Lesson 1: Generating Equivalent Expressions

Classwork

Opening Exercise

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let $\boldsymbol{t}$ represent the number of triangles, and let $\boldsymbol{q}$ represent the number of quadrilaterals.

a. Write an expression using t and q that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

d. Use the given values of t and q and your expression from part (a) to determine the number of sides that should be found in your envelope.

e. Use the same values for t and q and your expression from part (b) to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

f. Use the same values for t and q and your expression from part (c) to determine the number of sides that should be contained in all of the envelopes combined.

g. What do you notice about the various expressions in parts (e) and (f)?

Example 1: Any Order, Any Grouping Property with Addition

a. Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

b. Find the sum of $2x + 1$ and $5x$.

c. Find the sum of $-3a + 2$ and $5a - 3$.

Example 2: Any Order, Any Grouping with Multiplication

Find the product of $2x$ and 3.

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication

Use any order, any grouping to write equivalent expressions.

a. $3(2x)$

b. $4y(5)$

c. $4 \cdot 2 \cdot z$

d. $3(2x) + 4y(5)$

e. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$

f. Alexander says that $3x + 4y$ is equivalent to $(3)(4) + xy$ because of any order, any grouping. Is he correct? Why or why not?

Relevant Vocabulary

VARIABLE (DESCRIPTION): A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

NUMERICAL EXPRESSION (DESCRIPTION): A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

VALUE OF A NUMERICAL EXPRESSION: The *value of a numerical expression* is the number found by evaluating the expression.

EXPRESSION (DESCRIPTION): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

EQUIVALENT EXPRESSIONS: Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions.

AN EXPRESSION IN EXPANDED FORM: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form.* A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

TERM (DESCRIPTION): Each summand of an expression in expanded form is called a *term.* For example, the expression $2x + 3x + 5$ consists of three terms: $2x$, $3x$, and 5.

COEFFICIENT OF THE TERM (DESCRIPTION): The number found by multiplying just the numbers in a term together. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is $8x$. The number 8 is called the coefficient of the term $8x$.

AN EXPRESSION IN STANDARD FORM: An expression in expanded form with all its like terms collected is said to be in *standard form.* For example, $2x + 3x + 5$ is an expression written in expanded form; however, to be written in standard form, the like terms $2x$ and $3x$ must be combined. The equivalent expression $5x + 5$ is written in standard form.

Lesson Summary

Terms that contain exactly the same variable symbol can be combined by addition or subtraction because the variable represents the same number. Any order, any grouping can be used where terms are added (or subtracted) in order to group together like terms. Changing the orders of the terms in a sum does not affect the value of the expression for given values of the variable(s).

Problem Set

For Problems 1–9, write equivalent expressions by combining like terms. Verify the equivalence of your expression and the given expression by evaluating each for the given values: $a = 2$, $b = 5$, and $c = -3$.

1. $3a + 5a$
2. $8b - 4b$
3. $5c + 4c + c$
4. $3a + 6 + 5a$
5. $8b + 8 - 4b$
6. $5c - 4c + c$
7. $3a + 6 + 5a - 2$
8. $8b + 8 - 4b - 3$
9. $5c - 4c + c - 3c$

Use any order, any grouping to write equivalent expressions by combining like terms. Then, verify the equivalence of your expression to the given expression by evaluating for the value(s) given in each problem.

10. $3(6a)$; for $a = 3$
11. $5d(4)$; for $d = -2$
12. $(5r)(-2)$; for $r = -3$
13. $3b(8) + (-2)(7c)$; for $b = 2$, $c = 3$
14. $-4(3s) + 2(-t)$; for $s = \frac{1}{2}$, $t = -3$
15. $9(4p) - 2(3q) + p$; for $p = -1$, $q = 4$
16. $7(4g) + 3(5h) + 2(-3g)$; for $g = \frac{1}{2}$, $h = \frac{1}{3}$

The problems below are follow-up questions to Example 1, part (b) from Classwork: Find the sum of $2x + 1$ and $5x$.

17. Jack got the expression $7x + 1$ and then wrote his answer as $1 + 7x$. Is his answer an equivalent expression? How do you know?

18. Jill also got the expression $7x + 1$, and then wrote her answer as $1x + 7$. Is her expression an equivalent expression? How do you know?