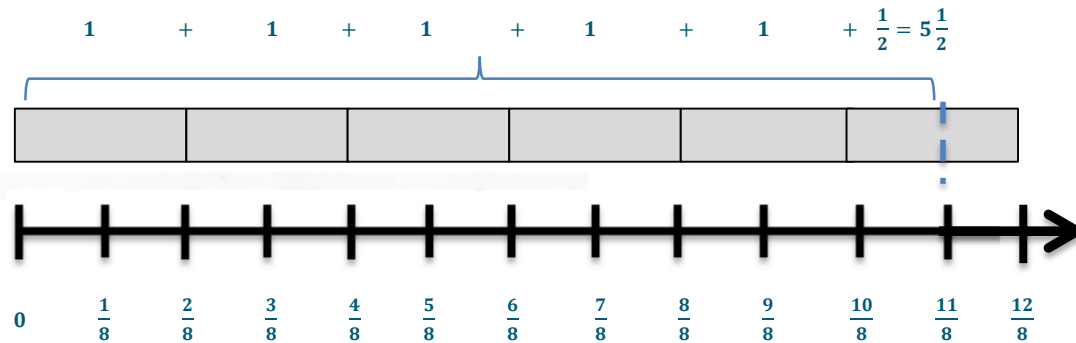
MP.1 **Example 2 (3 minutes)**

- Now imagine that Xavier, Molly's friend, purchased $\frac{11}{8}$ cups of strawberries, and he eats $\frac{3}{4}$ cups per serving. How many servings has he purchased?

    □ *He has purchased* $\frac{11}{6}$ *servings, or* 1 *and* $\frac{5}{6}$ *servings.* (This would be answered last after a brief discussion using the questions that follow.)

- What is this question asking us to do?

    □ *I am being asked to divide* $\frac{11}{8}$ *cups into* $\frac{3}{4}$ *cup servings.*

- How does the problem differ from the first example?

    □ *The denominators are different.*

- What are some possible ways that we could divide these two fractions?

    □ *I could change* $\frac{3}{4}$ *to* $\frac{6}{8}$. *These fractions are equivalent. I scaled up from* $\frac{3}{4}$ *by multiplying the numerator and denominator by* 2.

Example 2

Now imagine that Xavier, Molly's friend, purchased $\frac{11}{8}$ cups of strawberries. If he eats $\frac{3}{4}$ cups of strawberries per serving, how many servings will he have? Use a model to prove your answer.

There are 1 and $\frac{5}{6}$ servings.
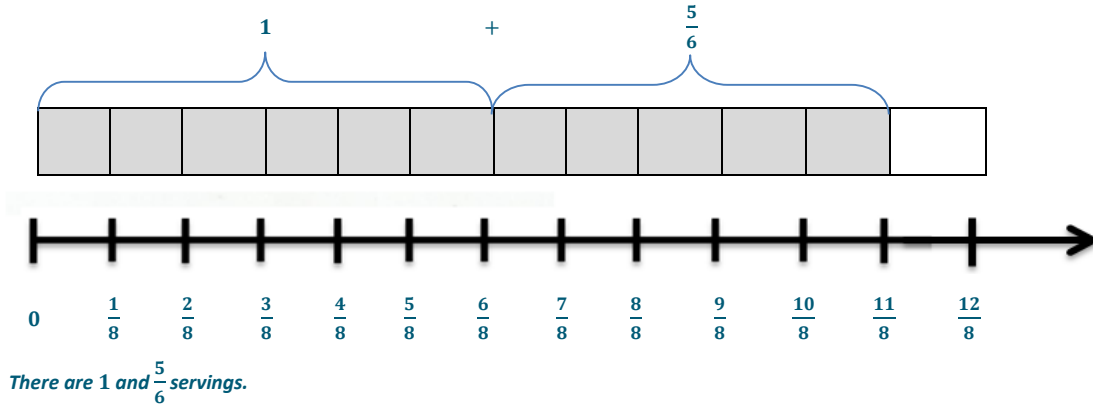
Example 3 (3 minutes)

- $\frac{3}{4} \div \frac{2}{3}$

- What is this question asking?

  - *$\frac{2}{3}$ of what is $\frac{3}{4}$? Or how many $\frac{2}{3}$ are in $\frac{3}{4}$?*

Lead students through a brief discussion about this example:

- Is your answer larger or smaller than one? Why?

  - *Since $\frac{2}{3}$ is less than $\frac{3}{4}$, we will have an answer that is larger than 1.*

- Why is this question more difficult to model than the questions in Lesson 3?

  - *The fractions do not have common denominators.*

- How can we rewrite this question to make it easier to model?

  - *We can create equivalent fractions with like denominators and then model and divide.*

  - *We can also think of this as $\frac{9}{12} \div \frac{8}{12}$, or nine twelfths divided by eight twelfths. 9 units $\div$ 8 units $= \frac{9}{8}$ or $1\frac{1}{8}$ units.*

MP.1

Example 3

Find the quotient: $\frac{3}{4} \div \frac{2}{3}$. **Use a model to show your answer.**

$$\frac{0 \quad \frac{1}{12} \quad \frac{2}{12} \quad \frac{3}{12} \quad \frac{4}{12} \quad \frac{5}{12} \quad \frac{6}{12} \quad \frac{7}{12} \quad \frac{8}{12} \quad \frac{9}{12} \quad \frac{10}{12} \quad \frac{11}{12}}$$

$$\frac{8}{8} \qquad \frac{1}{8}$$
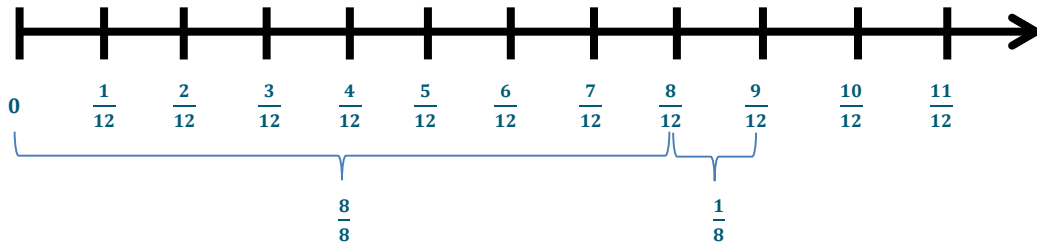
We could rewrite this problem to ask $\frac{9}{12} \div \frac{8}{12} = \frac{9}{8}$.

Exercises 1–5 (19 minutes)

Students will work in pairs or alone to solve more questions about division of fractions with unlike denominators.

Exercises 1–5

A model should be included in your solution.

1. $\frac{6}{2} \div \frac{3}{4}$

   We could rewrite this problem to ask $\frac{12}{4} \div \frac{3}{4} = \frac{12}{3} = 4.$

$$0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad \frac{4}{4} \quad \frac{5}{4} \quad \frac{6}{4} \quad \frac{7}{4} \quad \frac{8}{4} \quad \frac{9}{4} \quad \frac{10}{4} \quad \frac{11}{4} \quad \frac{12}{4}$$

$$1 \quad + \quad 1 \quad + \quad 1 \quad + \quad 1$$
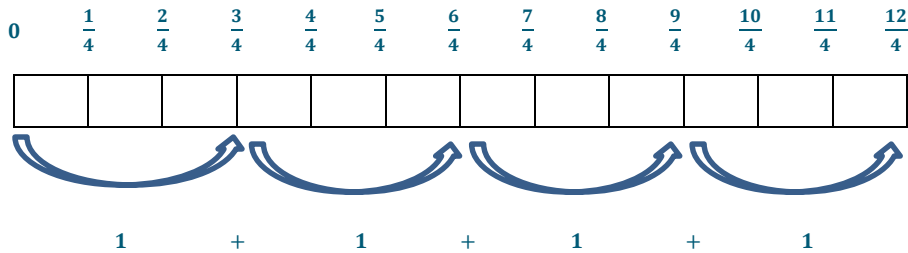
2. $\frac{2}{3} \div \frac{2}{5}$

   We could rewrite this problem to ask $\frac{10}{15} \div \frac{6}{15} = \frac{10}{6}$ or $1\frac{4}{6}$.

| 0 | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{4}{15}$ | $\frac{5}{15}$ | $\frac{6}{15}$ | $\frac{7}{15}$ | $\frac{8}{15}$ | $\frac{9}{15}$ | $\frac{10}{15}$ | $\frac{11}{15}$ |

1   + $\frac{4}{6}$
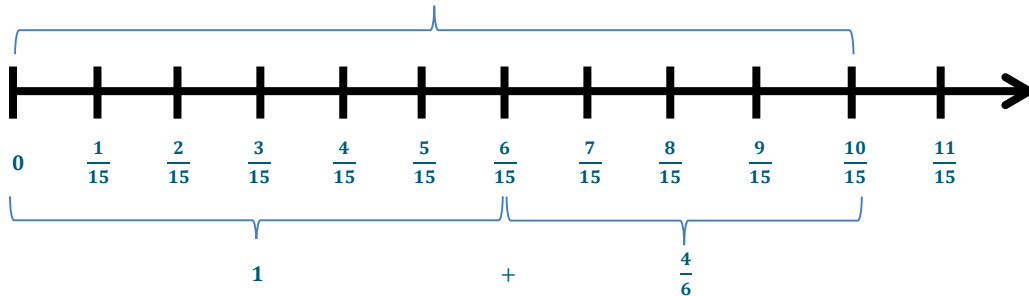
3. $\frac{7}{8} \div \frac{1}{2}$

   We could rewrite this as $\frac{7}{8} \div \frac{4}{8} = \frac{7}{4}$, or $1\frac{3}{4}$.

$\frac{4}{8}$ $\frac{8}{8}$

1   + $\frac{3}{4}$

4. $\frac{3}{5} \div \frac{1}{4}$

   This can be rewritten as $\frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2\frac{2}{5}$.

1   +   1   + $\frac{2}{5}$

COMMON CORE | **Lesson 4:**   Interpreting and Computing Division of a Fraction by a Fraction—More Models
Date: 7/3/14

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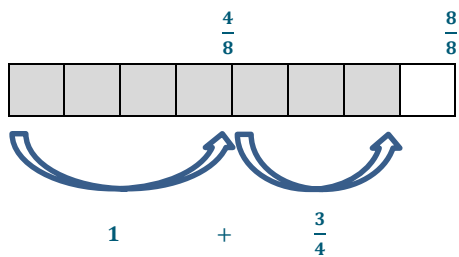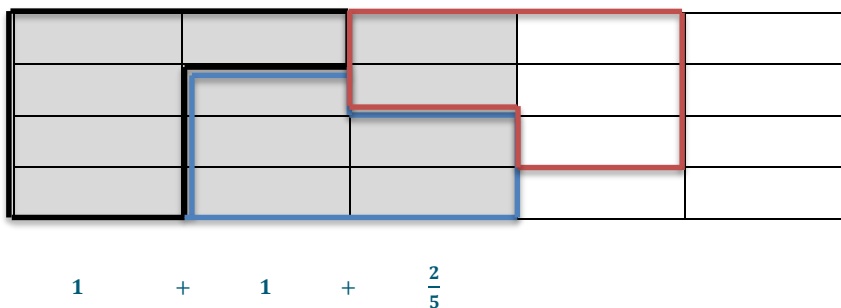
5. $\frac{5}{4} \div \frac{1}{3}$

We can rewrite this as $\frac{15}{12} \div \frac{4}{12} = \frac{15}{4} = 3\frac{3}{4}.$

| 0 | $\frac{1}{12}$ | $\frac{2}{12}$ | $\frac{3}{12}$ | $\frac{4}{12}$ | $\frac{5}{12}$ | $\frac{6}{12}$ | $\frac{7}{12}$ | $\frac{8}{12}$ | $\frac{9}{12}$ | $\frac{10}{12}$ | $\frac{11}{12}$ | 1 | $\frac{13}{12}$ | $\frac{14}{12}$ | $\frac{15}{12}$ | $\frac{16}{12}$ | $\frac{17}{12}$ |
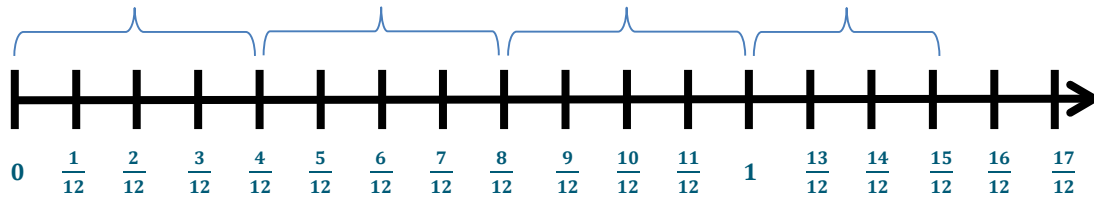
Closing (10 minutes)

- When dividing fractions, is it possible to get a whole number quotient?
  - *It is possible to get a whole number quotient when dividing fractions.*
- When dividing fractions, is it possible to get an answer that is larger than the dividend?
  - *It is possible to get a quotient that is larger than the dividend when dividing fractions.*
- When you are asked to divide two fractions with different denominators, what is one possible way to solve?
  - *To divide fractions with different denominators, we can use equivalent fractions with like denominators in order to solve.*

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 4: Interpreting and Computing Division of a Fraction by a Fraction—More Models

Exit Ticket

Draw a model to support your answer to the division questions.

1. $\frac{9}{4} \div \frac{3}{8}$

2. $\frac{3}{5} \div \frac{2}{3}$

Exit Ticket Sample Solutions

Draw a model to support your answer to the division questions.

1. $\frac{9}{4} \div \frac{3}{8}$

This can be rewritten as $\frac{18}{8} \div \frac{3}{8}$ = eighteen eighths divided by three eighths $= \frac{18}{3} = 6$.
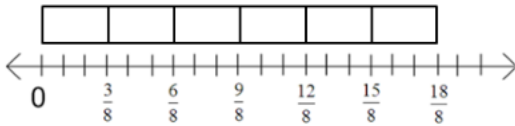
2. $\frac{3}{5} \div \frac{2}{3}$

This can be rewritten as $\frac{9}{15} \div \frac{10}{15}$ = nine fifteenths divided by ten fifteenths, or 9 units ÷ 10 units.

So, this is equal to $\frac{9}{10}$.
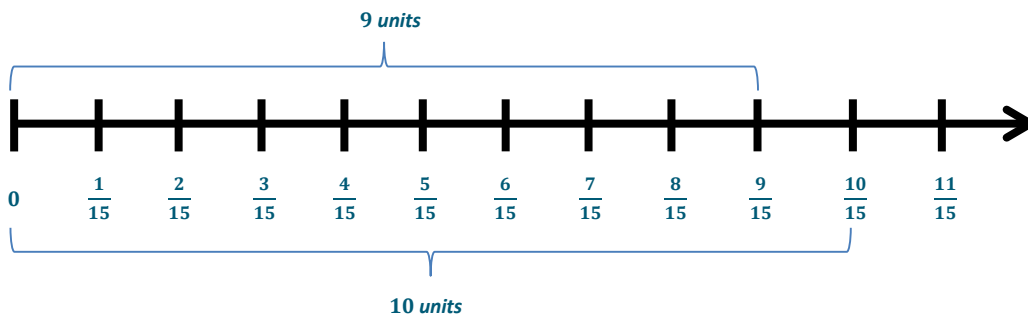
Problem Set Sample Solutions

The following problems can be used as extra practice or a homework assignment.

Draw a model to support your answer to the division questions.

1. $\frac{8}{9} \div \frac{4}{9}$

Eight ninths ÷ four ninths = 2.

COMMON CORE

Lesson 4:	Interpreting and Computing Division of a Fraction by a Fraction—More Models
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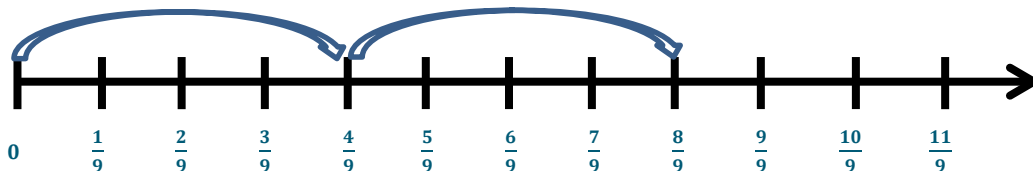
2. $\dfrac{9}{10} \div \dfrac{4}{10}$

Nine tenths ÷ four tenths = $2\dfrac{1}{4}$.

1	1	$\dfrac{1}{4}$
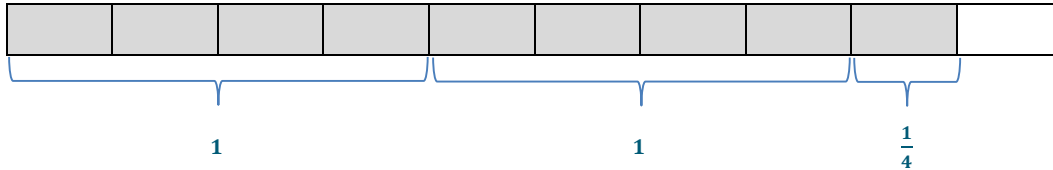
3. $\dfrac{3}{5} \div \dfrac{1}{3}$

$\dfrac{9}{15} \div \dfrac{5}{15} =$ *nine fifteenths ÷ five fifteenths* $= \dfrac{9}{5} = 1\dfrac{4}{5}$.

0	$\dfrac{1}{15}$	$\dfrac{2}{15}$	$\dfrac{3}{15}$	$\dfrac{4}{15}$	$\dfrac{5}{15}$	$\dfrac{6}{15}$	$\dfrac{7}{15}$	$\dfrac{8}{15}$	$\dfrac{9}{15}$	$\dfrac{10}{15}$	$\dfrac{11}{15}$

1 + $\dfrac{4}{5}$
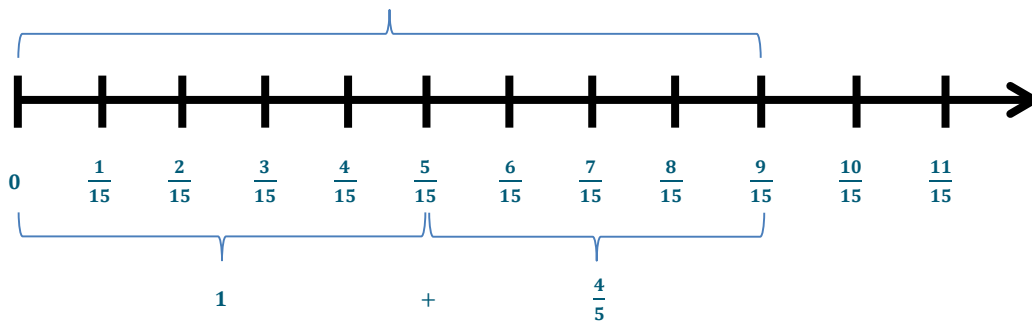
4. $\dfrac{3}{4} \div \dfrac{1}{5}$

$\dfrac{15}{20} \div \dfrac{4}{20} =$ *fifteen twentieths ÷ four twentieths* $= \dfrac{15}{4}$.

1 + 1 + 1 + $\dfrac{3}{4}$ $= 3\dfrac{3}{4}$

$\dfrac{4}{4}$ + $\dfrac{4}{4}$ + $\dfrac{4}{4}$ + $\dfrac{3}{4}$ $= \dfrac{15}{4}$

Lesson 5: Creating Division Stories

Student Outcomes

- Students demonstrate further understanding of division of fractions by creating their own word problems.
- Students select a measurement division problem, draw a model, find the answer, choose a unit, and set up a situation. They discover that they must try several situations and units before finding which ones are realistic with given numbers.

Lesson Notes

MP.1

During this lesson, students make sense of problems by creating them and persevering in solving them. They use concrete representations (fraction tiles or paper cutouts) and pictorial representations to show their understanding of the meaning of division and apply that understanding to the division of fractions.

There is a system for creating a division word problem. This sequence, which is noted in the second Student Outcome, is to be followed in order. Visual models showing examples of measurement division (e.g., $\frac{1}{2} \div \frac{1}{8}$ or how many $\frac{1}{8}$ are there in $\frac{1}{2}$?) should be available, visible, and referenced throughout the lesson. These can be from previous lessons, student work, chart paper, or board examples saved from Lessons 1–4 in this module. During the next lesson, partitive division problems ($50 \div \frac{2}{3}$ or 50 is $\frac{2}{3}$ of what number?) will be the focus.

Before class, make a display of both types of problems (measurement and partitive).

It will be helpful to put the "5 Steps for Creating Division Word Problems" on chart paper posted around the room.

Through previous lessons, students gained experience solving problems involving division with fractions and creating the corresponding models. Students will now extend this work to choosing units and creating "division" stories. Pair up the students before the lesson so that they can collaborate on the exercises.

Classwork

Opening Exercises (5 minutes)

The classwork and problem sets from Lessons 1–4 include visual models made by the students which will serve as the first three steps in writing division stories about the problems. The visual models include fraction bars, number lines, and area models. Examples are as follows:

Opening Exercises

Fraction Bar:

$\frac{8}{9} \div \frac{2}{9}$

$\frac{8}{9}$

$\frac{2}{9}$ $\frac{2}{9}$ $\frac{2}{9}$ $\frac{2}{9}$
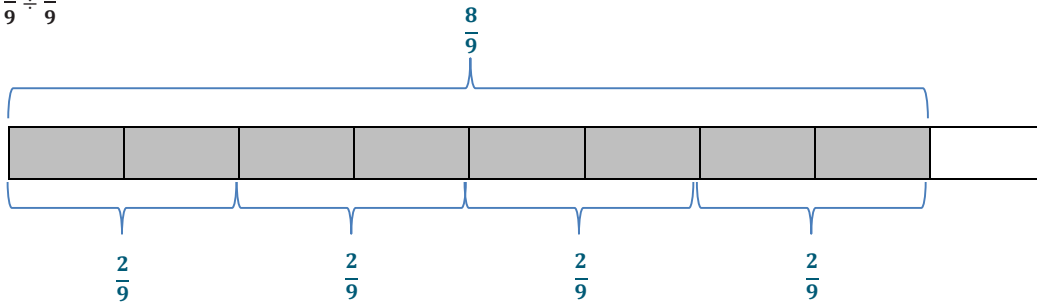
Here we have 4 groups of $\frac{2}{9}$. Therefore, the answer is 4.

Number Line:

Xavier, Molly's friend, purchased $\frac{11}{8}$ cups of strawberries. If he eats $\frac{3}{4}$ of a cup of strawberries per serving, how many servings will he have?

1 $\frac{5}{6}$

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$ $\frac{9}{8}$ $\frac{10}{8}$ $\frac{11}{8}$ $\frac{12}{8}$
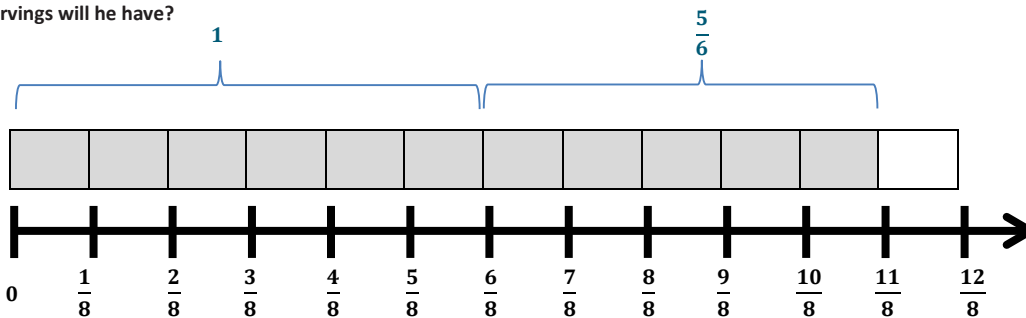
He will have 1 and $\frac{5}{6}$ servings.

Area Model:

$\frac{3}{5} \div \frac{1}{4}$

This can be rewritten as $\frac{12}{20} \div \frac{5}{20} = \frac{12}{5} = 2\frac{2}{5}$.

1 + 1 + $\frac{2}{5}$
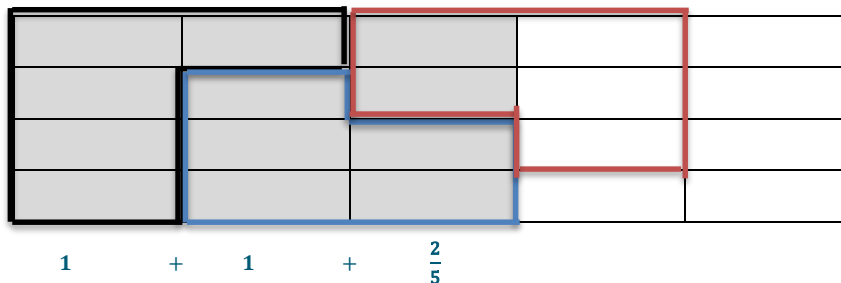
Use a few minutes to review the problem set from the last lesson. Ask students to add another model to each answer.

Discussion (5 minutes)

- When we know the original amount and the size or measure of one part, we use measurement division to find the number of parts. You can tell when a question is asking for measurement division because it asks, "How many _____ are in _____?"
- Writing division stories involves five steps that are to be done in order. The steps are as follows: (1) decide on an interpretation (measurement or partitive); (2) draw a model; (3) find the answer; (4) choose a unit; and (5) set up a situation.
- Today, we will work only on measurement division problems, and tomorrow, we will work on partitive division problems.
- By looking at all the work we have posted around the room, which of these steps are already done?
  - *Steps 1, 2, and 3 are already done.*
- Today, we will also focus on extending our work involving division with fractions to choosing units and setting up a story situation.

Example 1 (10 minutes)

MP.2

In this example, students reason abstractly and quantitatively. They make sense of quantities and their relationships in problems and understand "how many" as it pertains to the divisor when finding the quotients of fractions.

Encourage students to work each step in order.

- Let's look at an example of measurement division: $\frac{1}{2} \div \frac{1}{8}$ or how many $\frac{1}{8}$ are there in $\frac{1}{2}$?
- Our first step in writing a story problem is to decide which division interpretation to use. Do we need to decide this, or is it already given in this problem?

Example 1

$\frac{1}{2} \div \frac{1}{8}$

Step 1: Decide on an interpretation.

The interpretation, measurement division, is given in this problem.

- Our second step is to sketch out a model. This should be done neatly and fairly accurately, but should not take too long to do. Use a tape diagram or fraction bar to model this problem on your paper.

Step 2: Draw a model.

Scaffolding:

Plastic fraction tile sets should be available for students who prefer a concrete model to handle. These are also useful for students who have difficulty making accurate sketches to show their work.

Note: This drawing can be cut and pasted onto an interactive whiteboard document and then labeled.

$\frac{1}{2}$				$\frac{1}{2}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	

▪ The third step is to find the answer to the problem. Do this on your paper.

Step 3: Find the answer.

$$\frac{1}{2} \div \frac{1}{8} = \frac{4}{8} \div \frac{1}{8} = \frac{4}{1} = 4$$

So, the answer is 4. There are four $\frac{1}{8}$ in $\frac{1}{2}$.

Step 4: Choose a unit.

Answers will vary, but pounds will be used throughout the discussion below.

Step 5: Set up a situation.

Answers will vary, but there is a story problem provided in the discussion.

▪ Now that we have the answer, we can move on to the fourth step, choosing a unit. For measurement division, both divisor and dividend must be the same unit.

Note: Choosing the unit and using it in both the divisor and dividend consistently preserves the story situation clearly and precisely. With enough repetition, students will learn to interpret and write division story problems more clearly.

▪ Let's use pounds for this example. We are asking how many $\frac{1}{8}$ pounds are in $\frac{1}{2}$ pound.

▪ Step 5 is to set up a situation. This means writing a story problem that includes all of the information necessary to solve it and that is also interesting, realistic, short, and clear. It may take several attempts before you find a story that works well with the given dividend and divisor.

▪ One story problem that might go well with these numbers is the following: Bonnie Baker has a total of $\frac{1}{2}$ pound of chocolate. She needs $\frac{1}{8}$ pound of chocolate for each batch of brownies she bakes. How many batches of brownies can Bonnie bake with $\frac{1}{2}$ pound of chocolate?

Exercise 1 (5 minutes)

Allow students to work with a partner to create the story problem. Also, take time to share and discuss their work.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as ounces, yards, or miles if you prefer.

Possible story problems:

1. *Tina uses $\frac{1}{8}$ oz. of cinnamon each time she makes a batch of coffee cake topping. How many batches can she make if she has $\frac{1}{2}$ oz. left in her spice jar?*

2. *Eugenia has $\frac{1}{2}$ yard of ribbon. For each party decoration, she needs $\frac{1}{8}$ yard. How many party decorations can she make?*

Example 2 (7 minutes)

▪ Let's look at another example of measurement division: $\frac{3}{4} \div \frac{1}{2}$ or how many $\frac{1}{2}$ are there in $\frac{3}{4}$?

▪ Our first step in writing a story problem is to decide which division interpretation to use. Do we need to decide this, or is it already given in this problem?

 □ *It is already given in this problem.*

Example 2

$\frac{3}{4} \div \frac{1}{2}$

Step 1: Decide on an interpretation.

The interpretation, measurement division, is given in this problem.

▪ Our second step is to sketch out a model. This should be done neatly and fairly accurately, but should not take too long to do. Use a tape diagram to model this problem on your paper.

Step 2: Draw a diagram.
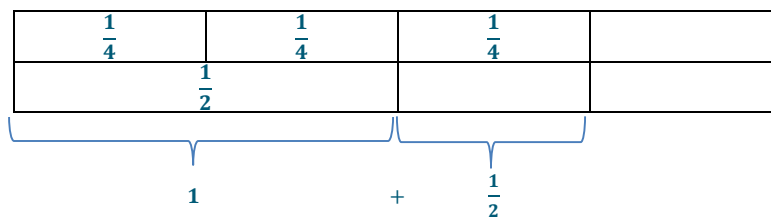
Step 3: Find the answer.

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4} = \frac{3}{2} = 1\frac{1}{2}.$

So, the answer is $1\frac{1}{2}$. There are $1\frac{1}{2}$ halves in $\frac{3}{4}$.

> **Step 4: Choose a unit.**
>
> *Answers will vary, but ounces will be used throughout the discussion below.*
>
>
> **Step 5: Set up a situation.**
>
> *Answers will vary, but there is a story problem provided in the discussion.*

- Step 4 is to choose a unit. Let's choose ounces.
- Step 5 is to set up a situation. This means writing a story problem that includes all of the information necessary to solve it and that is also interesting, realistic, short, and clear. It may take several attempts before you find a story that works well with the given dividend and divisor.
- One story problem that might go well with these numbers is the following: Tia has $\frac{3}{4}$ oz. of coffee left in her coffee can. She needs $\frac{1}{2}$ oz. to make one cup of coffee. How many cups of coffee can she make?

Exercise 2 (5 minutes)

Allow students to work with a partner to create the story problem. Also, take time to share and discuss their work.

> **Exercise 2**
>
> Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, but your situation must be unique. You could try another unit such as cups, yards, or miles if you prefer.
>
> *Possible story problems:*
>
> 1. *Tiffany uses $\frac{1}{2}$ cup of glycerin each time she makes a batch of soap bubble mixture. How many batches can she make if she has $\frac{3}{4}$ cup left in her glycerin bottle?*
>
> 2. *Theresa has $\frac{3}{4}$ yard of fabric. For each doll skirt she makes, she needs $\frac{1}{2}$ yard. Does she have enough fabric to make two doll skirts?*

Closing (5 minutes)

- How did we extend our work with division with fractions in this lesson?
  - *We took an answer to a measurement division problem, added a unit, and then wrote a realistic story that fit the numbers and the unit.*
- What were your biggest challenges when writing story problems involving division with fractions?
  - *Accept all answers.*

Lesson Summary

The method of creating division stories includes five steps:

Step 1: Decide on an interpretation (measurement or partitive). Today we used measurement division.

Step 2: Draw a model.

Step 3: Find the answer.

Step 4: Choose a unit.

Step 5: Set up a situation. This means writing a story problem that is interesting, realistic, and short. It may take several attempts before you find a story that works well with the given dividend and divisor.

Exit Ticket (3 minutes)

Name _____   Date_____

Lesson 5: Creating Division Stories

Exit Ticket

Write a story problem for the following measurement division: $\frac{3}{4} \div \frac{1}{8} = 6$.

1

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Exit Ticket Sample Solution

Write a story problem for the following measurement division: $\frac{3}{4} \div \frac{1}{8} = 6$.

1

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
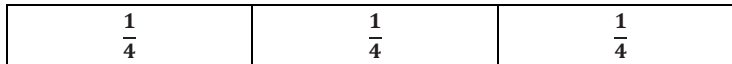
Arthur divided $\frac{3}{4}$ of his kingdom into parcels of land, each being $\frac{1}{8}$ of the entire kingdom. How many parcels did he make?

(Accept any other reasonable story problem showing $\frac{3}{4} \div \frac{1}{8} = 6$.)

Problem Set Sample Solutions

Please use each of the five steps of the process you learned. Label each step.

1. Write a measurement division story problem for $6 \div \frac{3}{4}$.

   Step 1: Measurement

   Step 2: Model, shown below (Students may have used a different type of model.)

1	2	3	4	5	6

| $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ | $\frac{3}{4}$ |
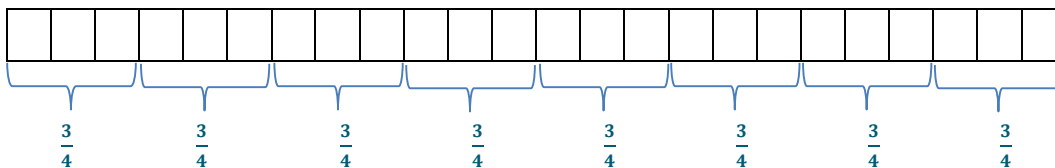
   Step 3: How many $\frac{3}{4}$ are there in 6?

   $6 \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4} = \frac{24}{3} = 8$

   Step 4: Feet (Answers may vary.)

   Step 5: Answers may vary, but this is one possible answer.

   Rafael has a 6-foot piece of wood. He has to cut shelves that are $\frac{3}{4}$ foot long. How many shelves can he cut from the 6-foot board?

2. Write a measurement division story problem for $\frac{5}{12} \div \frac{1}{6}$.

Step 1: Measurement

Step 2: Model, shown below (Students may have used a different type of model.)

X	X	X	X	X							

$$\frac{1}{6} \qquad \frac{1}{6} \qquad \frac{1}{6}$$
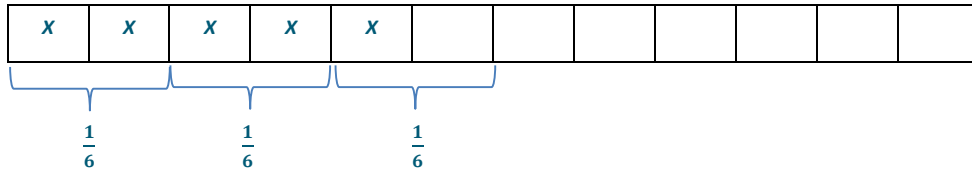
Step 3: How many $\frac{1}{6}$ are there in $\frac{5}{12}$?

$$\frac{5}{12} \div \frac{1}{6} = \frac{5}{12} \div \frac{2}{12} = \frac{5}{2} = 2\frac{1}{2}$$

Step 4: Feet (Answers may vary.)

Step 5: Answers may vary, but this is one possible answer.

There are 12 inches in a foot. A piece of wire is 5 inches ($\frac{5}{12}$ foot) long. Hector needs to cut pieces of wire that are 2 inches ($\frac{1}{6}$ foot) long. How many can he cut? $2\frac{1}{2}$

Lesson 6: More Division Stories

Student Outcomes

- Students demonstrate further understanding of division of fractions by creating their own word problems.
- Students choose a partitive division problem, draw a model, find the answer, choose a unit, and set up a situation. They also discover that they must try several situations and units before finding which ones are realistic with given numbers.

Lesson Notes

This lesson is a continuation of Lesson 5 and focuses on asking students to write fraction division story problems that are partitive in nature.

Classwork

Opening (5 minutes)

Provide students a few minutes to share the division stories they wrote for the previous lesson's problem set. Clarify any misconceptions that surface regarding the process of creating story problems when using measurement division.

Discussion (5 minutes)

- Partitive division is another interpretation of division problems. What do you recall about partitive division?

  - *We know that when we divide a whole number by a fraction, the quotient will be greater than the whole number we began with (the dividend). This is true regardless of whether we use a partitive approach or a measurement approach.*

  - *We know what the whole is and how many groups we are making, but must figure out what size the groups are.*

> *Scaffolding:*
> Paper fraction tile strips can be pre-cut for students who have difficulty making accurate sketches.

Example 1 (10 minutes)

- Today, we will work with partitive division.
- Step 1: Let's use an example that uses partitive division: $50 \div \frac{2}{3}$.

Example 1

Divide $50 \div \frac{2}{3}$.

Step 1: Decide on an interpretation.

The interpretation, partitive division, is given in this problem.

- Step 2 is to draw a model. How many equal sized rectangles do we need? How do you know?
  - *The denominator of the fraction tells us we are using thirds. We need 3 rectangles in the whole unit.*

> **Step 2: Draw a model.**
>
> [ ][ ][ ]

- The 50 accounts for how many of those 3 rectangles?
  - *It accounts for two because the numerator tells us how many thirds that the 50 represents.*
- So, the 50 must be spread out evenly between two thirds. How many would be in each box?
  - 25

> **Step 2: Draw a model.**
>
> 50
>
> | 25 | 25 | |
>
> ?
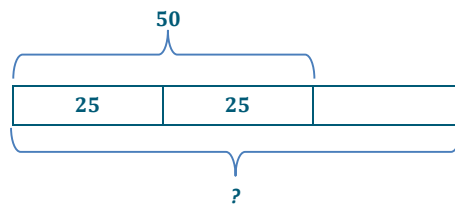
- Step 3 is to find the answer. $50 \div \frac{2}{3}$ means 50 is $\frac{2}{3}$ of some number that is greater than 50. By looking at our tape diagram, we can see that $25 = \frac{1}{3}$ of the number.

> **Step 3: Find the answer.**
>
> *By looking at our tape diagram, we see that $50 \div \frac{2}{3} = 25 \cdot 3 = 75$.*
>
> **Step 4: Choose a unit.**
>
> *Answers will vary, but dollars will be used throughout the discussion below.*
>
> **Step 5: Set up a situation.**
>
> *Answers will vary, but there is a story problem provided in the discussion.*

- Now that we have the answer, we can move on to the fourth step, choosing a unit. Let's choose dollars.
- Step 5 is to set up a situation. Remember that this means writing a story problem that includes all of the information necessary to solve it and that is also interesting, realistic, short, and clear. It may take several attempts before you find a story that works well with the given dividend and divisor.
- Spending money gives a "before and after" word problem. We are looking for a situation where $\frac{2}{3}$ of some greater dollar amount is $50.

- One story problem that might go well with these numbers is the following: Adam spent $50 on a new graphing calculator. This was $\frac{2}{3}$ of his money. How much money did he start with?

Exercise 1 (5 minutes)

Allow students to work with a partner to create the story problem. Also, take time to share and discuss their work.

Exercise 1

Using the same dividend and divisor, work with a partner to create your own story problem. You may use the same unit, dollars, but your situation must be unique. You could try another unit, such as miles, if you prefer.

Possible story problems:

1. Ronaldo has ridden 50 miles during his bicycle race and is $\frac{2}{3}$ of the way to the finish line. How long is the race?

2. Samantha used 50 tickets ($\frac{2}{3}$ of her total) to trade for a kewpie doll at the fair. How many tickets did she start with?

Example 2 (10 minutes)

- Step 1: Let's use an example that uses partitive division: $45 \div \frac{3}{8}$.

Example 2

Divide $45 \div \frac{3}{8}$.

Step 1: Decide on an interpretation.

The interpretation, partitive division, is given in this problem.

- Step 2 is to draw a model. How many equal sized rectangles do we need? How do you know?
    - *The denominator of the fraction tells us we are using eighths. We need to partition the whole into 8 equal sized rectangles.*

Step 2: Draw a model.

- The 45 accounts for how many of those 8 rectangles?
    - *It accounts for three because the numerator tells us how many eighths that the 45 represents.*
- So, the 45 must be spread out evenly among three eighths. How many would be in each box?
    - 15

Step 2: Draw a model.
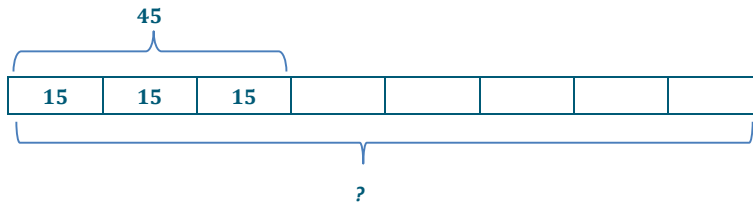
- Step 3 is to find the answer. $45 \div \frac{3}{8}$ means 45 is $\frac{3}{8}$ of some number that is greater than 45. By looking at our tape diagram, we can see that $15 = \frac{1}{8}$ of the number.

Step 3: Find the answer.

By looking at our tape diagram, $45 \div \frac{3}{8} = 15 \cdot 8 = 120.$

Step 4: Choose a unit.

Answers will vary, but carnival prize tickets will be used throughout the discussion below.

Step 5: Set up a situation.

Answers will vary, but there is a story problem provided in the discussion.

- Now that we have the answer, we can move on to the fourth step, choosing a unit. Let's choose carnival prize tickets.
- Step 5 is to set up a situation. Remember that this means writing a story problem that includes all of the information necessary to solve it and that is interesting, realistic, short, and clear. It may take several attempts before you find a story that works well with the given dividend and divisor.
- One story problem that might go well with these numbers is the following: Scott gave away 45 carnival prize tickets to his niece. This was $\frac{3}{8}$ of his tickets. How many tickets did he start with?

Exercise 2 (5 minutes)

Allow students to work with a partner to create the story problem. Also, take time to share and discuss their work.

Exercise 2

Using the same dividend and divisor, work with a partner to create your own story problem. Try a different unit. Remember, spending money gives a "before and after" word problem. If you use dollars, you are looking for a situation where $\frac{3}{8}$ of some greater dollar amount is $\$45$.

Possible story problems:

1. Daryl has been on a diet for 45 days and is $\frac{3}{8}$ of the way to the end of the diet program. How long is the program?

2. Amy collected 45 Box Tops for Education, which is $\frac{3}{8}$ of her goal. What is the total number she is trying to collect?

Closing (3 minutes)

- How did we extend our work with division with fractions in this lesson?
  - *We took an answer to a partitive division problem, added a unit, and then thought of a story problem that would fit it.*
- What were your biggest challenges when writing story problems involving division with fractions?
  - *Accept all answers.*

Exit Ticket (2 minutes)

Name _____   Date_____

Lesson 6: More Division Stories

Exit Ticket

Write a word problem for the following partitive division: $25 \div \frac{5}{8} = 40$.

$$25$$

```
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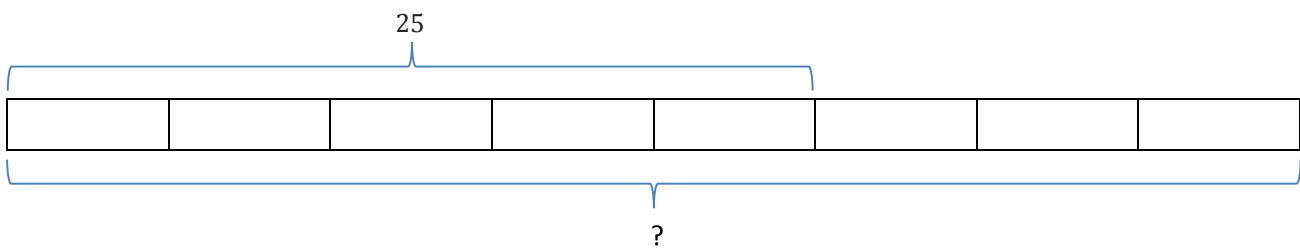
Exit Ticket Sample Solutions

Write a word problem for the following partitive division: $25 \div \frac{5}{8} = 40$.

25

?
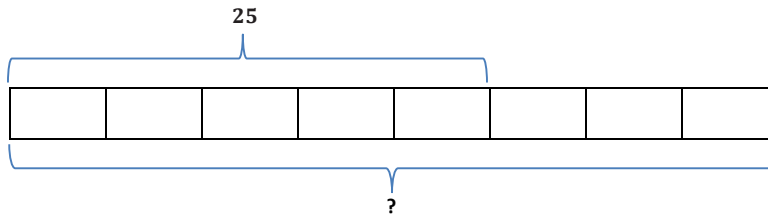
Zolanda spent $\frac{5}{8}$ of her class period, or 25 minutes, taking notes. How long was the class period? (Accept any other reasonable story problem showing $25 \div \frac{5}{8} = 40$.)

Problem Set Sample Solutions

1. Write a partitive division story problem for $45 \div \frac{3}{5}$.

    45

    | 15 | 15 | 15 | | |

    ?
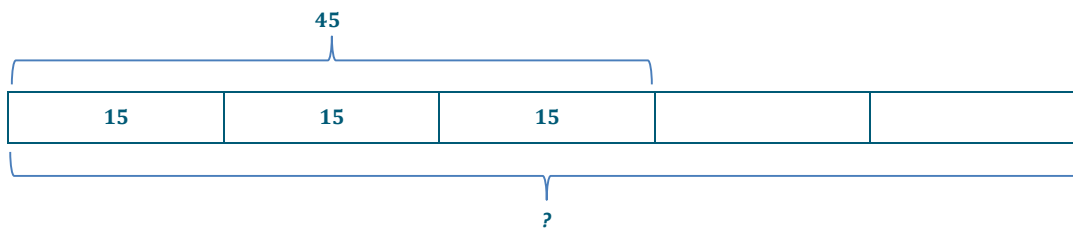
    Answers may vary, but an example of a division story is provided.

    Samantha Maria spent $\frac{3}{5}$ of her money, or $45, on a pair of earrings. How much money did she have before she bought the earrings?

2. Write a partitive division story problem for $100 \div \frac{2}{5}$.

    100

    | 50 | 50 | | | |

    ?
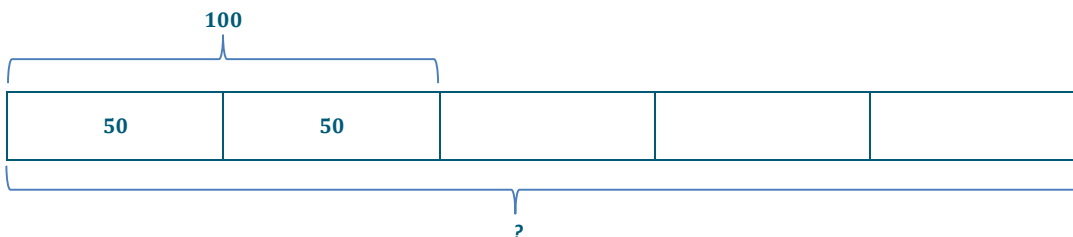
    Answers may vary, but an example of a division story is provided.

    There are 100 girls in the college marching band, which is $\frac{2}{5}$ of the total. How many members are there all together in the band?

Lesson 7: The Relationship Between Visual Fraction Models and Equations

Student Outcomes

- Students formally connect models of fractions to multiplication through the use of multiplicative inverses as they are represented in models.

Lesson Notes

Using pre-cut fraction strips saves time and assures more accurate models. Fraction strips are found at the end of this document and should be reproduced and cut prior to the lesson. With each example that students work through, the concept will be reinforced: dividing by a fraction yields the same result as multiplying by the inverse of that fraction.

The terms *multiplicative inverse* and *reciprocal* should be defined and displayed in the classroom, perhaps as part of a Word Wall.

Two numbers whose product is one are multiplicative inverses, or reciprocals, of one another.

$$\frac{2}{3} \implies \frac{3}{2}$$

$$\frac{5}{8} \implies \frac{8}{5}$$

$$4 \implies \frac{1}{4}$$

Classwork

Opening (4 minutes)

Introduce the definition of the term *multiplicative inverse*: Two numbers whose product is 1 are multiplicative inverses of one another. For example, $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

Point out and ask students to write several of their own examples of multiplicative inverses. The general form of the concept $\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$ should also be displayed in several places throughout the classroom.

- We already know how to make tape diagrams to model division problems. Today, we will use fraction tiles or fraction strips to model dividing fractions. This will help you understand this concept more completely.

MP.1

During this lesson, students continue to make sense of problems and persevere in solving them. They use concrete representations when understanding the meaning of division and apply it to the division of fractions. They ask themselves, "What is this problem asking me to find?" For instance, when determining the quotient of fractions, students may ask themselves how many sets or groups of the divisor are in the dividend. That quantity is the quotient of the problem.

Example 1 (15 minutes)

Consider the following, an example that we have worked with in previous lessons:

Example 1

$$\frac{3}{4} \div \frac{2}{5}$$

$\frac{2}{5}$ *of what number is* $\frac{3}{4}$?
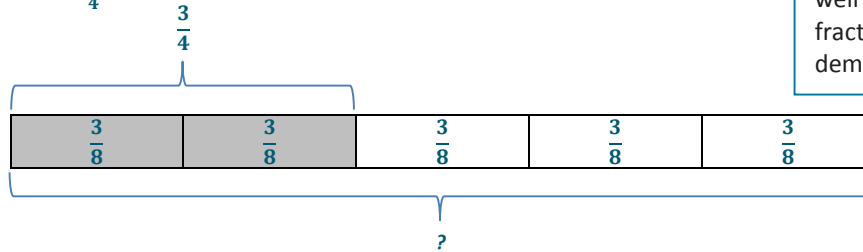
Shade 2 of the 5 sections $\left(\frac{2}{5}\right)$.

Label the part that is known $\left(\frac{3}{4}\right)$.

Make notes below on the math sentences needed to solve the problem.

MP.1

- How did we choose which fraction strip to use?
    - *We are dividing by* $\frac{2}{5}$. *There is a 5 in the denominator, which tells us what kind of fraction, so we chose fifths.*
- How did we know to shade in two sections?
    - *There is a 2 in the numerator. That tells us how many of the fifths to shade.*
- How did we know to put the $\frac{3}{4}$ in the brace that shows $\frac{2}{5}$?
    - *That is the part we know.*
- What about the bottom brace?
    - *That is unknown right now and is ready to be calculated.*
- Think about this problem like this:

2 units $= \frac{3}{4}$.

1 unit is half of $\frac{3}{4} = \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.

5 units $= \frac{3}{8} \cdot 5 = \frac{15}{8}$ or $1\frac{7}{8}$.

So, $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$ or $1\frac{7}{8}$.

- We have the answer to our initial question, but perhaps there is another way to solve the problem. Do you think it is possible to solve this problem without using division?
    - *Accept any answer.*

Extension: Allow time for students to inspect the problem and offer conjectures using the reciprocal of the divisor. If necessary, prompt them to consider another operation, such as multiplication. The following is a series of steps to explain how *invert and multiply* is justified.

- Using the any-order property, we can say the following:

$$\frac{1}{2} \cdot \frac{3}{4} \cdot 5 = \frac{3}{4} \cdot \left(\frac{1}{2} \cdot 5\right) = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}.$$

So, $\frac{3}{4} \div \frac{2}{5} = \frac{15}{8}$ and $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$.

Therefore, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \cdot \frac{5}{2}$.

- This method for dividing a fraction by a fraction is called *invert and multiply*. Dividing by a fraction is the same as multiplying by its inverse. It is important to invert the second fraction (the divisor) and **_not_** the first fraction (the dividend).

Example 2 (10 minutes)

During this part of the lesson, students look for and make use of structure. They continue to find patterns and connections when dividing fractions, and they recognize and make use of a related multiplication problem to determine the number of times the divisor is added to obtain the dividend.

- Does this always work? Let's find out using the example $\frac{1}{4} \div \frac{2}{3}$.

- How did we choose which fraction strip to use?

  - *We are dividing by $\frac{2}{3}$. There is a 3 in the denominator, which tells us what kind of fraction, so we chose thirds.*

- How did we know to shade in two sections?

  - *There is a 2 in the numerator. That tells us how many of the thirds to shade.*

MP.7

- How did we know to put the $\frac{1}{4}$ in the brace that shows $\frac{2}{3}$?

  - *That is the part we know.*

- What about the bottom brace?

  - *That is unknown right now and is ready to be calculated.*

- Think about this problem like this:

2 units $= \frac{1}{4}$.

1 unit is half of $\frac{1}{4}$, which means $\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$.

3 units $= \frac{1}{8} \cdot 3 = \frac{3}{8}$.

Example 2

$$\frac{1}{4} \div \frac{2}{3}$$

$$\frac{1}{4}$$

| $\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{8}$ |

?
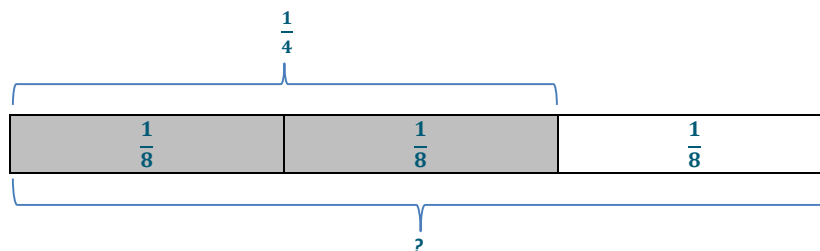
Show the number sentences below.

$$\frac{1}{4} \div \frac{2}{3} = \frac{2}{8} \div \frac{2}{3} = \frac{3}{8}$$

$$\frac{1}{2} \cdot \frac{1}{4} \cdot 3 = \frac{3}{8}$$

$$\frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$$

Therefore, $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{2}.$

Example 3 (8 minutes)

Ask students to solve the following problem with both a tape diagram and the *invert and multiply* rule. They should compare the answers obtained from both methods and find them to be the same.

Example 3

$$\frac{2}{3} \div \frac{3}{4}$$

$$\frac{2}{3}$$

| $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ |

?
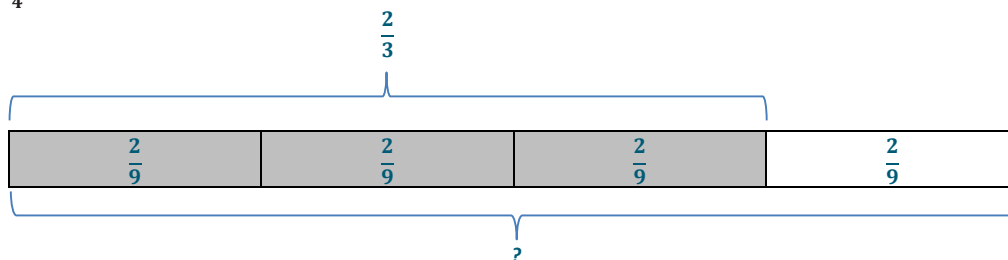
Show the number sentences below.

$$\frac{2}{3} \div \frac{3}{4} = \frac{8}{12} \div \frac{9}{12} = \frac{8}{9}$$

$$\frac{1}{3} \cdot \frac{2}{3} \cdot 4 = \frac{8}{9}$$

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \cdot \frac{4}{3} = \frac{8}{9}$$

- How did we choose which fraction strip to use?
  - *We are dividing by $\frac{3}{4}$. There is a 4 in the denominator, which tells us what kind of fraction, so we chose fourths.*
- How did we know to shade in three sections?
  - *There is a 3 in the numerator. That tells us how many of the fourths to shade.*
- How did we know to put the $\frac{2}{3}$ in the brace that shows $\frac{3}{4}$?
  - *That is the part we know.*
- What about the bottom brace?
  - *That is unknown right now and is ready to be calculated.*
- Think about this problem like this:

  3 units $= \frac{2}{3}$.

  1 unit is one third of $\frac{2}{3}$, which means $\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$.

  4 units $= \frac{2}{9} \cdot 4 = \frac{8}{9}$.

Closing (3 minutes)

- Dividing by a fraction is equivalent to multiplying by its reciprocal, or multiplicative inverse. Connecting the models of division by a fraction to multiplication by its inverse strengthens your understanding of the *invert and multiply* rule.

Lesson Summary

Connecting models of fraction division to multiplication through the use of reciprocals helps in understanding the *invert and multiply* rule.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 7: The Relationship Between Visual Fraction Models and Equations

Exit Ticket

1. Write the reciprocal of the following numbers.

Number	$\dfrac{7}{10}$	$\dfrac{1}{2}$	5
Reciprocal			

2. Rewrite this division problem as a multiplication problem: $\dfrac{5}{8} \div \dfrac{2}{3}$.

3. Solve Problem 2 using models.

Exit Ticket Sample Solutions

1. Write the reciprocal of the following numbers.

Number	$\frac{7}{10}$	$\frac{1}{2}$	5
Reciprocal	$\frac{10}{7}$	2	$\frac{1}{5}$

2. Rewrite this division problem as a multiplication problem: $\frac{5}{8} \div \frac{2}{3}$.

$$\frac{5}{8} \cdot \frac{3}{2} \quad or \quad \frac{1}{2} \cdot \frac{5}{8} \cdot 3$$

3. Solve Problem 2 using models.

$$\frac{5}{8}$$

$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$
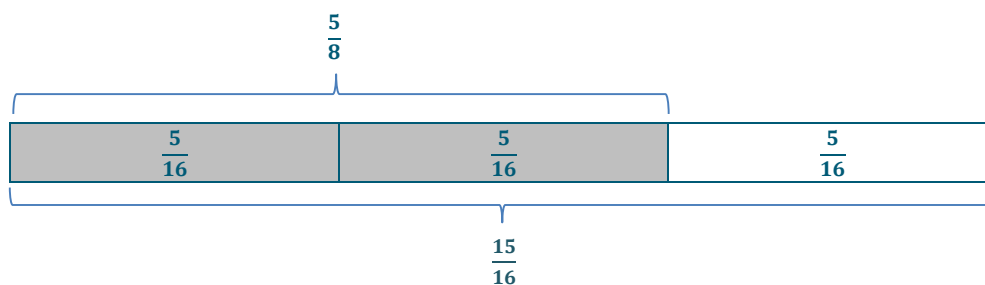
$$\frac{15}{16}$$

Answer:

$$\frac{5}{8} \div \frac{2}{3} = \frac{5}{8} \cdot \frac{3}{2} = \frac{15}{16}$$

Problem Set Sample Solutions

1. Draw a model that shows $\frac{2}{5} \div \frac{1}{3}$. Find the answer as well.

$$\frac{1}{3}$$

$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

$$\frac{6}{5}$$
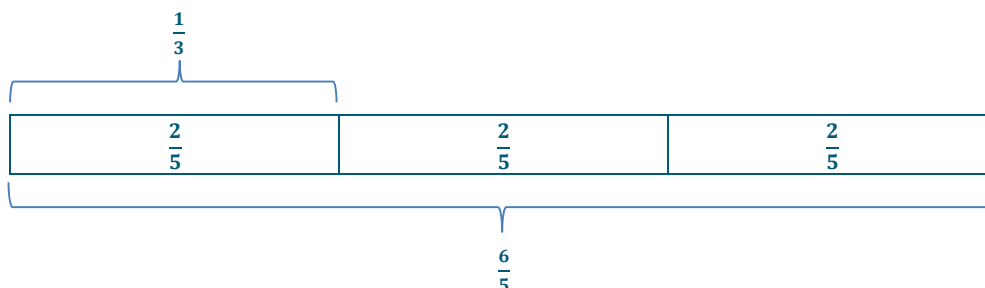
Answer:

$$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \cdot \frac{3}{1} = \frac{6}{5} \quad or \quad 1\frac{1}{6}$$

2.   Draw a model that shows $\frac{3}{4} \div \frac{1}{2}$. Find the answer as well.

$$\frac{1}{2}$$

| $\frac{3}{4}$ | $\frac{3}{4}$ |

$$\frac{6}{4}$$
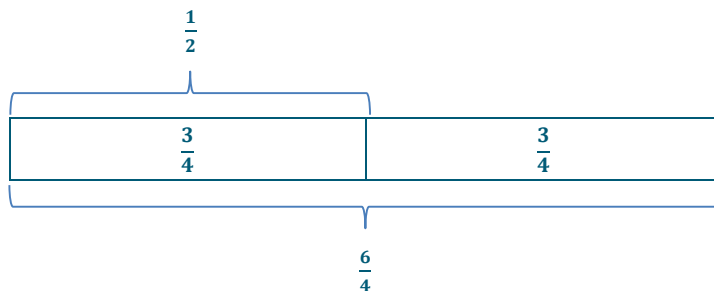
Answer:

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \cdot \frac{2}{1} = \frac{6}{4} \ \ or \ \ 1\frac{1}{2}$$

1 whole unit											

$\frac{1}{2}$	$\frac{1}{2}$

$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
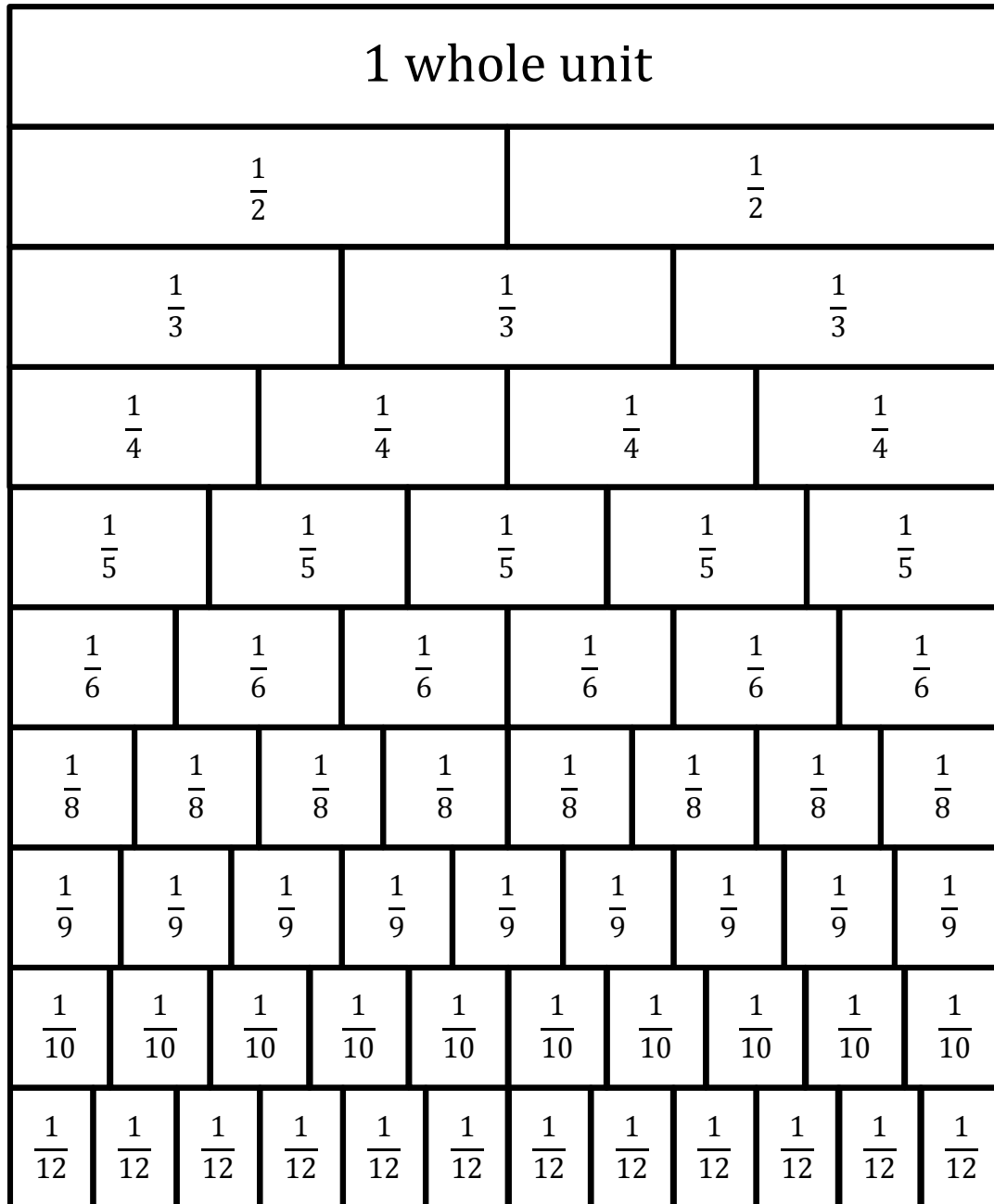
Lesson 8: Dividing Fractions and Mixed Numbers

Student Outcomes

- Students divide fractions by mixed numbers by first converting the mixed numbers into a fraction with a value larger than one.
- Students use equations to find quotients.

Lesson Notes

There is some mandatory prep work before teaching this lesson. The memory game that is included in this lesson needs to be cut and prepared for pairs or individual students.

Classwork

Example 1 (12 minutes): Introduction to Calculating the Quotient of a Mixed Number and a Fraction

- Carli has $4\frac{1}{2}$ walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has $\frac{5}{6}$ of a gallon left. How much paint can she use on each wall in order to have enough to paint the remaining walls?

- In order to solve the word problem, we must calculate the quotient of $\frac{5}{6} \div 4\frac{1}{2}$.

Before dividing, discuss how the answer must be less than one because you are dividing a smaller number by a larger number. Estimation could also be used to emphasize this point: $1 \div 5 = \frac{1}{5}$.

Explain that the mixed number must be converted into a fraction with a value larger than one. You may also emphasize that converting mixed numbers to fractions with a value larger than one is important for different division strategies. Have students complete this conversion on their own and share the process they followed.

Remind students about the formula they learned in the previous lesson; then, have them attempt to solve the problem. Have students show the process they used to find the quotient.

> *Scaffolding:*
>
> If students struggle with converting mixed numbers into fractions, a model may help. On a number line, show students how 9 steps of length $\frac{1}{2}$ will end up at $4\frac{1}{2}$.
>
>
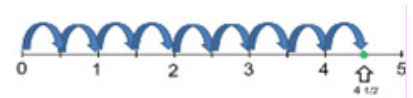
Example 1

Carli has $4\frac{1}{2}$ walls left to paint in order for all the bedrooms in her house to have the same color paint. However, she has used almost all of her paint and only has $\frac{5}{6}$ of a gallon left.

a. How much paint can she use on each wall in order to have enough to paint the remaining walls?

Calculate the quotient: $\quad \frac{5}{6} \div 4\frac{1}{2}$

Convert into a fraction: $\quad \frac{9}{2}$

Divide fractions: $\quad \frac{5}{6} \div \frac{9}{2} = \frac{5}{6} \times \frac{2}{9} = \frac{10}{54}$ or $\frac{5}{27}$

Carli can use $\frac{5}{27}$ of a gallon of paint on each of the remaining walls.

- Calculate the quotient. $\frac{2}{5} \div 3\frac{4}{7}$

Students solve this problem individually as the teacher walks around checking for understanding. Students then share their answers and processes used to find the quotients. Provide time for students to ask questions.

---

b. **Calculate the quotient.**

$\frac{2}{5} \div 3\frac{4}{7}$

Convert into a fraction: $\quad \frac{25}{7}$

Divide fractions: $\quad \frac{2}{5} \div \frac{25}{7} = \frac{2}{5} \times \frac{7}{25} = \frac{14}{125}$

Exercise (25 minutes)

Students complete the memory game individually or in small groups.

The following are some important directions for the teacher to share with students:

- The goal is to match each expression with the equivalent quotient. For the cards to match, it may be necessary to rewrite the quotient as a mixed number or equivalent fraction.

MP.1

- Each expression is assigned a letter. Students must show their work in that box on the student materials. Once they have solved a particular problem, they will not have to solve it again if they record their work in the correct place.

- Students are able to flip two cards over during each turn. If the expression and the quotient do not match, both cards should be flipped back over. If the expression and quotient do match, the student keeps the matches to determine a winner at the end.

If students are not clear about the directions, the teacher may choose two or three problems for students to do as a class to show how the memory game works.

Exercise

Show your work for the memory game in the boxes provided below.

A.	$\frac{3}{4} \div 6\frac{2}{3} = \frac{3}{4} \div \frac{20}{3} = \frac{3}{4} \times \frac{3}{20} = \frac{9}{80}$
B.	$\frac{1}{3} \div 4\frac{3}{4} = \frac{1}{3} \div \frac{19}{4} = \frac{1}{3} \times \frac{4}{19} = \frac{4}{57}$
C.	$\frac{2}{5} \div 1\frac{7}{8} = \frac{2}{5} \div \frac{15}{8} = \frac{2}{5} \times \frac{8}{15} = \frac{16}{75}$
D.	$7\frac{1}{2} \div \frac{5}{6} = \frac{15}{2} \div \frac{5}{6} = \frac{15}{2} \times \frac{6}{5} = \frac{90}{10} = 9$
E.	$3\frac{4}{7} \div \frac{5}{8} = \frac{25}{7} \div \frac{5}{8} = \frac{25}{7} \times \frac{8}{5} = \frac{200}{35} = 5\frac{5}{7}$
F.	$5\frac{5}{8} \div \frac{9}{10} = \frac{45}{8} \div \frac{9}{10} = \frac{45}{8} \times \frac{10}{9} = \frac{450}{72} = 6\frac{1}{4}$
G.	$\frac{1}{4} \div 10\frac{11}{12} = \frac{1}{4} \div \frac{131}{12} = \frac{1}{4} \times \frac{12}{131} = \frac{12}{524} = \frac{3}{131}$
H.	$5\frac{3}{4} \div \frac{5}{9} = \frac{23}{4} \div \frac{5}{9} = \frac{23}{4} \times \frac{9}{5} = \frac{207}{20} = 10\frac{7}{20}$
I.	$3\frac{1}{5} \div \frac{2}{3} = \frac{16}{5} \div \frac{2}{3} = \frac{16}{5} \times \frac{3}{2} = \frac{48}{10} = 4\frac{4}{5}$
J.	$\frac{3}{5} \div 3\frac{1}{7} = \frac{3}{5} \div \frac{22}{7} = \frac{3}{5} \times \frac{7}{22} = \frac{21}{110}$
K.	$\frac{10}{13} \div 2\frac{4}{7} = \frac{10}{13} \div \frac{18}{7} = \frac{10}{13} \times \frac{7}{18} = \frac{70}{234} = \frac{35}{117}$
L.	$2\frac{1}{4} \div \frac{7}{8} = \frac{9}{4} \div \frac{7}{8} = \frac{9}{4} \times \frac{8}{7} = \frac{72}{28} = 2\frac{4}{7}$

Closing (4 minutes)

- How does the process of dividing a fraction by a mixed number compare with our previous work with division of fractions? Discuss similarities and differences.
  - *Answers will vary, but some possibilities are listed below.*

    Similarities: We can use the invert and multiply method for both types of problems.

    Differences: It is necessary to change the mixed number into a fraction greater than one before applying the invert and multiply method.

Exit Ticket (4 minutes)

Name _____   Date_____

Lesson 8: Dividing Fractions and Mixed Numbers

Exit Ticket

Calculate the quotient.

1. $\dfrac{3}{4} \div 5\dfrac{1}{5}$

2. $\dfrac{3}{7} \div 2\dfrac{1}{2}$

3. $\dfrac{5}{8} \div 6\dfrac{5}{6}$

4. $\dfrac{5}{8} \div 8\dfrac{3}{10}$

Exit Ticket Sample Solutions

Calculate the quotient.

1. $\dfrac{3}{4} \div 5\dfrac{1}{5}$

$$\dfrac{3}{4} \div 5\dfrac{1}{5} = \dfrac{3}{4} \div \dfrac{26}{5} = \dfrac{3}{4} \times \dfrac{5}{26} = \dfrac{15}{104}$$

2. $\dfrac{3}{7} \div 2\dfrac{1}{2}$

$$\dfrac{3}{7} \div 2\dfrac{1}{2} = \dfrac{3}{7} \div \dfrac{5}{2} = \dfrac{3}{7} \times \dfrac{2}{5} = \dfrac{6}{35}$$

3. $\dfrac{5}{8} \div 6\dfrac{5}{6}$

$$\dfrac{5}{8} \div 6\dfrac{5}{6} = \dfrac{5}{8} \div \dfrac{41}{6} = \dfrac{5}{8} \times \dfrac{6}{41} = \dfrac{30}{328} \ or \ \dfrac{15}{164}$$

4. $\dfrac{5}{8} \div 8\dfrac{3}{10}$

$$\dfrac{5}{8} \div 8\dfrac{3}{10} = \dfrac{5}{8} \div \dfrac{83}{10} = \dfrac{5}{8} \times \dfrac{10}{83} = \dfrac{50}{664} \ or \ \dfrac{25}{332}$$

Problem Set Sample Solutions

Calculate each quotient.

1. $\dfrac{2}{5} \div 3\dfrac{1}{10}$

$$\dfrac{2}{5} \div \dfrac{31}{10} = \dfrac{2}{5} \times \dfrac{10}{31} = \dfrac{20}{155} \ or \ \dfrac{4}{31}$$

2. $4\dfrac{1}{3} \div \dfrac{4}{7}$

$$\dfrac{13}{3} \div \dfrac{4}{7} = \dfrac{13}{3} \times \dfrac{7}{4} = \dfrac{91}{12} \ or \ 7\dfrac{7}{12}$$

3. $3\dfrac{1}{6} \div \dfrac{9}{10}$

$$\dfrac{19}{6} \div \dfrac{9}{10} = \dfrac{19}{6} \times \dfrac{10}{9} = \dfrac{190}{54} \ or \ \dfrac{95}{27} \ or \ 3\dfrac{28}{54} \ or \ 3\dfrac{14}{27}$$

4. $\dfrac{5}{8} \div 2\dfrac{7}{12}$

$$\dfrac{5}{8} \div \dfrac{31}{12} = \dfrac{5}{8} \times \dfrac{12}{31} = \dfrac{60}{248} \ or \ \dfrac{15}{62}$$

Memory Game

A. $\dfrac{3}{4} \div 6\dfrac{2}{3}$	$\dfrac{9}{80}$	B. $\dfrac{1}{3} \div 4\dfrac{3}{4}$	$\dfrac{4}{57}$
C. $\dfrac{2}{5} \div 1\dfrac{7}{8}$	$\dfrac{16}{75}$	D. $7\dfrac{1}{2} \div \dfrac{5}{6}$	9
E. $3\dfrac{4}{7} \div \dfrac{5}{8}$	$5\dfrac{5}{7}$	F. $5\dfrac{5}{8} \div \dfrac{9}{10}$	$6\dfrac{1}{4}$
G. $\dfrac{1}{4} \div 10\dfrac{11}{12}$	$\dfrac{3}{131}$	H. $5\dfrac{3}{4} \div \dfrac{5}{9}$	$10\dfrac{7}{20}$
I. $3\dfrac{1}{5} \div \dfrac{2}{3}$	$5\dfrac{4}{5}$	J. $\dfrac{3}{5} \div 3\dfrac{1}{7}$	$\dfrac{21}{110}$
K. $\dfrac{10}{13} \div 2\dfrac{4}{7}$	$\dfrac{35}{117}$	L. $2\dfrac{1}{4} \div \dfrac{7}{8}$	$2\dfrac{4}{7}$

Topic B:

Multi-Digit Decimal Operations—Adding, Subtracting, and Multiplying

6.NS.B.3

Focus Standard:	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
Instructional Days:	3	
Lesson 9:	Sums and Differences of Decimals (P)[1]	
Lesson 10:	The Distributive Property and the Products of Decimals (P)	
Lesson 11:	Fraction Multiplication and the Products of Decimals (E)	

Prior to division of decimals, students will revisit all decimal operations in Topic B. Students have had extensive experience with decimal operations to the hundredths and thousandths (**5.NBT.B.7**), which prepares them to easily compute with more decimal places. Students begin by relating the first lesson in this topic to mixed numbers from the last lesson in Topic A. They find that sums and differences of large mixed numbers can be more efficiently determined by first converting to a decimal and then applying the standard algorithms (**6.NS.B.3**). Within decimal multiplication, students begin to practice the distributive property. Students use arrays and partial products to understand and apply the distributive property as they solve multiplication problems involving decimals. Place value enables students to determine the placement of the decimal point in products and recognize that the size of a product is relative to each factor. Students discover and use connections between fraction multiplication and decimal multiplication.

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[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 9: Sums and Differences of Decimals

Student Outcomes

- Students relate decimals to mixed numbers and round addends, minuends, and subtrahends to whole numbers in order to predict reasonable answers.
- Students use their knowledge of adding and subtracting multi-digit numbers to find the sums and differences of decimals.
- Students understand the importance of place value and solve problems in real-world contexts.

Lesson Notes

Students gained knowledge of rounding decimals in Grade 5. Students have also acquired knowledge of all operations with fractions and decimals to the hundredths place in previous grades.

Classwork

Discussion (5 minutes)

It is important for students to understand the connection between adding and subtracting mixed numbers and adding and subtracting decimals.

- Can you describe circumstances when it would be easier to add and subtract mixed numbers by converting them to decimals first?
  - *When fractions have large denominators, it would be difficult to find common denominators in order to add or subtract.*
  - *When a problem is solved by regrouping, it may be easier to borrow from decimals than fractions.*
- How can estimation be used to help solve addition and subtraction problems with rational numbers?
  - *Using estimation can help predict reasonable answers. It is a way to check to see if an answer is reasonable or not.*

Example 1 (8 minutes)

MP.6

Use this example to show students how rounding addends, minuends, and subtrahends can help predict reasonable answers. Also, have students practice using correct vocabulary (addends, sum, minuends, subtrahends, and difference) when talking about different parts of the expressions.

Example 1

$$25\frac{3}{10} + 376\frac{77}{100}$$

- Convert the mixed numbers into decimals.
  - $25.3 + 376.77$
- Round the addends to the nearest whole number. Then, find the estimated sum.
  - $25 + 377 = 402$
- Line up the addends appropriately using place value, and add.
  - $$\begin{array}{r} 25.3 \\ + \, 376.77 \\ \hline 402.07 \end{array}$$

MP.2 Show students that the sum is close to the estimation. Also show how the place value is important by completing the problem without lining up the place values. If this mistake is made, the actual sum is not close to the estimated sum.

Example 2 (8 minutes)

This example will be used to show that changing mixed numbers into decimals may be the best choice to solve a problem.

Divide the class in half. Have students solve the same problem, with one half of the class solving the problem using fractions and the other half of the class solving using decimals. Encourage students to estimate their answers before completing the problem.

Example 2

$426\dfrac{1}{5} - 275\dfrac{1}{2}$

Each group should get the same value as their answer; however, the fraction group will have $150\dfrac{7}{10}$, and the decimal group will have 150.7.

It is important for students to see that these numbers have the same value. Students solving the problem using fractions will most likely take longer to solve the problem and make more mistakes. Point out to students that the answers represent the same value, but using decimals made the problem easier to solve.

MP.2 When discussing the problem, use the correct vocabulary. $426\dfrac{1}{5}$ is the minuend, $275\dfrac{1}{2}$ is the subtrahend, and $150\dfrac{7}{10}$ is the difference.

Exercises 1–5 (14 minutes)

Students may work in pairs or individually to complete the following problems. Encourage students to write an expression and then round the addends, minuends, and subtrahends to the nearest whole number in order to predict a reasonable answer. Also, remind students that it is not always easier to change fractions to decimals before finding the sum or difference. Discuss the use of the approximation symbol when rounding decimals that repeat.

MP.2 & MP.6

Exercises 1–5

Calculate each sum or difference.

1. Samantha and her friends are going on a road trip that is $245\frac{7}{50}$ miles long. They have already driven $128\frac{53}{100}$. How much farther do they have to drive?

    Expression: $245\frac{7}{50} - 128\frac{53}{100}$ *Estimated answer:* $245 - 129 = 116$

    Actual answer: $245.14 - 128.53 = 116.61$

2. Ben needs to replace two sides of his fence. One side is $367\frac{9}{100}$ meters long, and the other is $329\frac{3}{10}$ meters long. How much fence does Ben need to buy?

    Expression: $367\frac{9}{100} + 329\frac{3}{10}$ *Estimated answer:* $367 + 329 = 696$

    Actual answer: $367.09 + 329.3 = 696.39.$

3. Mike wants to paint his new office with two different colors. If he needs $4\frac{4}{5}$ gallons of red paint and $3\frac{1}{10}$ gallons of brown paint, how much paint does he need in total?

    This problem is an example of where it may not be easiest to convert mixed numbers into decimals. Either method would result in a correct answer, but discuss with students why it may just be easier to find the sum by keeping the addends as mixed numbers.

    Expression: $4\frac{4}{5} + 3\frac{1}{10}$ *Estimated answer:* $5 + 3 = 8$

    Actual answer: $4\frac{8}{10} + 3\frac{1}{10} = 7\frac{9}{10}$

4. After Arianna completed some work, she figured she still had $78\frac{21}{100}$ pictures to paint. If she completed another $34\frac{23}{25}$ pictures, how many pictures does Arianna still have to paint?

    Expression: $78\frac{21}{100} - 34\frac{23}{25}$ *Estimated answer:* $78 - 35 = 43$

    Actual answer: $78.21 - 34.92 = 43.29$

Use a calculator to convert the fractions into decimals before calculating the sum or difference.

5. Rahzel wants to determine how much gasoline he and his wife use in a month. He calculated that he used $78\frac{1}{3}$ gallons of gas last month. Rahzel's wife used $41\frac{3}{8}$ gallons of gas last month. How much total gas did Rahzel and his wife use last month? Round your answer to the nearest hundredth.

    Expression: $78\frac{1}{3} + 41\frac{3}{8}$ *Estimated answer:* $78 + 41 = 119$

    Actual answer: $78.333 + 41.375 \approx 119.71$

Closing (5 minutes)

- Have students share their answers and processes for each of the exercise problems.
- Discuss which exercises would be easiest if the addends, minuends, or subtrahends were converted to decimals.
  - *Exercises 1, 2, and 4 would be easiest if the terms were converted to decimals before finding the sum or difference.*
  - *The only way to calculate the exact sum for Exercise 5 would be to leave it as a fraction. However, it would be easiest to solve by converting the mixed numbers to decimals.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 9: Sums and Differences of Decimals

Exit Ticket

Solve each problem. Show that the placement of the decimal is correct through either estimation or fraction calculation.

1. $382\frac{3}{10} - 191\frac{87}{100}$

2. $594\frac{7}{25} + 89\frac{37}{100}$

engage^{ny} 82

Exit Ticket Sample Solutions

Solve each problem. Show that the placement of the decimal is correct through either estimation or fraction calculation.

1. $382\frac{3}{10} - 191\frac{87}{100}$

   Estimation: $382 - 192 = 190$

   $382.3 - 191.87 = 190.43$

2. $594\frac{7}{25} + 89\frac{37}{100}$

   Estimation: $594 + 89 = 683$

   $594.28 + 89.37 = 683.85$

Problem Set Sample Solutions

1. **Find each sum or difference.**

   a. $381\frac{1}{10} - 214\frac{43}{100}$

      $381.1 - 214.43 = 166.67$

   b. $32\frac{3}{4} - 12\frac{1}{2}$

      $32\frac{3}{4} - 12\frac{2}{4} = 20\frac{1}{4}$ *or* 20.25

   c. $517\frac{37}{50} + 312\frac{3}{100}$

      $517.74 + 312.03 = 829.77$

   d. $632\frac{16}{25} + 32\frac{3}{10}$

      $632.64 + 32.3 = 664.94$

   e. $421\frac{3}{50} - 212\frac{9}{10}$

      $421.06 - 212.9 = 208.16$

2. **Use a calculator to find each sum or difference. Round your answer to the nearest hundredth.**

   a. $422\frac{3}{7} - 367\frac{5}{9}$

      $422.428571 - 367.555556 \approx 54.87$

   b. $23\frac{1}{5} + 45\frac{7}{8}$

      $23.2 + 45.875 \approx 69.08$

Lesson 10: The Distributive Property and the Products of Decimals

Student Outcomes

- Through the use of arrays and partial products, students use place value and apply the distributive property to find the product of decimals.

Lesson Notes

Stations are used in this lesson. Therefore, some prep work needs to be completed. Prepare stations before class and have a stopwatch available.

Classwork

Opening Exercise (3 minutes)

The Opening Exercise should be solved using the multiplication of decimals algorithm. These problems will be revisited in Examples 1 and 2 to show how partial products can assist in finding the product of decimals.

Opening Exercise

Calculate the product.

a. 200×32.6

$6,520$

b. 500×22.12

$11,060$

Example 1 (5 minutes): Introduction to Partial Products

Show students how the distributive property can assist in calculating the product of decimals. Use this example to model the process.

Example 1: Introduction to Partial Products

Use partial products and the distributive property to calculate the product.

200×32.6

$200(32) + 200(0.6) = 6,400 + 120 = 6,520$

MP.7

Separate 32.6 into an addition expression with two addends, 32 and 0.6. Emphasize the importance of the place value. The problem will now be $200 \times (32 + 0.6)$.

When the distributive property is applied, the problem will be $200(32) + 200(0.6)$.

MP.7

It is ideal for students to to be able to solve these problems mentally using the distributive property, but we understand if additional scaffolding is needed for struggling students. Remind students that they need to complete the multiplication before adding. After giving students time to solve the problem, ask for their solutions.

Show students that the answer to this example is the same as the Opening Exercise but that most of the calculations in this example could be completed mentally.

Example 2 (7 minutes): Introduction to Partial Products

Have students try to calculate the product by using partial products. After they complete the problem, encourage students to check their answers by comparing it to the product of the second problem in the Opening Exercise. When a majority of the class has completed the problem, have some students share the processes they used to find the product. Answer all student questions before moving on to the Exercises.

Scaffolding:

Possible extension: Have students complete more than two partial products. An example would be $500(20 + 2 + 0.1 + 0.02)$.

Example 2

Use partial products and the distributive property to calculate the area of the rectangular patio shown below.

22.12 ft.

500 ft.

$500 \times 22.12 = 500(22 + 0.12) = 500(22) + 500(0.12) = 11,000 + 60 = 11,060$ *square feet*

22.12 ft. 60 ft²   — 0.12 ft.

$11,000$ ft²   — 22 ft.
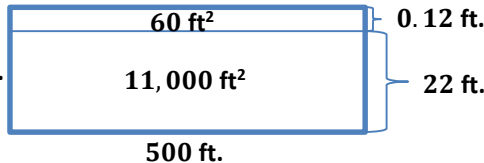
500 ft.

The area of the patio would be $11,060$ square feet.

Exercises (20 minutes)

MP.7

Students complete stations individually or in pairs. Encourage students to use partial products in order to solve the problems. Students are to write the problem and their processes in the space provided in the student materials. Remind students to record each station in the correct place because not everyone will start at station one.

Exercises

Use the boxes below to show your work for each station. Make sure that you are putting the solution for each station in the correct box.

Station One:

Calculate the product of $300 \times 25.4.$

$300(25) + 300(0.4) = 7,500 + 120 = 7,620$

Station Two:

Calculate the product of $45.9 \times 100.$

$100(45) + 100(0.9) = 4,500 + 90 = 4,590$

Station Three:

Calculate the product of $800 \times 12.3.$

$800(12) + 800(0.3) = 9,600 + 240 = 9,840$

Station Four:

Calculate the product of $400 \times 21.8.$

$400(21) + 400(0.8) = 8,400 + 320 = 8,720$

Station Five:

Calculate the product of $32.6 \times 200.$

$200(32) + 200(0.6) = 6,400 + 120 = 6,520$

Closing (6 minutes)

Students share their answers to the stations and ask any unanswered questions.

Exit Ticket (4 minutes)

Name _____   Date_____

Lesson 10: The Distributive Property and the Products of Decimals

Exit Ticket

Complete the problem using partial products.

500×12.7

Exit Ticket Sample Solutions

Complete the problem using partial products.

500×12.7

$500 \times 12.7 = 500(12) + 500(0.7) = 6,000 + 350 = 6,350$

Problem Set Sample Solutions

Calculate the product using partial products.

1. 400×45.2

     $400(45) + 400(0.2) = 18,000 + 80 = 18,080$

2. 14.9×100

     $100(14) + 100(0.9) = 1,400 + 90 = 1,490$

3. 200×38.4

     $200(38) + 200(0.4) = 7,600 + 80 = 7,680$

4. 900×20.7

     $900(20) + 900(0.7) = 18,000 + 630 = 18,630$

5. 76.2×200

     $200(76) + 200(0.2) = 15,200 + 40 = 15,240$

Lesson 11: Fraction Multiplication and the Products of Decimals

Student Outcomes

- Students use estimation and place value to determine the placement of the decimal point in products and to determine that the size of the product is relative to each factor.
- Students discover and use connections between fraction multiplication and decimal multiplication.
- Students recognize that the sum of the number of decimal digits in the factors yields the decimal digits in the product.

Lesson Notes

To complete this lesson, students will need large poster paper and markers so that they can present their detailed solutions to the Exploratory Challenge.

Classwork

Exploratory Challenge (20 minutes)

Students work in small groups to complete the two given problems. After finding each product, group members will use previous knowledge to convince their classmates that the product has the decimal in the correct location.

MP.1 MP.6 & MP.7

- Students will solve their problems on poster paper using the markers provided.
- On the poster paper, students will include all work that supports their solutions and the placement of the decimal in the answer. You may need to prompt students about their previous work with rounding and multiplication of mixed numbers.
- All groups, even those whose solutions or supporting work contain errors, will present their solutions and explain their supporting work. Having the decimal in the wrong place will allow for a discussion on why the decimal placement is incorrect. Since all groups are presenting, allow each group to present only one method of proving where the decimal should be placed.

Exploratory Challenge

You will not only solve each problem, but your groups will also need to prove to the class that the decimal in the product is located in the correct place. As a group, you will be expected to present your informal proof to the class.

1. Calculate the product. 34.62×12.8

   $34.62 \times 12.8 = 443.136$

   Some possible proofs:

   Using estimation: $35 \times 13 = 455$. *If the decimal was located in a different place, the product would not be close to* 455.

   Using fractions: $34\frac{62}{100} \times 12\frac{8}{10} = \frac{3,462}{100} \times \frac{128}{10} = \frac{443,136}{1,000}$. *Because the denominator is* $1,000$, *the last digit should be in the thousandths place when writing the fraction as a decimal. Therefore, the answer would be* 443.136.

2. Xavier earns $\$11.50$ per hour working at the nearby grocery store. Last week, Xavier worked 13.5 hours. How much money did Xavier earn last week? Remember to round to the nearest penny.

   $11.5 \times 13.5 = 155.25$

   Some possible proofs:

   Using estimation: $12 \times 14 = 168$. *If the decimal was located in a different place, the product would not be close to* 168.

   Using fractions: $11\frac{5}{10} \times 13\frac{5}{10} = \frac{115}{10} \times \frac{135}{10} = \frac{15,525}{100}$. *Because the denominator is* 100, *the last digit should be in the hundredths place when writing the fraction as a decimal. Therefore, the answer would be* $\$155.25$.

Discussion (5 minutes)

- Do you see a connection between the number of decimal digits in the factors and the product?
  - *In the first problem, there are two decimal digits in the first factor and one decimal digit in the second factor, which is a total of three decimal digits. The product has three decimal digits.*
  - *In the second problem, both factors have one decimal digit for a total of two decimal digits in the factors. The product also has two decimal digits.*

Show students that this is another way to determine if their decimal point is in the correct place. If this point was brought up by students in their presentations, the discussion can reiterate this method to find the correct placement of the decimal. Remind students to place the decimal before eliminating any unnecessary zeros from the answer.

At the end of the discussion, have students record notes on decimal placement in the student materials.

Discussion

Record notes from the discussion in the box below.

Exercises 1–4 (10 minutes)

MP.6 Students work individually to solve the four practice problems. Emphasize the importance of decimal placement to hold place value.

Exercises 1–4

1. Calculate the product: 324.56×54.82.

   $324.56 \times 54.82 = 17,792.3792$

2. Kevin spends $\$11.25$ on lunch every week during the school year. If there are 35.5 weeks during the school year, how much does Kevin spend on lunch over the entire school year? Remember to round to the nearest penny.

   $11.25 \times 35.5 = 399.375 \cong 399.38$

   Kevin would spend $\$399.38$ on lunch over the entire school year.

3. Gunnar's car gets 22.4 miles per gallon, and his gas tank can hold 17.82 gallons of gas. How many miles can Gunnar travel if he uses all of the gas in the gas tank?

   $22.4 \times 17.82 = 399.168$

   Gunnar can drive 399.168 miles on an entire tank of gas.

4. The principal of East High School wants to buy a new cover for the sand pit used in the long jump competition. He measured the sand pit and found that the length is 29.2 feet and the width is 9.8 feet. What will the area of the new cover be?

   $29.2 \times 9.8 = 286.16$

   The cover should have an area of 286.16 square feet.

Closing (5 minutes)

- How can we use information about the factors to determine the place value of the product and the number of decimal digits in the product?
  - *Calculate the sum of decimal digits in the factors. This sum represents the number of decimal digits in the product.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 11: Fraction Multiplication and the Product of Decimals

Exit Ticket

Use estimation or fraction multiplication to determine if your answer is reasonable.

1. Calculate the product: 78.93×32.45.

2. Paint costs $29.95 per gallon. Nikki needs 12.25 gallons to complete a painting project. How much will Nikki spend on paint? Remember to round to the nearest penny.

Exit Ticket Sample Solutions

> 1. **Calculate the product:** $78.93 \times 32.45.$
>
> $2,561.2785$
>
> 2. **Paint costs** $\$29.95$ **per gallon. Nikki needs** 12.25 **gallons to complete a painting project. How much will Nikki spend on paint? Remember to round to the nearest penny.**
>
>   *Nikki would spend* $\$366.89$ *on paint to complete her project.*

Problem Set Sample Solutions

> **Solve each problem. Remember to round to the nearest penny when necessary.**
>
> 1. **Calculate the product:** $45.67 \times 32.58.$
>
> $1,487.9286$
>
> 2. **Deprina buys a large cup of coffee for** $\$4.70$ **on her way to work every day. If there are** 24 **work days in the month, how much does Deprina spend on coffee throughout the entire month?**
>
> $4.70 \times 24 = 112.80$
>
>   *Deprina would spend* $\$112.80$ *a month on coffee.*
>
> 3. **Krego earns** $\$2,456.75$ **every month. He also earns an extra** $\$4.75$ **every time he sells a new gym membership. Last month, Krego sold** 32 **new gym memberships. How much money did Krego earn last month?**
>
> $2,456.75 + (4.75 \times 32) = 2,608.75$
>
>   *Krego earned* $\$2,608.75$ *last month.*
>
> 4. **Kendra just bought a new house and needs to buy new sod for her backyard. If the dimensions of her yard are** 24.6 **feet by** 14.8 **feet, what is the area of her yard?**
>
> $24.6 \times 14.8 = 364.08$
>
>   *The area of Kendra's yard is* 364.08 *square feet.*

Name _____   Date _____

1. Yasmine is having a birthday party with snacks and activities for her guests. At one table, five people are sharing three-quarters of a pizza. What equal-sized portion of the whole pizza will each of the five people receive?

   a. Use a model (e.g., picture, number line, or manipulative materials) to represent the quotient.

   b. Write a number sentence to represent the situation. Explain your reasoning.

   c. If three-quarters of the pizza provided 12 pieces to the table, how many pieces were in the pizza when it was full? Support your answer with models.

2. Yasmine needs to create invitations for the party. She has $\frac{3}{4}$ of an hour to make the invitations. It takes her $\frac{1}{12}$ of an hour to make each card. How many invitations can Yasmine create?

    a. Use a number line to represent the quotient.

    b. Draw a model to represent the quotient.

    c. Compute the quotient without models. Show your work.

3. Yasmine is serving ice cream with the birthday cake at her party. She has purchased $19\frac{1}{2}$ pints of ice cream. She will serve $\frac{3}{4}$ of a pint to each guest.

    a. How many guests can be served ice cream?

    b. Will there be any ice cream left? Justify your answer.

4. L.B. Johnson Middle School held a track and field event during the school year. Miguel took part in a four-person shot put team. Shot put is a track and field event where athletes throw (or "put") a heavy sphere, called a "shot," as far as possible. To determine a team score, the distances of all team members are added. The team with the greatest score wins first place. The current winning team's final score at the shot put is 52.08 ft. Miguel's teammates threw the shot put the following distances: 12.26 ft., 12.82 ft., and 13.75 ft. Exactly how many feet will Miguel need to throw the shot put in order to tie the current first place score? Show your work.

5. The sand pit for the long jump has a width of 2.75 meters and a length of 9.54 meters. Just in case it rains, the principal wants to cover the sand pit with a piece of plastic the night before the event. How many square meters of plastic will the principal need to cover the sand pit?

6. The chess club is selling drinks during the track and field event. The club purchased water, juice boxes, and pouches of lemonade for the event. They spent $138.52 on juice boxes and $75.00 on lemonade. The club purchased three cases of water. Each case of water costs $6.80. What is the total cost of the drinks?

| A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem,** <u>OR</u> **an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** **6.NS.A.1**	Student response is incorrect and is not supported by a visual model. <u>OR</u> Student did not answer the question.	Student response is incorrect, but some evidence of reasoning is presented with a flawed visual model.	Student visual model is correct; however, the quotient of $\frac{3}{20}$ is not determined. <u>OR</u> Student response is correct, and the answer is supported with a visual model, but the model is inaccurate.	Student response is correct. The visual model is appropriate and supports the quotient of $\frac{3}{20}$. The student may have chosen to support the quotient with the use of more than one visual model.
	b **6.NS.A.1**	Student response is incorrect. <u>OR</u> Student did not answer the question.	Student response is incorrect, but a portion of the equation has reasoning. For example, the student may have figured out to divide by five but did not multiply by $\frac{1}{5}$ to determine the quotient.	Student response is incorrect; however, the equation shows reasoning. The equation supports dividing by 5 and makes connection to multiplying by $\frac{1}{5}$ to determine the quotient of $\frac{3}{20}$, but computation is incorrect.	Student response of $\frac{3}{20}$ is correct. The equation depicts the situation and makes connections between division and multiplication. All calculations are correct.
	c **6.NS.A.1**	Student response is incorrect. Student found the product of $\frac{3}{4} \times 12$ to arrive at 9 as the solution. <u>OR</u> Student response is incorrect and is not supported with visual models.	Student response of 16 pieces is correct, but is not supported with visual models. <u>OR</u> Student response is incorrect with no support but shows general understanding of the equation.	Student response of 16 is correct. Student arrived at the answer using an equation, but did not support reasoning with a model. <u>OR</u> Student calculation is incorrect, but visual models support reasoning.	Student response of 16 is correct. Student supported the solution with appropriate visual models and determined the amount of each portion in order to determine the full amount.

2	a 6.NS.A.1	Student response is incorrect or missing. OR Student found the product of $\frac{3}{4} \times \frac{1}{12}$ to reach the response of $\frac{1}{16}$. A number line diagram does not support the response.	Student response is incorrect, but depicts some reasoning in an incomplete number line diagram. OR Student response of 9 invitations is correct without a supporting number line diagram.	Student response of 9 invitations is correct. Reasoning is evident through the use of a number line diagram, but the response is in terms of time, such as $\frac{9}{12}$ or $\frac{3}{4}$ of an hour, and not in the number of cards. OR Student response is correct through the use of calculation but is not supported by the number line diagram.	Student response of 9 invitations is correct. Reasoning is evident through the depiction of an accurately designed number line diagram.
	b 6.NS.A.1	Student response is incorrect or missing. OR Student computed the product of $\frac{3}{4} \times \frac{1}{2}$ to reach the response of $\frac{3}{8}$. No visual representation supports the student response.	Student response is incorrect, but depicts some reasoning in an incomplete visual model. OR Student response is correct but reasoning is unclear through the misuse of a visual model.	Student response is correct. Reasoning is evident through the use of visual models, but the response is in terms of time, such as $\frac{9}{12}$ or $\frac{3}{4}$ of an hour, and not in the number of cards. OR Student response is correct through the use of calculation but is not supported by the visual model.	Student response of 9 invitations is correct. Reasoning is evident through the depiction of an accurately designed visual model.
	c 6.NS.A.1	Student response is incorrect or missing. OR Student computed the product of the given fractions instead of determining the quotient.	Student response is correct but includes no computation to support reasoning.	Student response is correct. Student computed the quotient as 9 invitations but showed minimal computation.	Student response of 9 invitations is correct. Student demonstrated evidence of reasoning through concise application of an equation with accurate calculations.
3	a 6.NS.A.1	Student response is incorrect or missing. OR Student determined the product of $19\frac{1}{2}$ and $\frac{3}{4}$.	Student response is correct but shows no computation or reasoning. OR Student response is incorrect, but reasoning is evident through calculations.	Student response of 26 people is correct and represents some reasoning through calculation. OR Student response shows reasoning and application of mixed number conversion but includes errors in calculation.	Student response is correct. Reasoning is evident through correct mixed number conversion. The quotient of 26 people is determined using apparent understanding of factors.

	b **6.NS.A.1**	Student response is missing.	Student response is incorrect and does not depict understanding of whole and mixed numbers.	Student response correctly determines that there will be no leftover ice cream but is not supported with a clear understanding of whole and mixed numbers.	Student response is correct. Student explanation and reasoning include the understanding that a mixed number response will provide left over ice cream where a whole number response would not.
4	**6.NS.B.3**	Student response is incorrect. Justification does not include adding the given throw distances and determining the difference of that sum and the distance needed to tie for first place. The student response may show only addition.	Student response is incorrect but attempts to determine the sum of the throw distances first and then the difference of the sum and the distance needed to tie first place.	Student response is incorrect due to slight miscalculations when adding or subtracting. It is evident that the student understands the process of adding the decimals first, then subtracting the sum from the other team's final score.	Student response is correct. Student accurately determines the sum of the throw distances as 38.83 feet and the differences between that sum and the score needed to tie as 13.25 feet. It is evident that the student understands the process of adding the decimals first, then subtracting the sum from the other team's final score.
5	**6.NS.B.3**	Student response is incorrect or missing. The response depicts the use of an incorrect operation, such as addition or subtraction.	Student response is incorrect. The response shows understanding of multi-digit numbers but lacks precision in place value, resulting in a product less than 3 or more than 262.	Student response of 26.235 square meters is correct but shows little to no reasoning that multiplication is the accurate operation to choose to find the area of plastic to cover the sand pit.	Student response is correct and shows complete understanding of place value. The response of 26.235 square meters includes a picture that depicts finding the area through multiplication of the length and width of the sand pit.
6	**6.NS.B.3**	Student response is incorrect or missing. The response disregards finding the total price of the water.	Student response is incorrect. Student finds the total price of the water only.	Student response is incorrect. Student finds the total price of the water and adds it to the price of the lemonade and juice but makes minor computation errors.	Student response is correct. The student finds the total price of the water to be $20.40 and accurately adds it to the price of the lemonade and juice to determine a total cost of $233.92.

COMMON CORE™

Module 2:   Arithmetic Operations Including Division of Fractions
Date: 7/3/14

engageny

Name _____   Date _____

1. Yasmine is having a birthday party with snacks and activities for her guests. At one table, five people are sharing three-quarters of a pizza. What equal-sized portion of the pizza will each of the five people receive?

    a. Use a model (e. g., picture, number line, or manipulative materials) to represent the quotient.
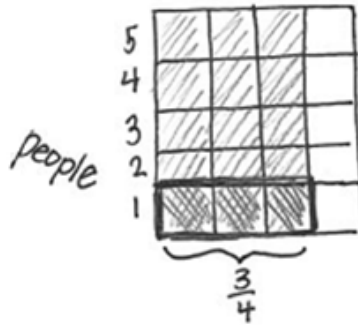
    b. Write a number sentence to represent the situation. Explain your reasoning.

    Because there are 5 people, we found 1 out of the 5, which is $\frac{1}{5}$. I can represent the situation as:

    $$\frac{3}{4} \div 5 = \frac{3}{4} \cdot \frac{1}{5} = \frac{3}{20}$$

    c. If three-quarters of the pizza provided 12 pieces to the table, how many pieces were in the pizza when it was full? Support your answer with models.
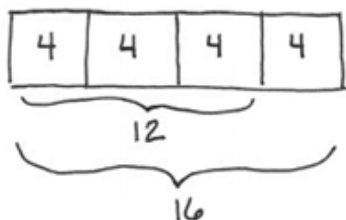
    $\frac{12}{3} = 4$, each portion is 4,

    $4 \cdot 4 = 16$ pieces

2. Yasmine needs to create invitations for the party. She has $\frac{3}{4}$ of an hour to make the invitations. It takes her $\frac{1}{12}$ of an hour to make each card. How many invitations can Yasmine create?

   a. Use a number line to represent the quotient.
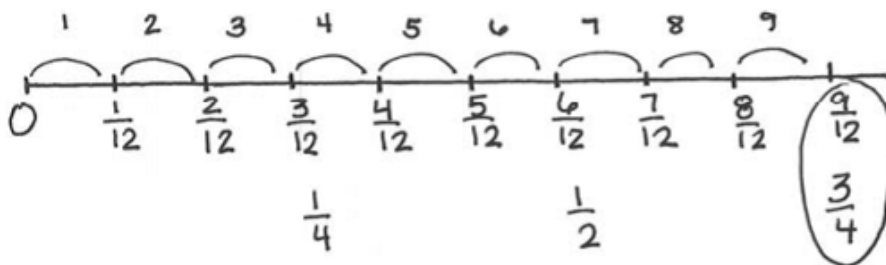
   Yasmine can make 9 invitations.

   b. Draw a model to represent the quotient.
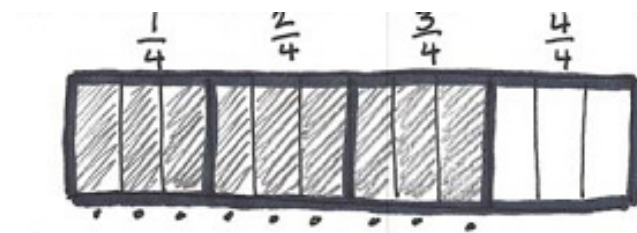
   $$\frac{3}{4} \div \frac{1}{12} = 9$$

   c. Compute the quotient without models. Show your work.

   $$\frac{3}{4} \div \frac{1}{12} = \frac{9}{12} \div \frac{1}{12} = \frac{9 \text{ twelfths}}{1 \text{ twelfth}} = \frac{9}{1} = 9$$

3. Yasmine is serving ice cream with the birthday cake at her party. She has purchased $19\frac{1}{2}$ pints of ice cream. She will serve $\frac{3}{4}$ of a pint to each guest.

    a. How many guests can be served ice cream?

$$\begin{array}{r} 19 \\ \times\ 2 \\ \hline 38 \end{array} + 1 = 39$$

$$19\tfrac{1}{2} \div \tfrac{3}{4}$$

$$\frac{39}{2} \div \frac{3}{4} = \frac{39}{2} \cdot \frac{4}{3} = 13 \times 2 = 26$$

Yasmine can serve 26 people.

    b. Will there be any ice cream left? Justify your answer.

My answer, 26, is a whole number, so there will be no ice cream left over. If my answer was $26\frac{1}{4}$ or any mixed number, there would be ice cream left over.

4. L.B. Johnson Middle School held a track and field event during the school year. Miguel took part in a four-person shot put team. Shot put is a track and field event where athletes throw (or "put") a heavy sphere, called a "shot," as far as possible. To determine a team score, the distances of all team members are added. The team with the greatest score wins first place. The current winning team's final score at the shot put is 52.08 ft. Miguel's teammates threw the shot put the following distances: 12.26 ft., 12.82 ft., and 13.75 ft. Exactly how many feet will Miguel need to throw the shot put in order to tie the current first place score? Show your work.

$$12.26$$
$$12.82$$
$$+\ 13.75$$
$$\overline{38.83}$$

$$52.08$$
$$-\ 38.83$$
$$\overline{13.25}$$

Miguel will need to throw the shot put 13.25 feet to tie the current first place score.
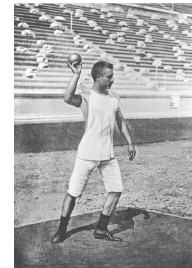
5. The sand pit for the long jump has a width of 2.75 meters and a length of 9.54 meters. Just in case it rains, the principal wants to cover the sand pit with a piece of plastic the night before the event. How many square meters of plastic will the principal need to cover the sand pit?

$$9.54$$
$$\times\ 2.75$$
$$\overline{4770}$$
$$67780$$
$$+\ 190800$$
$$\overline{26.2350}$$

The principal needs 26.235 m² of plastic to cover the sand pit.

6. The chess club is selling drinks during the track and field event. The club purchased water, juice boxes, and pouches of lemonade for the event. They spent $138.52 on juice boxes and $75.00 on lemonade. The club purchased three cases of water. Each case of water costs $6.80. What is the total cost of the drinks?

water

$6.80
× 3
———
$20.40

juice $138.52
lemonade   75.00
water   20.40
———————
$233.92

The total cost
of the drinks
was $233.92

Topic C:

Dividing Whole Numbers and Decimals

6.NS.B.2, 6.NS.B.3

Focus Standard:	6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.
	6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
Instructional Days:	4	
Lesson 12:	Estimating Digits in a Quotient (P)[1]	
Lesson 13:	Dividing Multi-Digit Numbers Using the Algorithm (P)	
Lesson 14:	The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions (P)	
Lesson 15:	The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math (P)	

In Topic C, students build upon previous lessons to fluently divide numbers and decimals. They apply estimation to place value and determine that the standard algorithm is simply a tally system arranged in place value columns (**6.NS.B.2**). Students understand that when they "bring down" the next digit in the algorithm, they are distributing, recording, and shifting to the next place value. They understand that the steps in the algorithm continually provide better approximations to the answer. Students further their understanding of division as they develop fluency in the use of the standard algorithm to divide multi-digit decimals (**6.NS.B.3**). They make connections to division of fractions and rely on mental math strategies in order to implement the division algorithm when finding the quotients of decimals.

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[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Lesson 12: Estimating Digits in a Quotient

Student Outcomes

▪ Students connect estimation with place value in order to determine the standard algorithm for division.

Classwork

Opening Exercise (5 minutes)

Opening Exercise

Show an example of how you would solve $5,911 \div 23$. You can use any method or model to show your work. Just be sure that you can explain how you arrived at your solution.

There are many possible models. Here is one possible solution:

	23
200	4600
50	1150
5	115
2	46

$5,911 \div 23 = 257$

We may want to check our work to see if our answer is reasonable. One way to do this is to estimate our answer.

▪ Estimate the quotient of 5,911 and 23.

  □ *Answers will vary. Sample solution: First I would round the numbers to $6,000$ and 20.*

  □ *Answer will vary. Sample solution: $6,000 \div 20 = 300$.*

▪ Using your estimation, would you say that the answer you came up with in your model is reasonable?

  □ *Answers will vary. Sample solution: Yes, 257 is close to 300. This shows that my answer is reasonable.*

▪ Would you expect your estimated answer to be greater than or less than the actual answer? Why?

  □ *Answers will vary. Sample solutions: Since I chose to round 23 to 20, I expect that the estimated answer will be greater than the actual quotient. If I had rounded up to 25, the estimated answer would be smaller than the actual quotient. (Students could also respond that $5,911$ was rounded to $6,000$. Because the number was rounded up, we know that the estimate will be larger than the actual answer.)*

- What other ways can we check our solution?

  □ *I can use the inverse of division (multiplication) to check my work.*

Example 1 (10 minutes)

> **Example 1**
>
> **We can also use estimates before we divide to help us solve division problems. In this lesson, we will be using estimation to help us divide two numbers using the division algorithm.**
>
> **Estimate the quotient of $8,085 \div 33$. Then, divide.**
>
> **$8,100 \div 30 = 270.$**

- How could I round these numbers to get an estimate?

  □ *There are many possible solutions. For example,* $8,000 \div 30$*;* $8,000 \div 35$*;* $8,100 \div 30$*;* $8,100 \div 35$*.*

- Why is 8,100 and 30 the best option?

  □ 3 *is not a factor of* 8*, but it is a factor of* 81*.*

- How can we use this to help us divide 8,085 by 33?

  □ *When I begin to divide, I use* 270 *to help me choose what numbers to divide by. My actual answer should be near* 270*. The first number I used in my area model will be* 200*. Then, I will see that the remainder is* $1,485$*. I know that* $30 \times 50 = 1,500$*, which is too big. So, I will choose one less ten and try* 40*.*

> **Create a model to show the division of $8,085$ by 33.**
>
>

- We can keep track of the areas in the model and what we have left by making a list and subtracting. We will create a list to keep track of the amounts the same way we created the diagram.

- Now, we can relate this model to the standard division algorithm.

$$\require{enclose} \begin{array}{r} 245 \\ 33 \enclose{longdiv}{8085} \\ -\,6600 \\ \hline 1485 \\ -\,1320 \\ \hline 165 \\ -\,165 \\ \hline 0 \end{array}$$

At this point, you are just showing how this work is the same as the work that is shown in the model.

- What does the 2 represent on top of the division bar?
    - *We divided* 80 *hundreds by* 33*, so the* 2 *represents* 2 *hundreds.*
- How did we use the 200 in the previous model?
    - $33 \times 200 = 6{,}600$
- What does the 4 represent on top of the division bar?
    - *We divided* 148 *tens by* 33*, so the* 4 *represents* 4 *tens.*
- How was the 40 used in the previous model?
    - $33 \times 40 = 1{,}320.$

MP.8
- Now let's check our division. How can we use the quotient to check our work?
    - $245 \times 33 = 8{,}085.$

Example 2 (10 minutes)

> **Example 2**
>
> **Use estimation and the standard algorithm to divide:** $1{,}512 \div 27.$

Students will estimate the quotient first and use the estimate to help them divide using the algorithm.

- Share an estimate that can be used to help us divide.
    - *Answers may vary.* $1{,}500 \div 30 = 50.$
- In the algorithm, we can show that there are fifty 27s by placing a 5 over the tens place. We know that 5 tens is 50. This is really showing that 151 tens $\div 27 = 5$ tens.

$$\require{enclose} \begin{array}{r} 5 \\ 27 \enclose{longdiv}{1512} \end{array}$$

- What would typically be the next step if you were creating a model? (Students can create the model while solving to further solidify the connection.)

  □ *I would multiply* 50×27, *which is* $1{,}350$. *Then, I would subtract* $1{,}350$ *from* $1{,}512$.

```
  1512
-  1350     27 × 50 ⎤
   ────               ⎬─ 27 × 56
    162     27 × 6  ⎦
-   162
   ────
      0
```

We will show these same steps in the algorithm.

MP.8

```
         5           │ 27
       ┌──────       ├──────────┐
    27 │ 1512     50 │  1350
       │ -1350       │
       │  ────       ├──────────┤
       │   162       │
```

- What would we do next?

  □ 162 *ones* $\div 27$. *I know that* 30×5 *is* 150. *So, I am going to estimate that the answer is greater than* 5. *Maybe there are six* 27s *in* 162.

- We will show the same steps again where we check our work by multiplying and subtracting.

```
        56          │ 27
      ┌──────       ├──────────┐
   27 │ 1512     50 │  1350
      │ -1350       │
      │  ────       ├──────────┤
      │   162     6 │   162
      │  -162       │
      │  ────
      │     0
```

- Finally, we will check our work. Remind me one more time how we can check our quotient.

  □ *We can multiply* 27×56 *and see if the product is* $1{,}512$.

Exercises 1–4 (10 minutes)

> **Exercises 1–4**
>
> 1. $1{,}008 \div 48$
>
>   a. **Estimate the quotient.**
>
>   *Answers may vary.*
>
> $1{,}000 \div 50 = 20$

b. Use the algorithm to divide. Draw a model to show how the steps relate to the steps used in the algorithm.

$$48 \overline{)\begin{array}{r} 21 \\ 1008 \\ -960 \\ \hline 48 \\ -48 \\ \hline 0 \end{array}}$$

	48
20	960
1	48
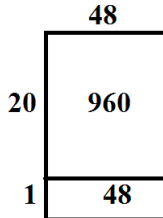
c. Check your work.

$48 \times 21 = 1,008$

2. $2,508 \div 33$

a. Estimate the quotient.

Answers may vary.

$2,400 \div 30 = 80$

b. Use the algorithm to divide. Draw a model to show how the steps relate to the steps used in the algorithm.

$$33 \overline{)\begin{array}{r} 76 \\ 2508 \\ -2310 \\ \hline 198 \\ -198 \\ \hline 0 \end{array}}$$

	33
70	2310
6	198
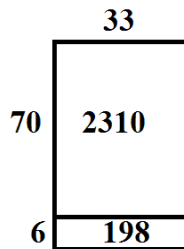
c. Check your work.

$33 \times 76 = 2,508$

3. $2,156 \div 28$

a. Estimate the quotient.

Answers may vary.

$2,100 \div 30 = 70$

b. Use the algorithm to divide.

$$28 \overline{)\begin{array}{r} 77 \\ 2156 \\ -1960 \\ \hline 196 \\ -196 \\ \hline 0 \end{array}}$$

c. Check your work.

$28 \times 77 = 2,156$

4. $4{,}732 \div 52$

    a.   **Estimate the quotient.**

       Answers may vary.

       $5{,}000 \div 50 = 100$

    b.   **Use the algorithm to divide.**

$$
\begin{array}{r}
91 \\
52\overline{)4732} \\
\underline{-4680} \\
52 \\
\underline{-\;52} \\
0
\end{array}
$$

    c.   **Check your work.**

       $52 \times 91 = 4{,}732$

Closing (5 minutes)

- How does estimation help you with the process of finding the exact quotient?
  - *My estimate gives me an idea of what number my answer should be close to. For example, if I have an estimate of* 75*, I know not to try a number in the hundreds.*
- In the previous problem, we used 100 to approximate the quotient 4,732 ÷ 52. How did we know that our actual quotient would be in the 90s and not 100 as our approximation suggested? When using your estimate, how do you know if your estimate is too big?
  - *I know that* $100 \times 52 = 5{,}200$*, and this is greater than* 4,732*. This tells me to start with a* 9 *in the tens place.*
- When using your estimate, how do you know if your estimate is too small?
  - *When I subtract, the difference is bigger than the divisor.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 12: Estimating Digits in a Quotient

Exit Ticket

1.   Estimate the quotient: $1,908 \div 36$.

2.   Use the division algorithm and your estimate to find the quotient: $1,908 \div 36$.

3.   Use estimation to determine if $8,580 \div 78$ has a quotient in the 10s, 100s, or 1000s.

Exit Ticket Sample Solutions

1.   **Estimate the quotient:** $1,908 \div 36$.

$2,000 \div 40 = 50$

2.   **Use the division algorithm and your estimate to find the quotient:** $1,908 \div 36$.

```
        53
36) 1908
   -1800
     108
   -  108
        0
```

3.   **Use estimation to determine if** $8,580 \div 78$ **has a quotient in the 10s, 100s, or 1000s.**

I would round $8,580$ *to* $8,800$ *and* 78 *to* 80. $8,800 \div 80 = 110$. *I know that the quotient should be in the* 100*s.*

Problem Set Sample Solutions

Complete the following steps for each problem:

   a.   **Estimate the quotient.**

   b.   **Use the division algorithm to solve.**

   c.   **Show a model that supports your work with the division algorithm.**

   d.   **Check your work.**

1. $3,312 \div 48$

$3,500 \div 50 = 70$

```
        69
48) 3312
   -2880
     432
   - 432
        0
```

	48
60	2880
9	432

$48 \times 69 = 3,312$

2. $3,125 \div 25$

$3,000 \div 30 = 100$

$$
\begin{array}{r}
125 \\
25 \overline{)\ 3125} \\
-2500 \\
\hline
625 \\
-500 \\
\hline
125 \\
-\ 125 \\
\hline
0
\end{array}
$$

	25
100	2500
20	500
5	125

$25 \times 125 = 3,125$

3. $1,344 \div 14$

$1,400 \div 14 = 100$

$$
\begin{array}{r}
96 \\
14 \overline{)\ 1344} \\
-1260 \\
\hline
84 \\
-84 \\
\hline
0
\end{array}
$$

	14
90	1260
6	84

$14 \times 96 = 1,344$

📝 **Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm**

Student Outcomes

- Students understand that the standard algorithm of division is simply a tally system arranged in place value columns.

Classwork

Example 1 (6 minutes)

The first example is a review from Lesson 12 that will prepare students to start taking a deeper look at the division algorithm.

Example 1

a.   **Create a model to divide: $1,755 \div 27$.**

Answers may vary. One possible solution:

	27
60	1620
5	135
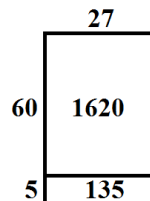
b.   **Use the division algorithm to show $1,755 \div 27$.**

```
        65
   27) 1755
      - 162
        135
       -135
          0
```

- Looking at your division work, where did the numbers 162 and 135 come from?

```
        65
   27) 1755
      (- 162)
        135
       (-135)
          0
```

  □   *The 162 comes from 27×6. This is really showing 27×60, which is 1,620. The 135 comes from 27×5.*

- If you had to describe what is happening underneath the division bar in your own words, what would you say? (For struggling students, have them think back to what they did when they used a model.)
  - *The work under the long division bar shows how I keep track of the parts that I have already divided out. Then, I can see what remains to continue the process.*

---

c. **Check your work.**

$$27 \times 65 = 1,755$$

Example 2 (6 minutes)

Example 2

Find the quotient of $205,276 \div 38$.

MP.6 & MP.8

- How can we use estimation to start solving this problem?
  - *Answers will vary. I can start by determining how many times* 40 *will divide into* 200 *thousands. I know that* $40 \times 5 = 200$. *So, I will start the division process by placing a* 5 *over the* 5 *in* 205,276. *This* 5 *represents* 5 *thousands. So, we will show in the long division that* 200 *thousands* $\div 38 =$ 5 *thousands.*
- Why did you divide 40 into 200 instead of 200,000?
  - *I thought of this problem as* 40 *into* 200 *thousands so that I could divide* 40 *into* 200 *to make the division simpler.*

Give students a chance to complete the division. Students should also be creating a model to show the connection between the algorithm and the model.

- After you divided by 400, what did you do next?
  - *I had brought down the seven to complete the next step. However,* 38 *does not divide into* 7 *one or more times. This told me to put a* 0 *in the tens place and bring down the* 6 *to continue dividing.*

- Imagine that your friend wrote 542 as the answer. How could you prove to your friends that 542 is not the solution?
  - *Answers may vary. I could use estimation. I would round the divisor and quotient and multiply them to see if I get an estimate that is close to the dividend, $40 \times 500 = 20,000$. The estimate is about 10 times too small. So, I can tell that the numbers are in the wrong place. I should have an estimate around 200,000.*

Now, let's use the algorithm to work through a division question that involves a much larger number.

Example 3 (6 minutes)

> **Example 3**
>
> **Find the quotient of $17,216,673 \div 23$.**

- When working with a dividend as large as this, what would happen if we tried to solve this question using a model?
  - *Answers will vary. The model would be difficult to make because it could have many parts for the different place values.*

MP.6 & MP.8

- It might be difficult to figure out how many times 23 goes in to this whole number. So, we can break this into parts. Does 23 go into 1 one or more times? Does 23 go into 17 one or more times? Will 23 divide into 172 one or more times?
  - *23 does not go into 1 or 17 one or more times. However, 23 will go into 172 one or more times.*
- Could we use estimation to help us start the problem?
  - *We could think about $175 \div 25$ or $180 \div 20$ (or other possible estimations that are backed by mathematical reasoning).*

$$
\begin{array}{r}
7 \\
23{\overline{\smash{\big)}\,17216673}} \\
\underline{-161} \\
11
\end{array}
$$

Remind students that what we are actually doing is 172 hundred thousands $\div$ 23 = 7 hundred thousands.

Each step can be represented using the units.

- Why would we place the 7 over the 2 and not somewhere else? What does the 7 represent?
  - *The 7 shows how many times 23 goes into 172, but it really represents how many times 23 goes into 17,200,000. Because the 7 represents 700,000, we place the 7 over the 2 in the hundred thousands place.*
- When we subtracted, we got an 11. What does this 11 represent?
  - *The 11 shows the difference between 172 and 161, but it actually represents 1,100,000. This is the amount remaining after 23 groups of 700,000 are taken from 17,200,000.*

- After we have completed this first set of steps, where do we go next?
  - *We could keep repeating the process until we reach the ones place.*

$$
\begin{array}{r}
748551 \\
23 \overline{\smash{)}\,17216673} \\
\underline{-161} \\
111 \\
\underline{-\,92} \\
196 \\
\underline{-184} \\
126 \\
\underline{-\,115} \\
117 \\
\underline{-115} \\
23 \\
\underline{-23} \\
0
\end{array}
$$

When discussing the remaining steps, you can refer to them as follows:

111 ten thousands ÷ 23: 4 ten thousands

196 thousands ÷ 23: 8 thousands

126 hundreds ÷ 23: 5 hundreds

117 tens ÷ 23: 5 tens

23 ones ÷ 23 = 1

- How can we determine if the answer is reasonable?
  - *We could multiply the quotient with the divisor. $748{,}551 \times 23 = 17{,}216{,}673$.*
  - *We could also use an estimate to check our work. $20{,}000{,}000 \div 20 = 1{,}000{,}000$. Our estimate is slightly larger because we rounded the dividend up.*

Exercises 1–6 (20 minutes)

Give students a chance to practice using the division algorithm. Students may not be able to complete all questions in the time given.

Exercises 1–6

For each question, you need to do the following:

   a.   Solve the question. Next to each line, explain your work using place value.

   b.   Evaluate the reasonableness of your answer.

1. $891{,}156 \div 12$

$$
\begin{array}{r}
74263 \\
12 \overline{\smash{)}\,891156} \\
\underline{-84} \\
51 \\
\underline{-48} \\
31 \\
\underline{-24} \\
75 \\
\underline{-72} \\
36 \\
\underline{-36} \\
0
\end{array}
$$

   a.   89 *ten thousands ÷ 12: 7 ten thousands*
        51 *thousands ÷ 12: 4 thousands*
        31 *hundreds ÷ 12: 2 hundreds*
        75 *tens ÷ 12: 6 tens*
        36 *ones ÷ 12: 3 ones*

   b. $74{,}263 \times 12 = 891{,}156$

---

2. $484,692 \div 78$

```
        6214
   78 ) 484692
       -468
        166
       -156
        109
        - 78
        312
       - 312
          0
```

a.   484 *thousands* $\div$ 78: 6 *thousands*
166 *hundreds* $\div$ 78: 2 *hundreds*
109 *tens* $\div$ 78: 1 *ten*
312 *ones* $\div$ 78: 4 *ones*

b. $6,214 \times 78 = 484,692$

3. $281,886 \div 33$

```
        8542
   33 ) 281886
       -264
        178
       - 165
        138
       -132
         66
        - 66
          0
```

a.   281 *thousands* $\div$ 33: 8 *thousands*
178 *hundreds* $\div$ 33: 5 *hundreds*
138 *tens* $\div$ 33: 4 *tens*
66 *ones* $\div$ 33: 2 *ones*

b. $8,542 \times 33 = 281,886$

4. $2,295,517 \div 37$

```
        62041
   37 ) 2295517
       -222
         75
        - 74
        151
        - 148
          37
         -37
          0
```

a.   229 *ten thousands* $\div$ 37: 6 *ten thousands*
75 *thousands* $\div$ 37: 2 *thousands*
15 *hundreds* $\div$ 37: 0 *hundreds*
151 *tens* $\div$ 37: 4 *tens*
37 *ones* $\div$ 37: 1 *one*

b. $62,041 \times 37 = 2,295,517$

5. $952,448 \div 11$

```
         8504
   112 ) 952448
        -896
         564
        -560
         448
        - 448
           0
```

a.   952 *thousands* $\div$ 112: 8 *thousands*
564 *hundreds* $\div$ 112: 5 *hundreds*
44 *tens* $\div$ 112: 0 *tens*
448 *ones* $\div$ 112: 4 *ones*

b. $8,504 \times 112 = 952,448$

6. $1,823,535 \div 245$

```
         7443
   245 ) 1823535
        -1715
         1085
        - 980
         1053
        - 980
          735
         -735
           0
```

a.   1,823 *thousands* $\div$ 245: 7 *thousands*
1,085 *hundreds* $\div$ 245: 4 *hundreds*
1,053 *tens* $\div$ 245: 4 *tens*
735 *ones* $\div$ 245: 3 *ones*

b. $7,443 \times 245 = 1,823,535$

Closing (3 minutes)

- Explain in your own words how the division algorithm works.

  - *Answers will vary. Sample response: The division algorithm shows successive estimates of the quotient organized by place value, or the division algorithm breaks one large division problem into several smaller ones organized by place value.*

Exit Ticket (4 minutes)

Name _____   Date_____

Lesson 13: Dividing Multi-Digit Numbers Using the Algorithm

Exit Ticket

Divide using the division algorithm: $392{,}196 \div 87$.

Exit Ticket Sample Solutions

Divide using the division algorithm: $392,196 \div 87$.

```
            4508
      _____
   87 ) 392196
       -348
        441
       -435
         696
        -696
           0
```

Problem Set Sample Solutions

1. $459,054 \div 54$

```
             8501
        _____
     54 ) 459054
         -432
          270
         - 270
           054
          - 54
             0
```

2. $820,386 \div 102$

```
            8043
       _____
   102 ) 820386
        -816
         438
        -408
         306
        - 306
           0
```

3. $1,183,578 \div 227$

```
             5214
        _____
    227 ) 1183578
         -1135
          485
         -454
          317
         - 227
            908
          - 908
             0
```

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Student Outcomes

- Students use the algorithm to divide multi-digit numbers with and without remainders. Students compare their answer to estimates to justify reasonable quotients.
- Students understand that when they "bring down" the next digit in the algorithm, they are distributing, recording, and shifting to the next place value.

Classwork

Example 1 (5 minutes)

Students will review how to divide a whole number by a number that is not a factor resulting in a non-whole number quotient. They will first estimate the quotient. Then, they will use the division algorithm to get an exact answer. Finally, they will compare the two to decide if the answer is reasonable.

Example 1

Divide: $31,218 \div 132$.

```
            236.5
      132 ) 31218.0
          - 264
            481
          - 396
            858
          - 792
            660
          - 660
              0
```

As we divide, we can use our knowledge of place value to guide us.

312 *hundreds* $\div$ **132**: **2** *hundreds*

481 *tens* $\div$ **132**: **3** *tens*

858 *ones* $\div$ **132**: **6** *ones*

660 *tenths* $\div$ **132**: **5** *tenths*

MP.2

- Estimate the quotient.
    - *Answers may vary. Possible estimates include the following:* $30{,}000 \div 100 = 300$ *or* $30{,}000 \div 150 = 200$.
- How was solving this question similar to the questions you solved in Lessons 12 and 13?
    - *Answers may vary. To get the quotient in all questions, I used the division algorithm where I divided two whole numbers.*

- How was solving this question different from the questions you solved in Lessons 12 and 13?
    - *Answers may vary. In this example, the divisor is not a factor of the dividend. I know this because the quotient is not a whole number. When I got to the ones place, I still had a remainder, so I placed a zero in the tenths place to continue dividing. Then, I divided 660 tenths by 132 ones. The answer to this question has a decimal in the quotient, but the other lessons had whole number quotients.*

Example 2 (5 minutes)

- We have seen questions with decimals in the quotient. Now, let's discuss how we would divide when there are decimals in the dividend and divisor.

Please note that this question is quite difficult. Students will most likely struggle with this question for quite some time, so you may want to offer this question as a challenge.

> **Example 2**
>
> **Divide: $974.835 \div 12.45$.**

MP.2 Point out that all whole number division that students have done up until now has involved dividing two quantities that are ultimately counting with the same unit, ones (e.g., 32,218 ones divided by 132 ones).

- Now, let's take a look at what this question is asking, including the units.
    - 974 *ones and* 835 *thousandths,* 12 *ones and* 45 *hundredths*
- What do you notice about these two numbers?
    - *They do not have the same unit.*
- How could we rewrite these numbers so that they have the same units?
    - $974.835 \div 12.450$
    - 974,835 *thousandths,* 12 ,450 *thousandths*
- Now, the division problem that we need to solve is 974,835 thousandths ÷ 12,450 thousandths.

$$
\begin{array}{r}
78.3 \\
12450 \overline{)\ 974835.0} \\
-\ 87150 \\
\hline
103335 \\
-\ \ 99600 \\
\hline
37350 \\
-\ 37350 \\
\hline
0
\end{array}
$$

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Example 3 (5 minutes)

> **Example 3**
>
> A plane travels $3,625.26$ miles in 6.9 hours. What is the plane's unit rate?

- What is this question asking us to do?
    - *This question is asking me to divide the miles by hours so that I can find out how many miles the plane went in 1 hour, as we did in Module 1.*
- How can we rewrite 3,625.26 (362,526 hundredths) and 6.9 (69 tenths) using the same units?
    - *First, I would rewrite the question as $3,625.26 \div 6.90$. This is the same as 362,526 hundredths ÷ 690 hundredths.*
- Now we can solve by dividing $362,526 \div 690$.

$$
\begin{array}{r}
525.4 \\
690 \overline{\smash{)}\ 362526.0} \\
\underline{-\ 3450} \\
1752 \\
\underline{-\ 1380} \\
3726 \\
\underline{-\ 3450} \\
2760 \\
\underline{-\ 2760} \\
0
\end{array}
$$

- Let's check our answer to ensure that it is reasonable. What are some different ways that we can do this?
    - *We can multiply the quotient with the original divisor and see if we get the original dividend.* $6.9 \times 525.4 = 3,625.26.$
    - *We could also estimate to check our answer.* $3,500 \div 7 = 500.$ *Because we rounded down, we should expect our estimate to be a little less than the actual answer.*

Exercises 1–7 (20 minutes)

Students can work on the problem set alone or in partners. Students should be estimating the quotient first and using the estimate to justify the reasonableness of their answer.

> **Exercises 1–7**
>
> Estimate the quotient first. Use the estimate to justify the reasonableness of your answer.
>
> 1. Daryl spent $\$4.68$ on each pound of trail mix. He spent a total of $\$14.04$. How many pounds of trail mix did he purchase?
>
>   *Estimate:* $15 \div 5 = 3$
>
> $14.04 \div 4.68 \rightarrow 1,404$ *hundredths* ÷ 468 *hundredths*
>
> $1,404 \div 468 = 3$
>
>   *Our estimate of* 3 *shows that our answer of* 3 *is reasonable.*

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2.   Kareem purchased several packs of gum to place in gift baskets for $\$1.26$ each. He spent a total of $\$8.82$. How many packs of gum did he buy?

Estimate: $9 \div 1 = 9$

$8.82 \div 1.26$ ➔ 882 *hundredths* $\div$ 126 *hundredths*

$882 \div 126 = 7$ *packs of gum*

Our estimate of 9 *shows that our answer of* 7 *is reasonable.*

3.   Jerod is making candles from beeswax. He has 132.72 ounces of beeswax. If each candle uses 8.4 ounces of beeswax, how many candles can he make? Will there be any wax left over?

Estimate: $120 \div 8 = 15$

$132.72 \div 8.4$ ➔ $13,272$ *hundredths* $\div$ 84 *tenths* ➔ $13,272$ *hundredths* $\div$ 840 *hundredths*

$13,272 \div 840 = 15$ *candles; there is wax left over.*

```
                15.8
        840 ) 13272
             - 840
               4872
             - 4200
               6720
             - 6720
                  0
```

Our estimate of 15 *shows that our answer of* 15.8 *is reasonable.*

4.   There are 20.5 cups of batter in the bowl. If each cupcake uses 0.4 cups of batter, how many cupcakes can be made?

Estimate: $20 \div 0.5 = 40$

$20.5 \div 0.4$ ➔ 205 *tenths* $\div$ 4 *tenths*

```
              51.25
        4 ) 205.00
           - 20
              5
            - 4
             10
            - 8
             20
           - 20
              0
```

Only 51 *cupcakes can be made. There is not quite enough for* 52.

Our estimate of 40 *shows that our answer of* 51.25 *is reasonable.*

5.   In Exercises 3 and 4, how were the remainders, or extra parts, interpreted?

In both Exercises 3 and 4, the remainders show that there was not quite enough to make another candle or cupcake. In the candle example, there was wax left over that could be saved for the next time there is more wax. However, in the cupcake example, the leftover batter could be used to make a smaller cupcake, but it would not count as another whole cupcake.

6. $159.12 \div 6.8$

   Estimate: $160 \div 8 = 20$

   $159.12 \div 6.8 \rightarrow 15,912$ *hundredths* $\div 68$ *tenths* $\rightarrow 15,912$ *hundredths* $\div 680$ *hundredths*

   $$680 \overline{)\,15912.0\,} \quad 23.4$$
   ```
              23.4
       ┌─────────────
   680 )   15912.0
         - 1360
           ──────
            2312
          - 2040
            ──────
             2720
           - 2720
             ──────
                 0
   ```

   Our estimate of 20 *shows that our answer of* 23.4 *is reasonable.*

7. $167.67 \div 8.1$

   Estimate: $160 \div 8 = 20$

   $167.67 \div 8.1 \rightarrow 16,767$ *hundredths* $\div 81$ *tenths* $\rightarrow 16,767$ *hundredths* $\div 810$ *hundredths*

   ```
              20.7
       ┌─────────────
   810 )   16767.0
         - 1620
           ──────
            5670
          - 5670
            ──────
               0
   ```

   Our estimate of 20 *shows that our answer of* 20.7 *is reasonable.*

Closing (5 minutes)

- Describe the steps that you use to change a division question with decimals to a division question with whole numbers.

   - *If the divisor and or the dividend are not whole numbers, we find the largest common unit, smaller than one, which allows us to rewrite each as a whole number multiple of this common unit.*

   - *Example:*

     1,220.934 *ones* ÷ 54.34 *ones*

     12,209.34 *tenths* ÷ 543.4 *tenths*

     122,093.4 *hundredths* ÷ 5,434 *hundredths*

     1,220,934 *thousandths* ÷ 54,340 *thousandths*

     We could keep going, and both the dividend and divisor would still be whole numbers, but we were looking for the largest common unit that would make this happen.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 14: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Fractions

Exit Ticket

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $14.70. How many pounds of almonds did she purchase?

2. Divide: $125.01 \div 5.4$. Then, check your answer for reasonableness.

Exit Ticket Sample Solutions

1. Lisa purchased almonds for $3.50 per pound. She spent a total of $14.70. How many pounds of almonds did she purchase?

```
            4.2
350 )  1470.0
      - 1400
         700
       -  700
           0
```

Lisa purchased 4.2 pounds of almonds.

2. Divide: $125.01 \div 5.4$. Then, check your answer for reasonableness.

```
          23.15
540 )  12501.00
     -  1080
        1701
      -  1620
          810
        -  540
         2700
       -  2700
           0
```

The quotient of 125.01 and 5.4 is 23.15.

Estimate: $125 \div 5 = 25$

My estimate of 25 is near 23, which shows that my answer is reasonable.

Problem Set Sample Solutions

1. Aslan purchased 3.5 lb. of his favorite mixture of dried fruits to use in a trail mix. The total cost was $16.87. How much does the fruit cost per pound?

$16.87 \div 3.5$ → 1,687 hundredths ÷ 350 hundredths

```
           4.82
350 )  1687.00
      -1400
        2870
      - 2800
         700
       -  700
           0
```

The dried fruit costs $4.82 per pound.

2.   **Divide:** $994.14 \div 18.9$.

$994.14 \div 18.9 \;\rightarrow\; 99,414$ *hundredths* $\div\; 1,890$ *hundredths*

```
              52.6
      1890 ) 99414.0
            -9450
             4914
           - 3780
             11340
           - 11340
                 0
```

$994.14 \div 18.9 = 52.6$

Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

Student Outcomes

- Students use their knowledge of dividing multi-digit numbers to solve for quotients of multi-digit decimals.
- Students understand the mathematical concept of decimal placement in the divisor and the dividend and its connection to multiplying by powers of 10.

Classwork

Opening Exercises (10 minutes)

This is an optional section for those who would like to include a warm-up exercise in their lessons. Although the Opening Exercises are optional, the 10 minutes provided for these questions are included in the 45 minutes designated for lesson.

These questions can be asked in discussion with the whole group to introduce the students to the idea of multiplying the divisor or dividend by powers of ten.

- Let's take a look at what happens when we change our division problem.

MP.7

Opening Exercises

Start by finding the quotient of $1,728$ and 32.

$$
\begin{array}{r}
54 \\
32 \overline{) 1728} \\
-160 \\
\hline
128 \\
-128 \\
\hline
0
\end{array}
$$

What would happen if we multiplied the divisor by 10? $1,728 \div 320$

$$
\begin{array}{r}
5.4 \\
320 \overline{) 1728.0} \\
-1600 \\
\hline
1280 \\
-1280 \\
\hline
0
\end{array}
$$

When the divisor is ten times bigger, the quotient is ten times smaller.

What would happen if we multiplied the dividend by 10? $17,280 \div 32$

$$
\begin{array}{r}
540 \\
32 \overline{)\ 17280} \\
-160 \\
\hline
128 \\
-128 \\
\hline
00 \\
-\ 00 \\
\hline
0
\end{array}
$$

When the dividend is ten times bigger, the quotient is ten times bigger.

What would happen if we multiplied both the divisor and dividend by 10? $17,280 \div 320$

$$
\begin{array}{r}
54 \\
320 \overline{)\ 17280} \\
-1600 \\
\hline
1280 \\
-1280 \\
\hline
0
\end{array}
$$

When both the divisor and dividend are multiplied by 10, the quotient does not change.

MP.7

What would happen if we multiplied both the divisor and dividend by 100? $172,800 \div 3,200$

$$
\begin{array}{r}
54 \\
3200 \overline{)\ 172800} \\
-16000 \\
\hline
12800 \\
-12800 \\
\hline
0
\end{array}
$$

When both the divisor and dividend are multiplied by 100, the quotient does not change.

What would happen if we multiplied both the divisor and the dividend by $1,000$, $10,000$, or $100,000$? What do you predict would happen?

As long as both the divisor and the dividend are multiplied by the same amount, the quotient will not change.

How can we use this to help us divide when there are decimals in the divisor? For example, how can we use this to help us divide 172.8 and 3.2?

172.8 is the same as $1,728$ tenths, and 3.2 is the same as 32 tenths. So, to get the quotient, we could divide $1,728$ by 32. If we multiply both the divisor and the dividend by 10, we get the same two numbers that we need to divide, $1,728$ and 32. Because we multiplied both numbers by the same amount, the answer will not change.

Example 1 (3 minutes)

Example 1

Using our discoveries from the discussion, let's divide 537.1 by 8.2.

How can we rewrite this problem using what we learned in Lesson 14?

We can rewrite this problem as $5,371$ tenths $\div$ 82 tenths. These two numbers already have the same unit, so we would divide $5,371$ by 82.

How could we use the short cut from our discussion to change the original numbers to $5,371$ and 82?

We can multiply both numbers by 10, which changes the divisor from 8.2 to 82. Then, we will have a whole number to divide by. So, the new problem will become $5,371 \div 82$.

$$
\begin{array}{r}
65.5 \\
82 \overline{)\ 5371.0} \\
-492 \\
\hline
451 \\
-410 \\
\hline
410 \\
-\ 410 \\
\hline
0
\end{array}
$$

Example 2 (3 minutes)

Example 2

Now let's divide 742.66 by 14.2.

How can we rewrite this division problem so that the divisor is a whole number, but the quotient remains the same?

We can multiply both numbers by 10, which changes the divisor from 14.2 to 142. Then, we will have a whole number to divide by. So, the new problem will become $7,426.6 \div 142$.

$$
\begin{array}{r}
52.3 \\
142 \overline{)\ 7426.6} \\
-\ 710 \\
\hline
326 \\
-\ 284 \\
\hline
426 \\
-\ 426 \\
\hline
0
\end{array}
$$

Exercises (21 minutes)

Exercises

Students will participate in a game called Pass the Paper. Students will work in groups of no more than four. There will be a different paper for each player. When the game starts, each student solves the first problem on his paper and passes the paper clockwise to the second student, who uses multiplication to check the work that was done by the previous student. Then, the paper is passed clockwise again to the third student, who solves the second problem. The paper is then passed to the fourth student, who checks the second problem. This process continues until all of the questions on every paper are complete or time runs out.

Four pages, one for each player, are attached at the end of the lesson. Answers are provided below.

Player A Answers

1. $15.5 \div 6.2 = 2.5$
2. $28.08 \div 7.8 = 3.6$
3. $44.888 \div 3.62 = 12.4$
4. $3,912.99 \div 15.9 = 246.1$
5. $865.1475 \div 47.25 = 18.31$

Player B Answers

1. $32.4 \div 7.2 = 4.5$
2. $49.14 \div 6.3 = 7.8$
3. $39.321 \div 2.57 = 15.3$
4. $8,578.02 \div 24.6 = 348.7$
5. $439.0464 \div 35.18 = 12.48$

Player C Answers

1. $25.9 \div 7.4 = 3.5$
2. $25.48 \div 5.2 = 4.9$
3. $61.962 \div 4.49 = 13.8$
4. $16,437.42 \div 31.8 = 516.9$
5. $1,238.8048 \div 52.76 = 23.48$

Player D Answers

1. $63.7 \div 9.8 = 6.5$
2. $32.68 \div 8.6 = 3.8$
3. $142.912 \div 8.12 = 17.6$
4. $23,344.58 \div 57.4 = 406.7$
5. $2,498.743 \div 39.65 = 63.02$

Closing (3 minutes)

- Based upon our work today, discuss ways you would alter the problem $4,509 \div 0.03$ to make it easier to use the long division algorithm yet yield the same answer.
  - *As long as I multiply both dividend and divisor by the same number, the quotient will not change. If I multiply by powers of 10, I will be able to ultimately get to a point where both dividend and divisor are whole numbers. In this case, if I multiply by 10^2, the problem will become $450,900 \div 3$.*

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 15: The Division Algorithm—Converting Decimal Division into Whole Number Division Using Mental Math

Exit Ticket

State the power of 10 you would use to convert the given decimal division to whole number division. Then, complete the multiplication on the dividend and divisor.

1. $133.84 \div 5.6$

2. $12.4 \div 1.036$

3. $38.9 \div 2.91$

4. $45 \div 1.5$

Exit Ticket Sample Solutions

State the power of 10 you would use to convert the given decimal division to whole number division. Then, complete the multiplication on the dividend and divisor.

1. $133.84 \div 5.6$

    10, 1338.4 and 56

2. $12.4 \div 1.036$

    1,000 or 10^3, 12,400 and 1,036

3. $38.9 \div 2.91$

    100 or 10^2, 3,890 and 291

4. $45 \div 1.5$

    10 or 10^1, 450 and 15

Problem Set Sample Solutions

1. $118.4 \div 6.4$

    $1,184 \div 64$

    ```
                  18.5
        64 ) 1184.0
           -  64
              544
            - 512
              320
            - 320
                0
    ```

2. $314.944 \div 3.7$

    $3,149.44 \div 37$

    ```
                85.12
        37 ) 3149.44
           - 296
             189
           - 185
              44
             -37
              74
            - 74
               0
    ```

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Lesson 15:   The Division Algorithm—Converting Decimal Division into Whole
                  Number Division Using Mental Math
Date: 7/3/14

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3. $1,840.5072 \div 23.56$

   $184,050.72 \div 2,356$

```
                       78.12
           _____
    2356  )  184050.72
           - 16492
           _____
             19130
           - 18848
           _____
              2827
           -  2356
           _____
               4712
             - 4712
             _____
                   0
```

Player A

1. $15.5 \div 6.2$	Check:
2. $28.08 \div 7.8$	Check:
3. $44.888 \div 3.62$	Check:
4. $3{,}912.99 \div 15.9$	Check:
5. $865.1475 \div 47.25$	Check:

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Player B

1. 32.4 ÷ 7.2	Check:
2. 49.14 ÷ 6.3	Check:
3. 39.321 ÷ 2.57	Check:
4. 8,578.02 ÷ 24.6	Check:
5. 439.0464 ÷ 35.18	Check:

Player C

1. $25.9 \div 7.4$	Check:
2. $25.48 \div 5.2$	Check:
3. $61.962 \div 4.49$	Check:
4. $16,437.42 \div 31.8$	Check:
5. $1,238.8048 \div 52.76$	Check:

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Player D

1. 63.7 ÷ 9.8	Check:
2. 32.68 ÷ 8.6	Check:
3. 142.912 ÷ 8.12	Check:
4. 23,344.58 ÷ 57.4	Check:
5. 2,498.743 ÷ 39.65	Check:

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6
GRADE

Mathematics Curriculum

π

Topic D:

Number Theory—Thinking Logically About Multiplicative Arithmetic

6.NS.B.4

Focus Standard:	6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
Instructional Days:	4	
Lesson 16:	Even and Odd Numbers (S)[1]	
Lesson 17:	Divisibility Tests for 3 and 9 (S)	
Lesson 18:	Least Common Multiple and Greatest Common Factor (P)	
Lesson 19:	The Euclidean Algorithm as an Application of the Long Division Algorithm (P)	

Students have previously developed facility with multiplication and division. They now begin to reason logically about them in Topic D. Students apply odd and even number properties and divisibility rules to find factors and multiples. They extend this application to consider common factors and multiples and find greatest common factors and least common multiples. Students explore and discover that Euclid's Algorithm is a more efficient means of finding the greatest common factor of larger numbers and determine that Euclid's Algorithm is based on long division.

---

[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

![chat icon] **Lesson 16: Even and Odd Numbers**

Student Outcomes

- Students generalize rules for adding and multiplying even and odd numbers.

Lesson Notes

Students will need poster paper and markers to complete the exercises.

Classwork

Opening Exercise (15 minutes)

Present each question and then allow students to share their thinking. Also, have students record notes in their student materials.

Opening Exercise

What is an even number?

Possible student responses:

- *An integer that can be evenly divided by 2*

- *A number whose unit digit is $0, 2, 4, 6,$ or 8*

- *All the multiples of 2*

List some examples of even numbers.

Answers will vary.

What is an odd number?

Possible student responses:

- *An integer that CANNOT be evenly divided by 2*

- *A number whose unit digit is $1, 3, 5, 7,$ or 9*

- *All the numbers that are NOT multiples of 2*

List some examples of odd numbers.

Answers will vary.

Present each question and then discuss the answer using models.

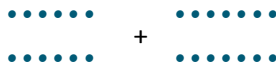**What happens when we add two even numbers? Will we always get an even number?**

Before holding a discussion about the process to answer the following questions, have students write or share their predictions.

Exercises 1–3

> **Exercises 1–3**
>
> 1. **Why is the sum of two even numbers even?**
>
>   a. **Think of the problem $12 + 14$. Draw dots to represent each number.**
>
>   • • • • • •   • • • • • • •
>   +
>   • • • • • •   • • • • • • •
>
>   b. **Circle pairs of dots to determine if any of the dots are left over.**
>
>
>
>   *There are no dots leftover; the answer will be even.*
>
>   c. **Will this be true every time two even numbers are added together? Why or why not?**
>
>   *Since 12 is represented by 6 sets of two dots, and 14 is represented by 7 sets of two dots, the sum will be 13 sets of two dots. This will be true every time two even numbers are added together because even numbers will never have dots left over when we are circling pairs. Therefore, the answer will always be even.*

Before holding a discussion about the process to answer the following questions, have students write or share their predictions.

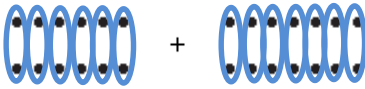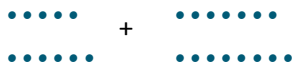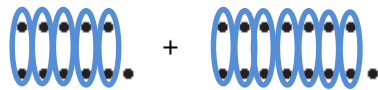- Now, decide what happens when we add two odd numbers.

> 2. **Why is the sum of two odd numbers even?**
>
>   a. **Think of the problem $11 + 15$. Draw dots to represent each number.**
>
>   • • • • •   • • • • • • •
>   +
>   • • • • • •   • • • • • • • •
>
>   b. **Circle pairs of dots to determine if any of the dots are left over.**
>
>
>
>   *When we circle groups of two dots, there is one dot remaining in each representation because each addend is an odd number. When we look at the sum, however, the two remaining dots can form a pair, leaving us with a sum that is represented by groups of two dots. The sum is, therefore, even. Since each addend is odd, there is one dot for each addend that does not have a pair. However, these two dots can be paired together, which means there are no dots without a pair, making the sum an even number.*

> c.   **Will this be true every time two odd numbers are added together? Why or why not?**
>
> *This will be true every time two odd numbers are added together because every odd number will have one dot remaining when we circle pairs of dots. Since each number will have one dot remaining, these dots can be combined to make another pair. Therefore, no dots will be remaining, resulting in an even sum.*

- Use the same method we used in the two prior examples to show that the sum of an odd number and an even number is odd. Use the problem $14 + 11$.

MP.3 & MP.8

> 3.   **Why is the sum of an even number and an odd number odd?**
>
> a.   **Think of the problem $14 + 11$. Draw dots to represent each number.**
>
> b.   **Circle pairs of dots to determine if any of the dots are left over.**

Students draw dots to represent each number. After circling pairs of dots, there will be one dot left for the number 11, and the number 14 will have no dots remaining. Since there is one dot left over, the sum will be odd because every dot does not have a pair.

> c.   **Will this be true every time an even number and an odd number are added together? Why or why not?**
>
> *This will always be true when an even number and an odd number are added together because only the odd number will have a dot remaining after we circle pairs of dots. Since this dot does not have a pair, the sum will be odd.*
>
> d.   **What if the first addend was odd and the second was even? Would the sum still be odd? Why or why not? For example, if we had $11 + 14$, would the sum be odd?**
>
> *The sum will still be odd for two reasons. First, the commutative property states that changing the order of an addition problem does not change the answer. Because an even number plus an odd number is odd, then an odd number plus an even number is also odd. Second, it does not matter which addend is odd; there will still be one dot remaining, making the sum odd.*

Scaffolding:

- The teacher could also ask students if the same rules apply to subtraction. Using the same method for addition, have students determine if the rules apply to subtraction.

- If students are struggling with the proofs, the teacher can present each proof as students take notes in their handbooks. Or, allow students time to explore, and have a few groups who did not struggle present at the end.

- Ask early finishers if the same rule applies to division.

If students are struggling, encourage them to draw dots to prove their answer. It may also be helpful to remind students of the commutative property to help them prove their answer.

Sum up the discussion by having students record notes in their handbooks.

> **Let's sum it up:**
>
> *Even + even = even*
>
> *Odd + odd = even*
>
> *Odd + even = odd*

Exploratory Challenge/Exercises 4–6 (20 minutes–12 minutes for group work; 8 minutes for gallery walk and discussion)

MP.7

Divide students into small groups. On poster paper, each group will be asked to determine whether one of the following products is odd or even: the product of two even numbers, the product of two odd numbers, or the product of an even number and an odd number. Encourage students to use previous knowledge about even and odd numbers, the connection between addition and multiplication, and visual methods (e.g., dots) in their proofs.

Exploratory Challenge/Exercises 4–6

4. **The product of two even numbers is even.**

    Answers will vary, but one example answer is provided.

    Using the problem 6×14, students know that this is equivalent to six groups of fourteen, or $14 + 14 + 14 + 14 + 14$. Students also know that the sum of two even numbers is even; therefore, when adding the addends two at a time, the sum will always be even. This means the sum of six even numbers will be even, making the product even since it will be equivalent to the sum.

    Using the problem 6×14, students can use the dots from previous examples.

    From here, students can circle dots and see that there will be no dots remaining, so the answer must be even.

5. **The product of two odd numbers is odd.**

    Answers will vary, but an example answer is provided.

    Using the problem 5×15, students know that this is equivalent to five groups of fifteen, or $15 + 15 + 15 + 15 + 15$. Students also know that the sum of two odd numbers is even, and the sum of an odd and even number is odd. When adding two of the addends together at a time, the answer will rotate between even and odd. When the final two numbers are added together, one will be even and the other odd. Therefore, the sum will be odd, which makes the product odd since it will be equivalent to the sum.

    Using the problem 5×15, students may also use the dot method.

    After students circle the pairs of dots, one dot from each set of 15 will remain, for a total of 5 dots. Students can group these together and circle more pairs, as shown below.

    Since there is still one dot remaining, the product of two odd numbers is odd.

---

6. The product of an even number and an odd number is even.

Answers will vary, but one example is provided.

Using the problem 6×7, students know that this is equivalent to the sum of six sevens, or $7 + 7 + 7 + 7 + 7 + 7$. Students also know that the sum of two odd numbers is even, and the sum of two even numbers is even. Therefore, when adding two addends at a time, the results will be an even number. Calculating the sum of these even numbers will also be even, which means the total sum will be even. This also implies the product will be even since the sum and product are equivalent.

Using the problem 6×7, students may also use the dot method.

After students circle the pairs of dots, one dot from each set of 7 will remain, for a total of 6 dots. Students can group these together and circle more pairs, as shown below.

Since there are no dots remaining, the product of an even number and an odd number is even.

After students complete their posters, hang the posters up around the room. Conduct a gallery walk to allow groups to examine each poster and take notes in their student materials. In the end, students should have a proof for all three exercises in their student handbook.

Allow time for a discussion and an opportunity for students to ask any unanswered questions.

Closing (5 minutes)

- How does knowing whether a sum or product will be even or odd assist in division?
  - *Possible student response: When dividing, it is helpful to know whether the sum or product of two numbers is even or odd because it narrows down the possible factors. For example, if a dividend is odd, then we know the factors must also be odd because the product of two odd numbers is odd.*

Lesson Summary

Adding:

- The sum of two even numbers is even.
- The sum of two odd numbers is even.
- The sum of an even number and an odd number is odd.

Multiplying:

- The product of two even numbers is even.
- The product of two odd numbers is odd.
- The product of an even number and an odd number is even.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 16: Even and Odd Numbers

Exit Ticket

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. $56{,}426 + 17{,}895$

2. $317{,}362 \times 129{,}324$

3. $10{,}481 + 4{,}569$

4. $32{,}457 \times 12{,}781$

5.   Show or explain why $12 + 13 + 14 + 15 + 16$ will result in an even sum.

Exit Ticket Sample Solutions

Determine whether each sum or product will be even or odd. Explain your reasoning.

1. $56,426 + 17,895$

    The sum is odd because the sum of an even number and an odd number is odd.

2. $317,362 \times 129,324$

    The product is even because the product of two even numbers is even.

3. $10,481 + 4,569$

    The sum is even because the sum of two odd numbers is even.

4. $32,457 \times 12,781$

    The product is odd because the product of two odd numbers is odd.

5. **Show or explain why $12 + 13 + 14 + 15 + 16$ will result in an even sum.**

    $12 + 13$ *will be odd because even + odd is odd.*

    Odd number $+14$ *will be odd because odd + even is odd.*

    Odd number $+15$ *will be even because odd + odd is even.*

    Even number $+16$ *will be even because even + even is even.*

    OR

    Students may group even numbers together, $12 + 14 + 16$, which would result in an even number. Then, when students combine the two odd numbers, $13 + 15$, the result would be another even number. We know that the sum of two evens would result in another even number.

Problem Set Sample Solutions

Without solving, tell whether each sum or product is even or odd. Explain your reasoning.

1. $346 + 721$

    The sum is odd because the sum of an even and an odd number is odd.

2. $4,690 \times 141$

    The product is even because the product of an even and an odd number is even.

3. $1,462,891 \times 745,629$

    The product is odd because the product of two odd numbers is odd.

4. $425,922 + 32,481,064$

   The sum is even because the sum of two even numbers is even.

5. $32 + 45 + 67 + 91 + 34 + 56$

   The first two addends will be odd because even and an odd is odd.

   Odd number $+67$ will be even because the sum of two odd numbers is even.

   Even number $+91$ will be odd because the sum of an even and an odd number is odd.

   Odd number $+34$ will be odd because the sum of an odd and an even number is odd.

   Odd number $+56$ will be odd because the sum of an odd and an even number is odd.

   Therefore, the final sum will be odd.

Lesson 17: Divisibility Tests for 3 and 9

Student Outcomes

- Students apply divisibility rules, specifically for 3 and 9, to understand factors and multiples.

Lesson Notes

Students already have knowledge on the divisibility rules of 2, 4, 6, 8, and 10. Although those rules are not a focus for this lesson, they are revisited throughout the lesson. Also, emphasize the difference between factors and multiples throughout the lesson.

Classwork

Opening Exercise (5 minutes)

The Opening Exercise will help students review the divisibility tests for the numbers 2, 4, 5, 8, and 10.

MP.1

Opening Exercise

Below is a list of 10 numbers. Place each number in the circle(s) that is a factor of the number. You will place some numbers in more than one circle. For example, if 32 were on the list, you would place it in the circles with 2, 4, and 8 because they are all factors of 32.

$$24; \ 36; \ 80; \ 115; \ 214; \ 360; \ 975; \ 4,678; \ 29,785; \ 414,940$$

2

24; 36; 80; 214; 360; 4,678; 414,940

4

24; 36; 80; 360; 414,940

5

80; 115; 360; 975; 29,785; 414,940

8

24; 80; 360

1

80; 360; 414,940

Discussion (8 minutes)

Discuss students' results from the opening exercise. Students can either share their answers, or the teacher can conduct a poll (raising hands, standing up, electronically) to determine where students placed each number.

MP.3

After sharing which numbers go in each circle, have students examine the numbers in the opening activity. Ask students to find shortcuts to determine in which group the number belongs just by looking at it.

Ask students to share their short cuts or rules and discuss the divisibility rules for each number. Have students take notes in their handbooks.

MP.8

> **Discussion**
>
> - **Divisibility rule for 2:** *If and only if its unit digit is $0, 2, 4, 6,$ or 8.*
>
> - **Divisibility rule for 4:** *If and only if its last two digits are a number divisible by 4.*
>
> - **Divisibility rule for 5:** *If and only if its unit digit is 0 or 5.*
>
> - **Divisibility rule for 8:** *If and only if its last three digits are a number divisible by 8.*
>
> - **Divisibility rule for 10:** *If and only if its unit digit is 0.*
>
> - **Decimal numbers with fraction parts do not follow the divisibility tests.**

Explain that students will learn two new divisibility rules today. The rules will be used to determine if numbers are divisible by 3 or 9. Start with a number students already know have factors of 3 and 9, so they can see that the rule works.

- What do the numbers 12, 18, 30, 66, and 93 all have in common?
    - *They are divisible by 3.*
- Calculate the sum of the digits for each given number. For example, the sum of the digits in the number 12 is 3 because $1 + 2 = 3$.

Provide time for students to find the sums. Record sums on the board.

- What do all these sums have in common?
    - *They are divisible by 3.*
- When the sum of a number's digits is divisible by 3, the entire number is divisible by 3. Now let's examine a different set of numbers: $27, 36, 54, 72,$ and 99. What do these numbers have in common?
    - *They are divisible by 9.*
- Calculate the sum of the digits for each given number.

Provide time for students to find the sums. Record sums on the board.

- What do all the sums have in common?
    - *They are divisible by 9.*
- When the sum of the digits is divisible by 3 and 9, the entire number is divisible by 9. Let's try to use this knowledge to determine if a large number is divisible by 3, 9, or both. The number 765 is divisible by both 3 and 9. (Show students on the calculator.) Find the sum of the digits.
    - $7 + 6 + 5 = 18.$

- Are 3 and 9 both factors of 18?
  - *Yes.*
- Calculating the sum of a number's digits helps us to determine if the number is divisible by 3 or 9 or both.

Introduce the divisibility rules for 3 and 9. Have students record the rules in their handbooks.

> - **Divisibility rule for 3:** *If the sum of the digits is divisible by 3, then the number is divisible by 3.*
> - **Divisibility rule for 9:** *If the sum of the digits is divisible by 9, then the number is divisible by 9.*

Through further discussion, explain to students that if a number is divisible by 9, it is also divisible by 3, but if a number is divisible by 3, it is not necessarily divisible by 9.

- Because $9 = 3 \times 3$, any number that is divisible by 9 will also be divisible by 3.

Example 1 (5 minutes)

Example 1

This example will show you how to apply the two new divisibility rules we just discussed.

Is 378 divisible by 3 or 9? Why or why not?

a. **What are the three digits in the number 378?**

   3, 7, and 8

b. **What is the sum of the three digits?**

   $3 + 7 + 8 = 18$; *the sum of the three digits is 18.*

c. **Is 18 divisible by 9?**

   Yes.

d. **Is the entire number 378 divisible by 9? Why or why not?**

   The number 378 is divisible by 9 because the sum of the digits is divisible by 9.

Scaffolding:

- If needed, the teacher can also ask if 18 is divisible by 3. Students may still struggle with the connection between the multiples of 3 and 9.
- If students struggled with the opening exercise, the divisibility rules for 2, 4, 5, 8, and 10 can be reviewed in this example as well.

MP.7 This may be the place to help students recognize the difference between factors and multiples. Nine is a factor of 378 because it is the product of 9 and 42; therefore, 378 is a multiple of 9.

e. **Is the number 378 divisible by 3? Why or why not?**

   Three is a factor of 378 because if 9 is a factor of 378, then 3 will also be a factor. OR

   The number 378 is divisible by 3 because the sum of the digits is divisible by 3.

Example 2 (5 minutes)

The students have now seen one example of the two new divisibility rules. Allow students to work with a partner to decide whether a given number is divisible by 3 and 9. If a majority of students are still struggling, the teacher may ask the same leading questions found in Example 1.

- Is 3,822 divisible by 3 or 9? Why or why not?

Encourage students to check 9 first because if 9 is a factor, then students know that 3 is also a factor. If 3,822 is not divisible by 9, then students must check to see if 3,822 is divisible by 3.

Example 2

Is 3,822 divisible by 3 or 9? Why or why not?

The number 3,822 is divisible by 3, but not by 9 because the sum of the digits is $3 + 8 + 2 + 2 = 15$, and 15 is divisible by 3, but not by 9.

MP.7

This is another opportunity to emphasize the difference between factors and multiples. Three is a factor of 3,822 because the product of 3 and 1,274 is 3,822; therefore, 3,822 is a multiple of 3.

Exercises 1–5 (13 minutes)

MP.8

Students may work with partners or individually to complete the exercises. Remind students that they may circle more than one answer.

Exercises 1–5

Circle ALL the numbers that are factors of the given number. Complete any necessary work in the space provided.

1. **Is 2,838 divisible by**

    (3)
    9
    4

    Explain your reasoning for your choices.

    The number 2,838 is divisible by 3 because 3 is a factor of 2,838. I know this because the sum of the digits is 21, which is divisible by 3. The number 2,838 is not divisible by 9 because 21 is not divisible by 9, and 2,838 is not divisible by 4 because the last two digits (38) are not divisible by 4.

2. **Is 34,515 divisible by**

    (3)
    (9)
    (5)

    Explain your reasoning for your choices.

    The number 34,515 is divisible by 3 and 9 because both 3 and 9 are factors of 34,515. I know this because the sum of the digits is 18, and 18 is divisible by both 3 and 9. The number 34,515 is also divisible by 5 because the unit digit is a 5.

---

3. **Is $10,534,341$ divisible by**

- ⟨3⟩
- 9
- 2

Explain your reasoning for your choices.

The number $10,534,341$ is divisible by 3, but not 9, because 3 is a factor of $10,534,341$, but 9 is not. I know this because the sum of the digits is 21, which is divisible by 3, but not 9. The number $10,534,341$ is not divisible by 2 because it does not end with 0, 2, 4, 6, or 8.

4. **Is $4,320$ divisible by**

- ⟨3⟩
- ⟨9⟩
- ⟨10⟩

Explain your reasoning for your choices.

The number $4,320$ is divisible by 3 and 9 because 3 and 9 are factors of $,320$. I know this because the sum of the digits is 9, which is divisible by 3 and 9. The number $4,320$ is also divisible by 10 because 10 is a factor of $,320$. I know this because the unit digit is 0.

5. **Is $6,240$ divisible by**

- ⟨3⟩
- 9
- ⟨8⟩

Explain your reasoning for your choices.

The number $6,240$ is divisible by 3, but not divisible by 9, because 3 is a factor of $6,240$, but 9 is not. I know this because the sum of the digits is 12, which is divisible by 3 but not divisible by 9. The number $6,240$ is divisible by 8 because the last three digits (240) is divisible by 8.

Closing (4 minutes)

- Without completing the division, how can you determine if a number is divisible by 3?
  - *Calculate the sum of the digits; if the sum of the digits is divisible by 3, the entire number is divisible by 3.*
- If a number is divisible by 9, will it be divisible by 3? Explain your answer.
  - *If a number is divisible by 9, the sum of the digits will be divisible by 9. Any number that is divisible by 9 is also divisible by 3 since $9 = 3 \times 3$.*
- If a number is divisible by 3, will it be divisible by 9? Explain your answer.
  - *If a number is divisible by 3, it may not be divisible by 9 because 3 has more multiples than 9.*

Lesson Summary

To determine if a number is divisible by 3 or 9:

- **Calculate the sum of the digits.**

- **If the sum of the digits is divisible by 3, the entire number is divisible by 3.**

- **If the sum of the digits is divisible by 9, the entire number is divisible by 9.**

Note: If a number is divisible by 9, the number is also divisible by 3.

Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 17: Divisibility Tests for 3 and 9

Exit Ticket

1.   Is 26,341 divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.

2.   Is 8,397 divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.

3.   Explain why 186,426 is divisible by both 3 and 9.

Exit Ticket Sample Solutions

1. **Is $26,341$ divisible by 3? If it is, write the number as the product of 3 and another factor. If not, explain.**

   The number $26,341$ is not divisible by 3 because the sum of the digits is 16, which is not divisible by 3.

2. **Is $8,397$ divisible by 9? If it is, write the number as the product of 9 and another factor. If not, explain.**

   The number $8,397$ is divisible by 9 because the sum of the digits is 27, which is divisible by 9. Nine is a factor of $8,397$ because $9 \times 933 = 8,397$.

3. **Explain why $186,426$ is divisible by both 3 and 9.**

   The number $186,426$ is divisible by both 3 and 9 because the sum of the digits is 27, which is divisible by both 3 and 9.

Problem Set Sample Solutions

1. **Is $32,643$ divisible by both 3 and 9? Why or why not?**

   The number $32,643$ is divisible by both 3 and 9 because the sum of the digits is 18, which is divisible by 3 and 9.

2. **Circle all the factors of $424,380$ from the list below.**

   (2)   (3)   (4)   (5)   8   9   (10)

3. **Circle all the factors of $322,875$ from the list below.**

   2   (3)   4   (5)   8   (9)   10

4. **Write a 3 digit number that is divisible by both 3 and 4. Explain how you know this number is divisible by 3 and 4.**

   Answers will vary. Possible student response: The sum of the digits is divisible by 3, and that's how I know the number is divisible by 3. The last 2 digits are divisible by 4, so the entire number is divisible by 4.

5. **Write a 4 digit number that is divisible by both 5 and 9. Explain how you know this number is divisible by 5 and 9.**

   Answers will vary. Possible student response: The number ends with a 5 or 0, so the entire number is divisible by 5. The sum of the digits is divisible by 9, so the entire number is divisible by 9.

Lesson 18: Least Common Multiple and Greatest Common Factor

Student Outcomes

▪ Students find the least common multiple and greatest common factor and apply knowledge of factors to use the distributive property.

Lesson Notes

Least common multiple and greatest common factor are terms that are easily confused by young learners. A clear definition of both phrases with several examples of each should be posted in the classroom before, during, and after the lesson. Furthermore, these skills should be practiced regularly so that the concepts do not fade or blend together from lack of use.

During this lesson, students will move around in groups to various stations where a topic is presented on chart paper. At each station, students read the directions, choose a problem, and then work collaboratively to solve the problem. Group students prior to the lesson using the most appropriate ability or social grouping.

There are four different topics: Factors and GCF; Multiples and LCM; Using Prime Factors to Determine GCF; and Applying Factors to the Distributive Property.

If two sets of chart paper are prepared for each topic, there will be eight stations. This makes manageable groups of 3 or 4 students. Further, if the stations are placed in order around the room (1, 2, 3, 4, 1, 2, 3, 4), it will not matter where a group starts, and the group will finish after only three rotations. Groups should spend five minutes at each station.

Suggested Student Roles:

Marker	This student records the group's work on the chart paper. Note: Each group should use a different (unique) color when adding its work to the chart paper.
Recorder	This student records the group's work in his/her student materials and later shares this work with the other members of the group, ensuring that all students finish the activity with their student materials completed.
Calculator Operator/Master Mathematician	This student uses a calculator when necessary and helps clarify the group's response.

Materials: Eight pieces of chart paper with directions attached; one calculator per group; a different colored chart marker for each group; a multiplication table posted at Stations 2 and 4

Classwork

Opening (4 minutes)

Point out the definitions on the student pages, and work through the examples before assigning and releasing groups.

> The <u>Greatest Common Factor</u> of two whole numbers a and b, written $\text{GCF}(a, b)$, is the greatest whole number, which is a factor of both a and b.
>
> The <u>Least Common Multiple</u> of two nonzero numbers a and b, written $\text{LCM}(a, b)$, is the least whole number (larger than zero), which is a multiple of both a and b.

Example 1 (3 minutes): Greatest Common Factor

> **Example 1: Greatest Common Factor**
>
> **Find the greatest common factor of 12 and 18.**
>
> - Listing the factor pairs in order will help you not miss any common factors. Start with one times the number.
> - Circle all factors that appear on both lists.
> - Place a triangle around the greatest of these common factors.
>
> $\text{GCF}\,(\mathbf{12}, \mathbf{18})$　　　6
>
> 12
>
> | | |
> |---|---|
> | ①　 | 12 |
> | ②　 | △6 |
> | ③　 | 4 |
>
> 18
>
> | | |
> |---|---|
> | ①　 | 18 |
> | ②　 | 9 |
> | ③　 | △6 |

Example 2 (5 minutes): Least Common Multiple

Example 2: Least Common Multiple

Find the least common multiple of 12 and 18.

LCM (12, 18)

Write the first 10 multiples of 12.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120

Write the first 10 multiples of 18.

18, 36, 54, 72, 90, 108, 126, 144, 162, 180

Circle the multiples that appear on both lists.

12, 24, (36) 48, 60, (72) 84, 96, (108) 120

18, (36) 54, (72) 90, (108) 126, 144, 162, 180

Put a rectangle around the least of these common multiples.

12, 24, [36] 48, 60, (72) 84, 96, (108) 120

18, [36] 54, (72) 90, 108, 126, 144, 162, 180

> **Scaffolding:**
> - Multiplication tables should be used by any learners who have automaticity issues. Naming this a "Multiples Table" is also effective with some students.

- Did you really need to write out 10 multiples of each number?
  - *No, we could have stopped as soon as a multiple appeared on both lists.*
- Should we start by writing the multiples of the larger or the smaller of the two numbers? Which will lead us to find the LCM most efficiently?
  - *If we start writing the multiples of the larger of the two numbers, we can stop when we find the first one that is a multiple of the smaller of the two numbers. In the example given, we would list the multiples of 18 first and stop at 36 because 36 is a multiple of 12. In this case, we will have found the LCM (12, 18) after listing only two numbers.*

Discussion (5 minutes)

- Today, you will be visiting several stations around the room. At each station, you will have five minutes to read and follow directions. Only the Recorder's paper will be used at the station. Later, the Recorder will share his copy of the work so each of you will leave with a record of today's classwork.
- Another person in the group, the Marker, will have the chart marker for writing on the chart paper, and a third person will serve as Calculator Operator/Master Mathematician, who will use a calculator when necessary and help clarify your group's response before putting it on the chart paper.
- At each station, the directions start out the same way: Choose one of the problems that has not yet been solved. Solve it together on the Recorder's page. The Marker should copy your group's work neatly on the chart paper and cross out the problem your group solved so that the next group solves a different problem.

Exercises (20 minutes; 5 minutes per station)

Station 1: Factors and GCF

Groups will choose from the following number problems:

Find the greatest common factor of one of these pairs: $30, 50$; $30, 45$; $45, 60$; $42, 70$; $96, 144$.

Next, groups will choose one of the following application problems:

a.   There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same ratio of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b.   The Ski Club members are preparing identical welcome kits for the new skiers. The Ski Club has 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of identical kits they can prepare using all of the hand and foot warmer packets? How many hand warmer packets and foot warmer packets will be in each welcome kit?

c.   There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representatives and senators could be formed from all of Congress if we want the largest groups possible? How many representatives and senators would be in each group?

d.   Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e.   Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

Station 1: Factors and GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

GCF $(30, 50)$

Factors of 30: $\ 1, 2, 3, 5, 6, 10, 15, 30$ *Factors of* 50: $\ 1, 2, 5, 10, 25, 50$
Common Factors: $\ 1, 2, 5, 10$ *Greatest Common Factor:* $\ 10$

GCF $(30, 45)$

Factors of 30: $\ 1, 2, 3, 5, 6, 10, 15, 30$ *Factors of* 45: $\ 1, 3, 5, 9, 15, 45$
Common Factors: $\ 1, 3, 5, 15$ *Greatest Common Factor:* $\ 15$

GCF $(45, 60)$

Factors of 45: $\ 1, 3, 5, 9, 15, 45$ *Factors of* 60: $\ 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$
Common Factors: $\ 1, 3, 5, 15$ *Greatest Common Factor:* $\ 15$

GCF $(42, 70)$

Factors of 42: $\ 1, 2, 3, 6, 7, 14, 21, 42$ *Factors of* 70: $\ 1, 2, 5, 7, 10, 14, 35, 70$
Common Factors: $\ 1, 2, 7, 14$ *Greatest Common Factor:* $\ 14$

GCF $(96, 144)$

Factors of 96: $\ 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$ *Factors of* 144: $\ 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144$
Common Factors: $\ 1, 2, 3, 4, 6, 8, 12, 16, 24, 48$ *Greatest Common Factor :* $\ 48$

Next, choose one of these problems that has not yet been solved:

a.   **There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?**

There will be 6 teams, each having 3 girls and 4 boys.

> **b.** The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand and foot warmer packets? How many hand warmer packets and foot warmer packets will be in each welcome kit?
>
> *There will be 12 welcome kits, each having 5 hand warmer packets and 4 foot warmer packets.*
>
> **c.** There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representative and senators could be formed from all of Congress if we want the largest groups possible? How many representatives and senators will be in each group?
>
> *There will be 5 groups, each with 87 representatives and 20 senators.*
>
> **d.** Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.
>
> *Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.*
>
> **e.** Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.
>
> *No. Factors are, by definition, less than or equal to the number. Therefore, the GCF cannot be greater than both numbers.*

Station 2: Multiples and LCM

Groups will choose from the following number problems:

Find the least common multiple of one of these pairs: $9, 12$; $8, 18$; $4, 30$; $12, 30$; $20, 50$.

Next, groups will choose one of the following application problems:

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?

b. Starting at $6\!:\!00$ a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at $6\!:\!00$ a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

> **Station 2: Multiples and LCM**
>
> **Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.**
>
> **LCM $(9, 12)$**
>
> *Multiples of 9: 9, 18, 27, 36*   *Multiples of 12: 12, 24, 36*
> *Least Common Multiple: 36*
>
> **LCM $(8. 18)$**
>
> *Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72*   *Multiples of 18: 36, 54, 72*
> *Least Common Multiple: 72*

LCM $(4, 30)$

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60   *Multiples of 30: 30, 60*
Least Common Multiple: 60

LCM $(12, 30)$

Multiples of 12: 12, 24, 36, 48, 60   *Multiples of 30: 30, 60*
Least Common Multiple: 60

LCM $(20, 50)$

Multiples of 20: 20, 40, 60, 80, 100   *Multiples of 50: 50, 100*
Least Common Multiple: 100

Next, choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy? How many items of each item would we have to buy?

Four packages of hot dogs = 40 hot dogs. Five packages of buns = 40 buns. LCM $(8, 10) = 40$.

b. Starting at $6:00$ a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at $6:00$ a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

Both a bus and a taxi will arrive at the corner at $7:00$ a.m., which is 60 minutes after $6:00$ a.m. LCM $(12, 15) = 60$.

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

The first gear will need five revolutions. During this time, 120 teeth will pass by. The second gear will revolve three times. LCM $(24, 40) = 120$.

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

Yes. Valid examples will consist of a pair of numbers where the lesser of the two numbers is a factor of the greater number; the greater of the two numbers is a multiple of the lesser number.

e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

No. Multiples are, by definition, equal to or greater than the number.

Station 3: Using Prime Factors to Determine GCF

Note: If the classroom has Internet access, a Factor Tree applet is available at:
http://nlvm.usu.edu/en/nav/frames_asid_202_g_3_t_1.html?from=category_g_3_t_1.html

Choose "Two Factor Trees" and "User Defined Problems." When both numbers are factored into prime factors, each common prime factor is dragged into the middle of a two-circle Venn diagram. Unique prime factors are separated into the other two regions of the Venn diagram. Introducing the applet before the lesson and allowing exploration time will strengthen understanding and make this lesson more cohesive.

Groups will choose from the following number problems:

Use prime factors to find the greatest common factor of one of the following pairs of numbers:
$30, 50$; $30, 45$; $45, 60$; $42, 70$; $96, 144$.

Next, groups will choose one of the following application problems:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

b. Find the GCF of your original pair of numbers.

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?

d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The prime factors of this number do not repeat, and all of the prime numbers are less than 12. What is Glenn's number? When was Sarah born?

Station 3: Using Prime Factors to Determine GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.
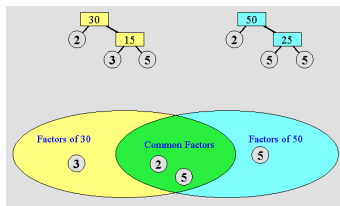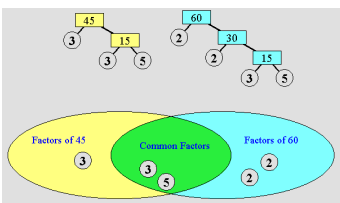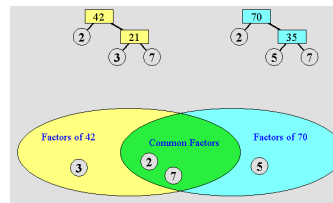
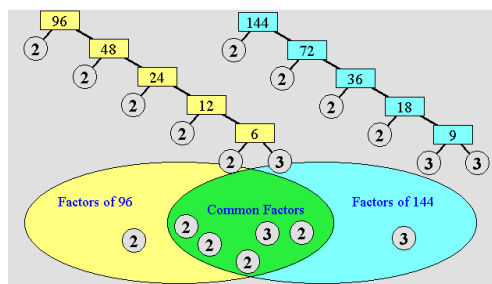**GCF $(30, 50)$**　　　　　　　　　　**GCF $(30, 45)$**
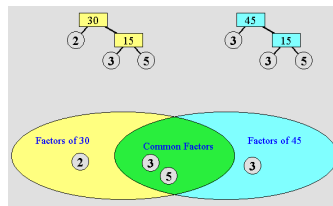
GCF $(45, 60)$　　　　　　　　　　**GCF $(42, 70)$**

GCF $(96, 144)$

Next, choose one of these problems that has not yet been solved:

a. **Would you rather find all the factors of a number or find all the prime factors of a number? Why?**

   Accept opinions. Students should defend their answer and use accurate mathematical terms in their response.

b. **Find the GCF of your original pair of numbers.**

   See answers listed in Exploratory Challenge 1.

c. **Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?**

   In all cases, GCF $(a, b) \cdot$ LCM $(a, b) = a \cdot b$.

d. **Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat, and all of the prime numbers are less than 12. What is Glenn's number? When was Sarah born?**

   $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2,310$ *Sarah's birthdate is 2/3/10.*

Station 4: Applying Factors to the Distributive Property

MP.7 Study these examples of how factors apply to the distributive property.

$$8 + 12 = 4(2) + 4(3) = 4(2 + 3) = 20 \qquad\qquad 15 + 25 = 5(3) + 5(5) = 5(3 + 5) = 40$$
$$4(2) + 4(3) = 4(5) = 20 \qquad\qquad\qquad 5(3) + 5(5) = 5(8) = 40$$

$$36 - 24 = 4(9) - 4(6) = 4(9 - 6) = 12$$
$$4(9) - 4(6) = 4(3) = 12$$

Students will factor out the GCF from the two numbers and rewrite the sum using the distributive property.

Groups will choose one of the following problems:

a. $12 + 18 =$
b. $42 + 14 =$
c. $36 + 27 =$
d. $16 + 72 =$
e. $44 + 33 =$

Next, students will add their own examples to one of two statements applying factors to the distributive property:

$n(a) + n(b) = n(a + b)$

$n(a) - n(b) = n(a - b)$

Station 4: Applying Factors to the Distributive Property

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Find the GCF from the two numbers, and rewrite the sum using the distributive property.

1. $12 + 18 =$

   $6(2) + 6(3) = 6(2 + 3) = 6(5) = 30$

2. $42 + 14 =$

   $7(6) + 7(2) = 7(6 + 2) = 7(8) = 56$

3. $36 + 27 =$

   $9(4) + 9(3) = 9(4 + 3) = 9(7) = 63$

4. $16 + 72 =$

   $8(2) + 8(9) = 8(2 + 9) = 8(11) = 88$

5. $44 + 33 =$

   $11(4) + 11(3) = 11(4 + 3) = 11(7) = 77$

Next, add another new example to one of these two statements applying factors to the distributive property.

Choose any numbers for n, a, and b.

$n(a) + n(b) = n(a + b)$

Accept all student responses that are mathematically correct.

$n(a) - n(b) = n(a - b)$

The distributive property holds for addition as well as subtraction. Accept all student responses that are mathematically correct.

Closing (4 minutes)

- Use this time to discuss each station. Assign the Problem Set which asks students to revisit each topic independently.

Exit Ticket (4 minutes)

Name _____   Date_____

Lesson 18: Least Common Multiple and Greatest Common Factor

Exit Ticket

1.   Find the LCM and GCF of 12 and 15.

2.   Write two numbers, neither of which is 8, whose GCF is 8.

3.   Write two numbers, neither of which is 28, whose LCM is 28.

Rate each of the stations you visited today. Use this scale:

3—Easy—I've got it; I don't need any help.

2 —Medium—I need more practice, but I understand some of it.

1—Hard—I'm not getting this yet.

Complete the following chart:

Station	Rating (3, 2, 1)	Comment to the Teacher
Station 1: Factors and GCF		
Station 2: Multiples and LCM		
Station 3: Using Prime Factors for GCF		
Station 4: Applying Factors to the Distributive Property		

Exit Ticket Sample Solutions

1. **Find the LCM and GCF of 12 and 15.**

   LCM: **60**; *GCF:* **3**

2. **Write two numbers, neither of which is 8, whose GCF is 8.**

   Answers will vary, i.e., **16** *and* **24**, *or* **24** *and* **32**.

3. **Write two numbers, neither of which is 28, whose LCM is 28.**

   Answers will vary, i.e., **4** *and* **14**, *or* **4** *and* **7**.

Rate each of the stations you visited today. Use this scale:

3—Easy—I've got it, I don't need any help.

2—Medium—I need more practice, but I understand some of it.

1—Hard—I'm not getting this yet.

Complete the following chart:

Station	Rating (3, 2, 1)	Comment to the Teacher
Station 1 Factors and GCF		
Station 2 Multiples and LCM		
Station 3 Using Prime Factors for GCF		
Station 4 Applying Factors to the Distributive Property		

Problem Set Sample Solutions

Students should complete the remaining stations from class.

Exploratory Challenge Reproducible

Station 1: Factors and GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Find the greatest common factor of one of these pairs: $30, 50$; $30, 45$; $45, 60$; $42, 70$; $96, 144$.

Next, choose one of these problems that has not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same number of girls and boys, what is the greatest number of teams that can enter? How many boys and girls will be on each team?

b. The Ski Club members are preparing identical welcome kits for the new skiers. They have 60 hand warmer packets and 48 foot warmer packets. What is the greatest number of kits they can prepare using all of the hand and foot warmer packets? How many hand warmer packets and foot warmer packets will be in each welcome kit?

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same number of representatives and senators could be formed from all of Congress if we want the largest groups possible? How many representatives and senators will be in each group?

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.
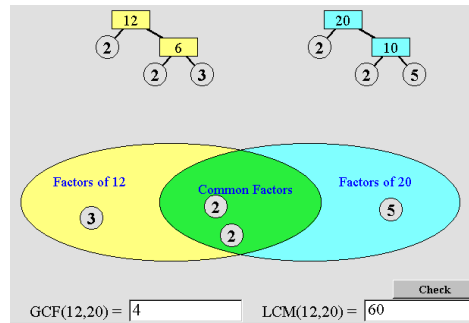
Station 2: Multiples and LCM

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Find the least common multiple of one of these pairs: $9, 12$; $8, 18$; $4, 30$; $12, 30$; $20, 50$.

Next, choose one of these problems that has not yet been solved:

a.   Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic, with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?

b.   Starting at $6\!:\!00$ a.m., a bus makes a stop at my street corner every 15 minutes. Also starting at $6\!:\!00$ a.m., a taxi cab comes by every 12 minutes. What is the next time there will be a bus and a taxi at the corner at the same time?

c.   Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

d.   Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

e.   Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

Solve it together on your student page. Then, use your marker to copy your work neatly on this chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Station 3: Using Prime Factors to Determine GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Use Prime Factors to find the Greatest Common Factor of one of the following pairs of numbers:

$30, 50$ $30, 45$ $45, 60$ $42, 70$ $96, 144$

Next, choose one of these problems that has not yet been solved:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

b. Find the GCF of your original pair of numbers.

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?

d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat, and all of the prime numbers are less than 12. What is Glenn's number? When was Sarah born?

Station 4: Applying Factors to the Distributive Property

Study these examples of how factors apply to the distributive property.

$$8 + 12 = 4(2) + 4(3) = 4(2 + 3) = 20 \qquad 15 + 25 = 5(3) + 5(5) = 5(3 + 5) = 40$$
$$4(2) + 4(3) = 4(5) = 20 \qquad\qquad 5(3) + 5(5) = 5(8) = 40$$

$$36 - 24 = 4(9) - 4(6) = 4(9 - 6) = 12$$
$$4(9) - 4(6) = 4(3) = 12$$

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

Find the GCF from the two numbers, and rewrite the sum using the distributive property.

1. $12 + 18 =$

2. $42 + 14 =$

3. $36 + 27 =$

4. $16 + 72 =$

5. $44 + 33 =$

Next, add another new example to one of these two statements applying factors to the distributive property.

Choose any numbers for n, a, and b.

$n(a) + n(b) = n(a + b)$

$n(a) - n(b) = n(a - b)$

Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Student Outcomes

- Students explore and discover that Euclid's Algorithm is a more efficient means to finding the greatest common factor of larger numbers and determine that Euclid's Algorithm is based on long division.

Lesson Notes

MP.7 Students look for and make use of structure, connecting long division to Euclid's Algorithm.

Students look for and express regularity in repeated calculations leading to finding the greatest common factor of a pair of numbers.

These steps are contained in the Student Materials and should be reproduced, so they can be displayed throughout the lesson:

Euclid's Algorithm is used to find the greatest common factor (GCF) of two whole numbers.

MP.8
1. Divide the larger of the two numbers by the smaller one.
2. If there is a remainder, divide it into the divisor.
3. Continue dividing the last divisor by the last remainder until the remainder is zero.
4. The final divisor is the GCF of the original pair of numbers.

In application, the algorithm can be used to find the side length of the largest square that can be used to completely fill a rectangle so that there is no overlap or gaps.

Classwork

Opening (5 minutes)

Lesson 18 Problem Set can be discussed before going on to this lesson.

Opening Exercise (4 minutes)

- There are three division warm-ups on your Student Material page today. Please compute them now. Check your answer to make sure it is reasonable.

> **Opening Exercise**
>
> **Euclid's Algorithm is used to find the greatest common factor (GCF) of two whole numbers.**
>
> 1. **Divide the larger of the two numbers by the smaller one.**
> 2. **If there is a remainder, divide it into the divisor.**
> 3. **Continue dividing the last divisor by the last remainder until the remainder is zero.**
> 4. **The final divisor is the GCF of the original pair of numbers.**
>
> $383 \div 4 = 95.75$ $432 \div 12 = 36$ $403 \div 13 = 11$

Discussion (2 minutes)

- The Opening Exercise was meant for you to recall how to determine the quotient of two numbers using long division. You have practiced the long division algorithm many times. Today's lesson is inspired by Euclid, a Greek mathematician who lived around 300 B.C. He discovered a way to find the greatest common factor of two numbers that is based on long division.

Example 1 (10 minutes): Euclid's Algorithm Conceptualized

- What is the GCF of 60 and 100?
  - 20
- What is the interpretation of the GCF in terms of area? Let's take a look.

  Project the following diagram:

> **Example 1: Euclid's Algorithm Conceptualized**
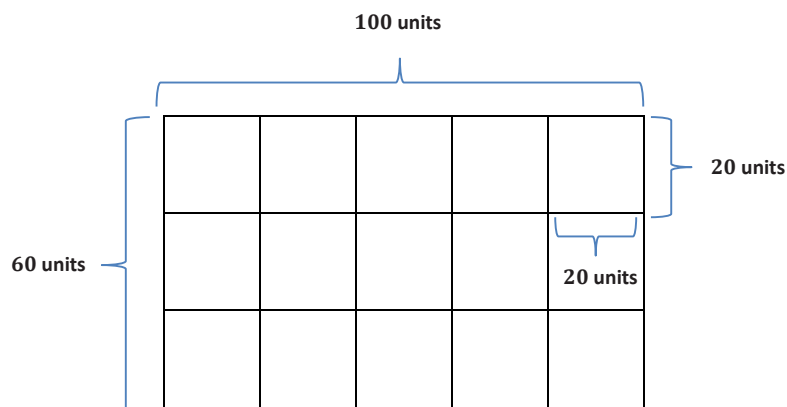
- Notice that we can use the GCF of 20 to create the largest square tile that will cover the rectangle without any overlap or gaps. We used a 20×20 tile.
- But, what if we didn't know that? We could start by guessing. What is the biggest square tile that we can guess?
  - 60×60

Display the following diagram:

100 units

60 units
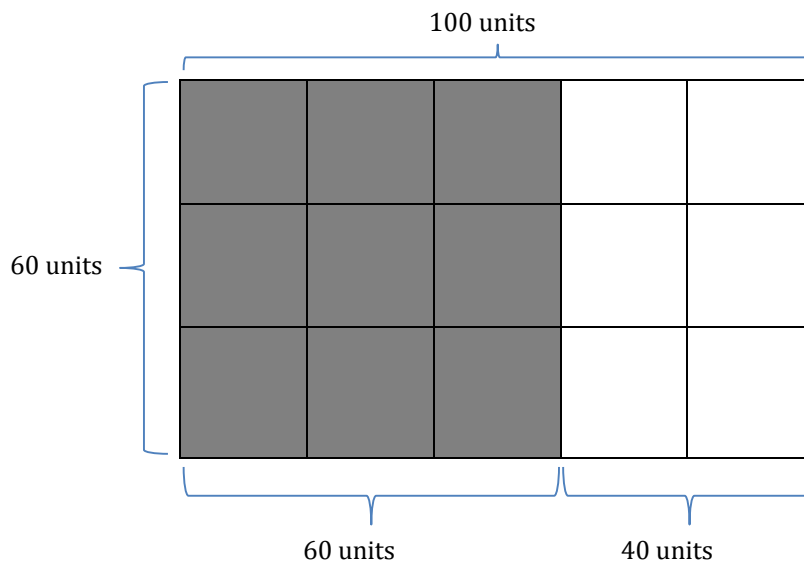
60 units   40 units

- It fits, but there are 40 units left over. Do the division problem to prove this.

Teacher note: With each step in this process, please write out the long division algorithm that accompanies it.

It is important for students to make a record of this as well. The remainder becomes the new divisor and continues until the remainder is zero.

$$\begin{array}{r} 1 \\ 60\overline{)100} \\ -\ 60 \\ \hline 40 \end{array}$$

- What is the leftover area?
  - 60×40
- What is the largest square tile that we can fit in the leftover area?
  - 40×40

Display the following diagram:
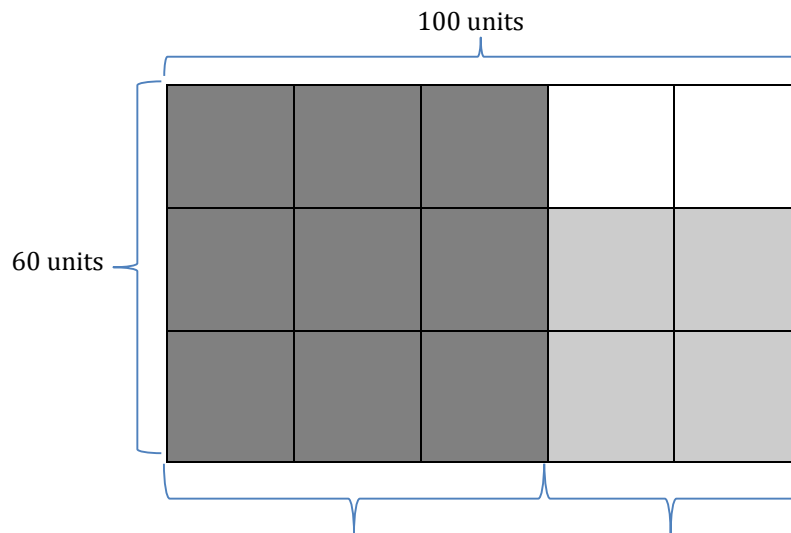
- ▪ What is the leftover area?
    - ▫ 20×40
- ▪ What is the largest square tile that we can fit in the leftover area?
    - ▫ 20×20
- ▪ When we divide 40 by 20, there is no remainder. So, we have tiled the entire rectangle.
- ▪ If we had started tiling the whole rectangle with squares, the largest square we could have used would be 20 by 20.

$$\begin{array}{r} 2 \\ 20{\overline{\smash{\big)}\,40}} \\ -\ 40 \\ \hline 0 \end{array}$$

Take a few minutes to allow discussion, questions, and clarification.

Example 2 (5 minutes): Lesson 18 Classwork Revisited

During Lesson 18, students found the GCF of several pairs of numbers.

- ▪ Let's apply Euclid's Algorithm to some of the problems from our last lesson.

Example 2: Lesson 18 Classwork Revisited

a. **Let's apply Euclid's Algorithm to some of the problems from our last lesson.**

   i. **What is the GCF of 30 and 50?**
      10

   ii. **Using Euclid's Algorithm, we follow the steps that are listed in the opening exercise.**

   $$\begin{array}{r} 1 \\ 30{\overline{\smash{\big)}\,50}} \\ -\ 30 \\ \hline 20 \end{array} \quad\longrightarrow\quad \begin{array}{r} 1 \\ 20{\overline{\smash{\big)}\,30}} \\ -\ 20 \\ \hline 10 \end{array} \quad\longrightarrow\quad \begin{array}{r} 2 \\ 10{\overline{\smash{\big)}\,20}} \\ -\ 20 \\ \hline 00 \end{array}$$

   When the remainder is zero, the final divisor is the GCF.

Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm
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engage^{ny}

> **b.** **Apply Euclid's Algorithm to find the GCF** $(30, 45)$.
>
> **15**
>
> $$\begin{array}{r} 1 \\ 30\overline{)45} \\ -30 \\ \hline 15 \end{array} \qquad \begin{array}{r} 2 \\ \boxed{15}\overline{)30} \\ -30 \\ \hline 00 \end{array}$$

Example 3 (5 minutes): Larger Numbers

Example 3: Larger Numbers

GCF $(96, 144)$

$$\begin{array}{r} 1 \\ 96\overline{)144} \\ -096 \\ \hline 048 \end{array} \qquad \begin{array}{r} 2 \\ \boxed{48}\overline{)96} \\ -96 \\ \hline 00 \end{array}$$

GCF $(660, 840)$

$$\begin{array}{r} 1 \\ 660\overline{)840} \\ -660 \\ \hline 180 \end{array} \quad \begin{array}{r} 3 \\ 180\overline{)660} \\ -540 \\ \hline 120 \end{array} \quad \begin{array}{r} 1 \\ 120\overline{)180} \\ -120 \\ \hline 060 \end{array} \quad \begin{array}{r} 2 \\ \boxed{60}\overline{)120} \\ -120 \\ \hline 000 \end{array}$$

Example 4 (5 minutes): Area Problems

Example 4: Area Problems

The greatest common factor has many uses. Among them, the GCF lets us find out the maximum size of squares that will cover a rectangle. When we solve problems like this, we cannot have any gaps or any overlapping squares. Of course, the maximum size squares will be the minimum number of squares needed.

A rectangular computer table measures 30 inches by 50 inches. We need to cover it with square tiles. What is the side length of the largest square tile we can use to completely cover the table so that there is no overlap or gaps?
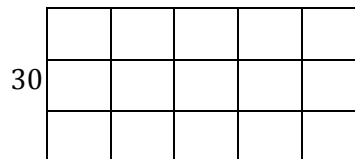
Direct students to consider the GCF $(30, 50)$, which they already calculated. It should become clear that squares of 10 inches by 10 inches will tile the area.

> **a.** **If we use squares that are 10 by 10, how many will we need?**
>
> **$3 \cdot 5$, or 15 squares**
>
> **b.** **If this were a giant chunk of cheese in a factory, would it change the thinking or the calculations we just did?**
>
> **No.**
>
> **c.** **How many 10 inch × 10 inch squares of cheese could be cut from the giant 30 inch × 50 inch slab?**
>
> **15**

Closing (4 minutes)

- Euclid's Algorithm is used to find the greatest common factor (GCF) of two whole numbers. The steps are listed on your Student Page and need to be followed enough times to get a zero remainder. With practice, this can be a quick method for finding the GCF.

- Let's use the steps to solve this problem: Kiana is creating a quilt made of square patches. The quilt is 48 inches in length and 36 inches in width. What is the largest size of square length of each patch?

- Divide the larger of the two numbers by the smaller one. What is the quotient?
   - 1

- If there is a remainder, divide it into the divisor. What is the remainder?
   - 12

- What is the divisor?
   - 36

- Divide 36 by 12. What is the quotient?
   - 3

- Continue dividing the last divisor by the last remainder until the remainder is zero. Is the remainder zero?
   - *Yes.*

- The final divisor is the GCF of the original pair of numbers. What is the final divisor?
   - 12

Exit Ticket (5 minutes)

Name _____ Date_____

Lesson 19: The Euclidean Algorithm as an Application of the Long Division Algorithm

Exit Ticket

Use Euclid's Algorithm to find the greatest common factor of 45 and 75.

Exit Ticket Sample Solutions

Use Euclid's Algorithm to find the greatest common factor of 45 and 75.

$$
\begin{array}{r}
1 \\
45\overline{)75} \\
-45 \\
\hline
30
\end{array}
\qquad
\begin{array}{r}
1 \\
30\overline{)45} \\
-30 \\
\hline
15
\end{array}
\qquad
\begin{array}{r}
2 \\
15\overline{)30} \\
-30 \\
\hline
00
\end{array}
$$

GCF $(45, 75) = 15$

Problem Set Sample Solutions

1. Use Euclid's Algorithm to find the greatest common factor of the following pairs of numbers:

    a. **GCF** $(12, 78)$

    $$
    \begin{array}{r}
    6 \\
    12\overline{)78} \\
    -72 \\
    \hline
    06
    \end{array}
    \qquad
    \begin{array}{r}
    2 \\
    6\overline{)12} \\
    -12 \\
    \hline
    00
    \end{array}
    $$

    GCF $(12, 78) = 6$

    b. **GCF** $(18, 176)$

    $$
    \begin{array}{r}
    9 \\
    18\overline{)176} \\
    -162 \\
    \hline
    014
    \end{array}
    \qquad
    \begin{array}{r}
    1 \\
    14\overline{)18} \\
    -14 \\
    \hline
    04
    \end{array}
    \qquad
    \begin{array}{r}
    3 \\
    4\overline{)14} \\
    -12 \\
    \hline
    02
    \end{array}
    \qquad
    \begin{array}{r}
    2 \\
    2\overline{)4} \\
    -4 \\
    \hline
    0
    \end{array}
    $$

    GCF $(18, 176) = 2$

2. Juanita and Samuel are planning a pizza party. They order a rectangular sheet pizza which measures 21 inches by 36 inches. They tell the pizza maker not to cut it because they want to cut it themselves.

    a. All pieces of pizza must be square with none left over. What is the length of the side of the largest square pieces into which Juanita and Samuel can cut the pizza?

    GCF $(21, 36) = 3$. *They can cut the pizza into 3 by 3 inch squares.*

    b. How many pieces of this size will there be?

    $7 \cdot 12 = 84$. *There will be 84 pieces.*

3. Shelly and Mickelle are making a quilt. They have a piece of fabric that measures 48 inches by 168 inches.

    a. All pieces of fabric must be square with none left over. What is the length of the side of the largest square pieces into which Shelly and Mickelle can cut the fabric?

    GCF $(48, 168) = 24$

    b. How many pieces of this size will there be?

 $2 \cdot 7 = 14$. *There will be 14 pieces.*

Name _____   Date _____

1. L.B. Johnson Middle School held a track and field event during the school year. The chess club sold various drink and snack items for the participants and the audience. All together, they sold 486 items that totaled $2,673.

    a. If the chess club sold each item for the same price, calculate the price of each item.

    b. Explain the value of each digit in your answer to 1(a) using place value terms.

2. The long jump pit was recently rebuilt to make it level with the runway. Volunteers provided pieces of wood to frame the pit. Each piece of wood provided measures 6 feet, which is approximately 1.8287 meters.

2.75 meters

9.54 meters
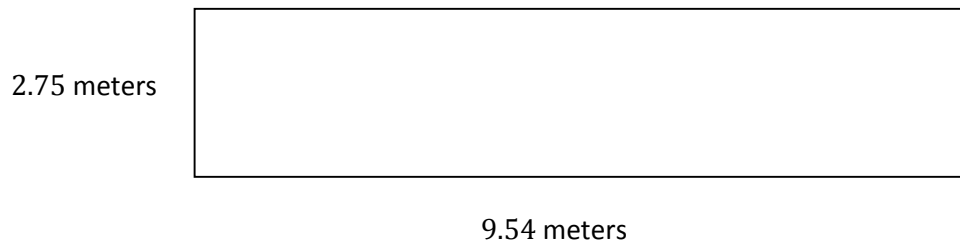
a. Determine the amount of wood, in meters, needed to rebuild the frame.

b. How many boards did the volunteers supply? Round your calculations to the nearest hundredth and then provide the whole number of boards supplied.

3. Andy runs 436.8 meters in 62.08 seconds.

    a. If Andy runs at a constant speed, how far does he run in one second? Give your answer to the nearest tenth of a second.

    b. Use place value, multiplication with powers of 10, or equivalent fractions to explain what is happening mathematically to the decimal points in the divisor and dividend before dividing.

    c. In the following expression, place a decimal point in the divisor and the dividend to create a new problem with the same answer as in 3(a). Then, explain how you know the answer will be the same.

$$4\ 3\ 6\ 8 \div 6\ 2\ 0\ 8$$

4. The PTA created a cross-country trail for the meet.

    a. The PTA placed a trail marker in the ground every four hundred yards. Every nine hundred yards the PTA set up a water station. What is the shortest distance a runner will have to run to see both a water station and trail marker at the same location?

                                     Answer: _____hundred yards

    b. There are 1,760 yards in one mile. About how many miles will a runner have to run before seeing both a water station and trail marker at the same location? Calculate the answer to the nearest hundredth of a mile.

    c. The PTA wants to cover the wet areas of the trail with wood chips. They find that one bag of wood chips covers a $3\frac{1}{2}$ yards section of the trail. If there is a wet section of the trail that is approximately $50\frac{1}{4}$ yards long, how many bags of wood chips are needed to cover the wet section of the trail?

5. The Art Club wants to paint a rectangle-shaped mural to celebrate the winners of the track and field meet. They design a checkerboard background for the mural where they will write the winners' names. The rectangle measures 432 inches in length and 360 inches in width. Apply Euclid's Algorithm to determine the side length of the largest square they can use to fill the checkerboard pattern completely without overlap or gaps.

A Progression Toward Mastery						
Assessment Task Item			STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 6.NS.B.2		Student response is missing or depicts inaccurate operation choice.	Student response is inaccurate and does not represent the correct place value.	Student response is inaccurate through minor calculation errors; however, place value is represented accurately.	Student response is correct. The price of each item is determined as $5.50, where place value is represented accurately.
	b 6.NS.B.2		Student response is incorrect or missing. Place value is not depicted in the response.	Student response depicts place value only in monetary denominations, such as dollars and cents.	Student response depicts place value accurately, but makes little to no correlation to monetary denominations.	Student response is accurate. Each place value is labeled accurately and shows correlation to the monetary denominations each place value represents. For example, 5 dollars is labeled with 5 ones and 5 dollars, 50 cents is labeled with 5 tenths and 5 dimes, and the zero in the hundredths place is labeled with zero hundredths and "no pennies."
2	a 6.NS.B.3		Student response is incorrect or missing. Students merely included one length and one side in the calculation.	Student response is incorrect based on place value.	Student response depicts understanding of the addition algorithm, but minor calculation errors hinder the correct sum of 24.58 meters.	Student calculations include all sides of the sand pit. Student applied the standard algorithm of addition of decimals to determine the correct sum of 24.58 meters.
	b 6.NS.B.3		Student response is incorrect or missing. Calculations disregard place value.	Student response is incorrect and depicts inaccurate place value.	Student response is incorrect. Student rounded the decimal quotient to the nearest	Student response is correct. Reasoning is evident through the use of place value. The final

				hundredth and determined the quotient to be 13.44. The student does not provide the whole number of boards supplied.	response is in terms of a whole number. Student determines that from the calculation of 13.44, the volunteers supplied 14 boards.
3	**a** **6.NS.B.3**	Student response is incorrect or missing. Calculations disregard place value.	Student response is incorrect. Response depicts inaccurate place value where the divisor is represented by a whole number, but the dividend remains a decimal.	Student response is correct, but the quotient of 7.03 is not rounded to the nearest tenth. OR Student calculations are incorrect, but represent knowledge of place value.	Student response is correct, depicting accurate place value in order to generate a whole number dividend. Calculations are flawless, and the answer, 7.0, is represented to the nearest tenth.
	b **6.NS.B.3**	Student response either incorrectly depicts place value or is missing.	Student response depicts some place value knowledge, but not enough to sufficiently describe why and how a whole number divisor is generated.	Student response correctly includes accurate place value through the use of equivalent fractions to demonstrate how to generate a whole number divisor.	Student response is correct and includes multiplying by a power of ten to determine an equivalent fraction with a whole number denominator. Student determines that the quotient of the decimals is equivalent to the quotient of the whole numbers generated through the use of place value.
	c **6.NS.B.3**	Student response is missing.	Student response is incorrect or indicates the same decimal placements from the previous problem.	Student response accurately places decimals in the divisor and dividend with no explanation or justification.	Student response accurately places decimals within the divisor (6.208) and dividend (43.68) to generate a quotient of 7.03 and justifies placement through the use of either place value, powers of ten, or equivalent fractions.
4	**a** **6.NS.B.4**	Student response is incorrect or missing. Response is a result of finding the sum of or the difference between 9 and 4.	Student response is incorrect or is simply the product of 4 and 9 with no justification.	Student response accurately finds the least common multiple of 4 and 9, but the response is determined as 36, instead of 36 hundred or 3,600 yards or the correct response reflects finding the LCM of 400 and 900.	Student response is accurately determined through finding the least common multiple. The response represents an understanding of the unit "hundred" as a means of efficiently determining LCM using 4 and 9, instead of 400 and 900.

	b **6.NS.B.2**	Student response is missing. <u>OR</u> Student response utilizes incorrect operations, such as addition, subtraction, or multiplication.	Student response shows little reasoning through the use of division to determine the quotient. Student response depicts division of 1,760 yards by a divisor of 2, derived from counting the two stations. Student response does not include values from the previous problem.	Student response is incorrect, but does include values from the previous problem. Instead of using 3,600, however, the response chooses 36 as the dividend, resulting in an incorrect quotient.	Student response is computed accurately and the solution is appropriately rounded to the hundredths place. The response reflects the correct divisor as 1,760 and the correct dividend as 3,600. The solution, 2.045, is accurately rounded to 2.05 miles.
	c **6.NS.A.1**	Student response is incorrect or missing. Response includes inappropriate operations, such as addition, subtraction, or multiplication.	Student response is incorrect due to inaccurate calculations when converting mixed numbers or when finding the quotients of the fractions.	Student response is correctly determined through mixed number conversion and division of fractions, but is inaccurately left as a mixed number ($14\frac{5}{14}$).	Student response is accurately demonstrated through the use of visual models, such as a number line. The response is confirmed through precise mixed number conversion and division of fractions. The need for 15 bags satisfies understanding that the quotient ($14\frac{5}{14}$) is not a whole number <u>AND</u> that 14 bags is not sufficient.
5	**6.NS.B.4**	Student response is incorrect or missing. Response includes inappropriate operations, such as addition, subtraction, or multiplication.	Student response is incorrect, but depicts reasoning leading to finding the greatest common factor. <u>OR</u> Student response incorrectly utilizes division to determine the quotient of $1\frac{5}{72}$.	Student response determines that the greatest common factor of 432 and 360 is 72, through means other than the Euclidean Algorithm.	Student response efficiently utilizes the Euclidean Algorithm to determine the greatest common factor of 432 and 360 as 72. Response correlates the GCF to the side length of the largest square.

Name _____   Date _____

1. L.B. Johnson Middle School held a track and field event during the school year. The chess club sold various drink and snack items for the participants and the audience. All together, they sold 486 items that totaled $2,673.

    a. If the chess club sold each item for the same price, calculate the price of each item.

$$
\begin{array}{r}
5.5 \\
486\overline{)2673.0} \\
-\ 2430 \\
\hline
2430 \\
-2430 \\
\hline
0
\end{array}
$$

Each item's price is $5.50.

    b. Explain the value of each digit in your answer to 1(a) using place value terms.

$ 5 . 50

5 ones
(five dollars)

5 tenths
(five dimes)

zero hundredths
(no pennies)

2. The long jump pit was recently rebuilt to make it level with the runway. Volunteers provided pieces of wood to frame the pit. Each piece of wood provided measures 6 feet, which is approximately 1.8287 meters.

2.75 meters

9.54 meters

a. Determine the amount of wood, in meters, needed to rebuild the frame.

$$9.54$$
$$9.54$$
$$2.75$$
$$+ \ 2.75$$
$$24.58 \text{m}$$

b. How many boards did the volunteers supply? Round your calculations to the nearest hundredth and then provide the whole number of boards supplied.

$$\frac{24.58 \cdot 10,000}{1.8287 \cdot 10,000} = \frac{245800}{18287}$$

13.441 boards.
To have enough, the volunteers supplied 14 boards.

$$\begin{array}{r}
13.4412 \\
18287 \overline{)245800.0000} \\
-18287 \\
\hline
62930 \\
-54861 \\
\hline
80690 \\
-73148 \\
\hline
75420 \\
-73148 \\
\hline
22720 \\
-18287 \\
\hline
44330 \\
-36574 \\
\hline
7756
\end{array}$$

3. Andy runs 436.8 meters in 62.08 seconds.

    a. If Andy runs at a constant speed, how far does he run in one second? Give your answer to the nearest tenth of a second.

$$\frac{436.8}{62.08}$$

$$6208\overline{)43680.00}$$
$$\begin{array}{r} 7.03 \\ -43456 \\ \hline 2240 \\ -0 \\ \hline 22400 \\ -18624 \\ \hline 3776 \end{array}$$

Andy ran 7.0 meters in one second.

    b. Use place value, multiplication with powers of 10, or equivalent fractions to explain what is happening mathematically to the decimal points in the divisor and dividend before dividing.

$$\frac{436.8 \cdot 100}{62.08 \cdot 100} = \frac{43680}{6,208}$$

When you write the problem as a fraction, multiply the numerator and denominator by 100. Multiplying each by 100 resulted in both numbers being whole numbers.

$436.8 \div 62.08$ is the same as $43,680 \div 6,208$.

    c. In the following expression, place a decimal point in the divisor and the dividend to create a new problem with the same answer as in 3(a). Then, explain how you know the answer will be the same. (6.NS.3 – Lesson 15)

$$4\ 3\ .6\ 8 \div 6\ .2\ 0\ 8$$

$$\frac{436.8}{62.08}\begin{array}{l}\div 10 \\ \div 10\end{array} = \frac{43.68}{6.208}$$

numerator and denominator by 1,000 or divide each by 10.

$$\frac{43.68}{6.208}\begin{array}{l}\cdot 1,000 \\ \cdot 1,000\end{array} = \frac{43680}{6,208}$$

4. The PTA created a cross-country trail for the meet.

   a. The PTA placed a trail marker in the ground every four hundred yards. Every nine hundred yards the PTA set up a water station. What is the shortest distance a runner will have to run to see both a water station and trail marker at the same location?

4 (hundred)
$/\backslash$
$2 \cdot 2$

9 (hundred)
$/\backslash$
$3 \cdot 3$

LCM $2 \cdot 2 \cdot 3 \cdot 3 = 36$ hundred

Answer: ____36____ hundred yards
3,600 yards

   b. There are 1,760 yards in one mile. About how many miles will a runner have to run before seeing both a water station and trail marker at the same location? Calculate the answer to the nearest hundredth of a mile.

```
         2.045
1,760 ) 36600.00
       -3520
        - 808
        - 8000
          7040
          9600
        - 8800
           800
```

2.05 miles

   c. The PTA wants to cover the wet areas of the trail with wood chips. They find that one bag of wood chips covers a $3\frac{1}{2}$ yards section of the trail . If there is a wet section of the trail that is approximately $50\frac{1}{4}$ yards long, how many bags of wood chips are needed to cover the wet section of the trail?
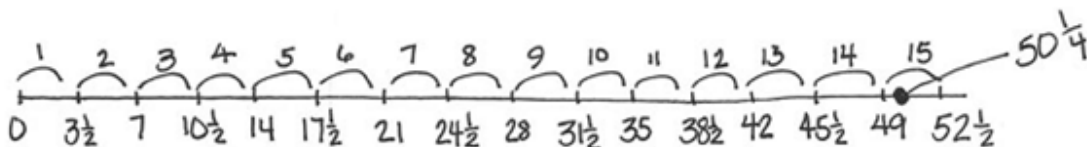
They need more than 14 but less than 15. They need 15 bags to have enough.

$$\frac{201}{4} \div \frac{7}{2} = \frac{201}{4} \div \frac{14}{4} = 201 \div 14 = \frac{201}{14} = 14\frac{5}{14}$$

5. The Art Club wants to paint a rectangle-shaped mural to celebrate the winners of the track and field meet. They design a checkerboard background for the mural where they will write the winners' names. The rectangle measures 432 inches in length and 360 inches in width. Use Euclid's Algorithm to determine the side length of the largest square they can use to fill the checkerboard pattern completely without overlap or gaps.

length – 432 inches
width – 360 inches

$$360 \overline{)432}$$
$$-360$$
$$\overline{72}$$

or

$$360 \overline{)432} \qquad 72 \overline{)360}$$
$$-360 \qquad\qquad -360$$
$$\overline{72} \qquad\qquad \overline{0}$$

$432 = 360 \cdot 1 + 72$

GCF $(432, 360) =$ GCF $(360, 72)$

$72 = 72 \checkmark$

The side length of the largest square they can use is 72 inches.