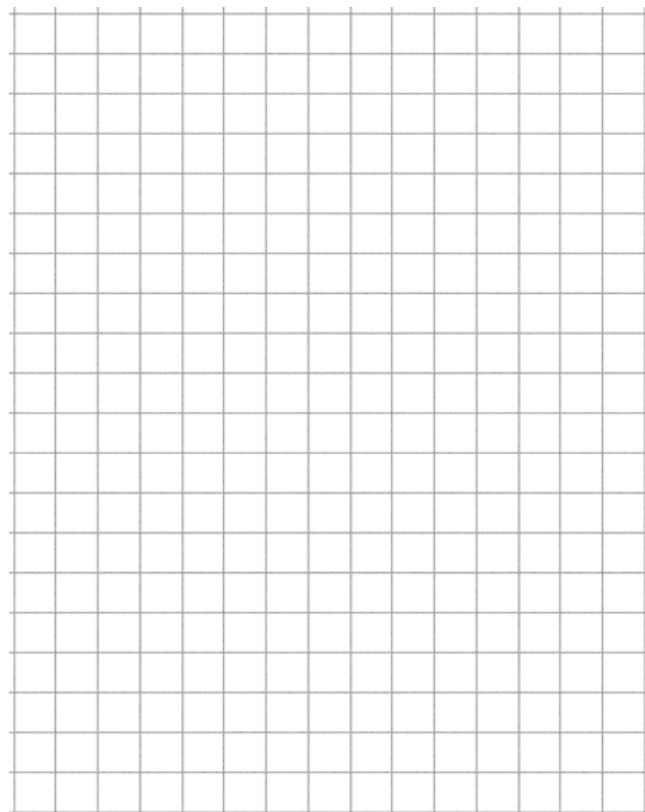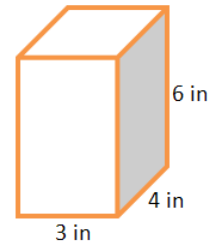# Lesson 21: Surface Area

Classwork

Opening Exercise: Surface Area of a Right Rectangular Prism

On the provided grid, draw a net representing the surfaces of the right rectangular prism (assume each grid line represents 1 inch). Then, find the surface area of the prism by finding the area of the net.

Exercise 1

Marcus thinks that the surface area of the right triangular prism will be half that of the right rectangular prism and wants to use the modified formula $SA = \frac{1}{2}(2lw + 2lh + 2wh)$. Do you agree or disagree with Marcus? Use nets of the prisms to support your argument.

5 in.

6 in.

4 in.

3 in.
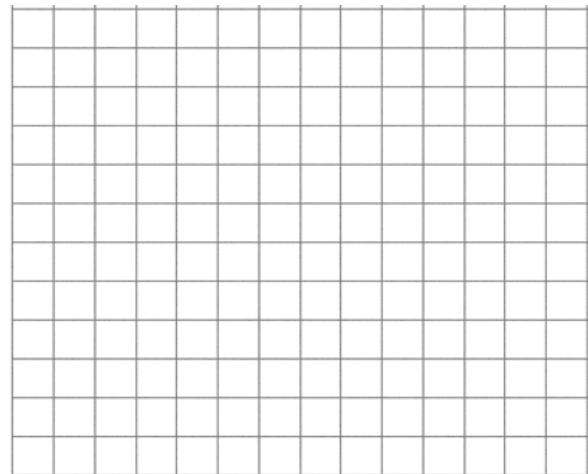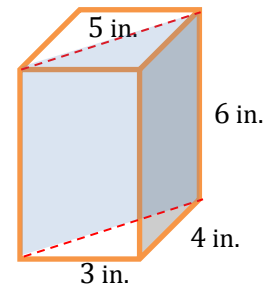
Example 1: Lateral Area of a Right Prism

A right triangular prism, a right rectangular prism, and a right pentagonal prism are pictured below, and all have equal heights of h.
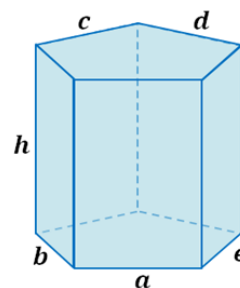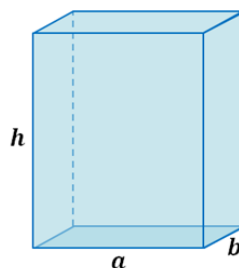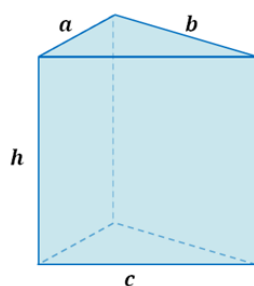
a. Write an expression that represents the lateral area of the right triangular prism as the sum of the areas of its lateral faces.

b.   Write an expression that represents the lateral area of the right rectangular prism as the sum of the areas of its lateral faces.

c.   Write an expression that represents the lateral area of the right pentagonal prism as the sum of the areas of its lateral faces.

d.   What value appears often in each expression and why?

e.   Rewrite each expression in factored form using the distributive property and the height of each lateral face.

f.   What do the parentheses in each case represent with respect to the right prisms?

g.   How can we generalize the lateral area of a right prism into a formula that applies to all right prisms?

Relevant Vocabulary

RIGHT PRISM: Let E and E' be two parallel planes. Let B be a triangular or rectangular region or a region that is the union of such regions in the plane E. At each point P of B, consider the segment PP' perpendicular to E, joining P to a point P' of the plane E'. The union of all these segments is a solid called a *right prism*.

There is a region B' in E' that is an exact copy of the region B. The regions B and B' are called the base faces (or just bases) of the prism. The rectangular regions between two corresponding sides of the bases are called lateral faces of the prism. In all, the boundary of a right rectangular prism has 6 *faces*: 2 base faces and 4 lateral faces. All adjacent faces intersect along segments called edges (base edges and lateral edges).
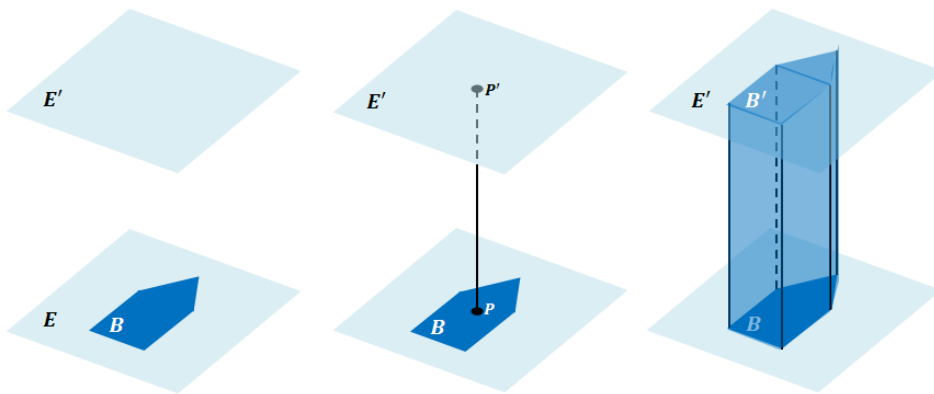
CUBE: A *cube* is a right rectangular prism all of whose edges are of equal length.

SURFACE: The *surface of a prism* is the union of all of its faces (the base faces and lateral faces).

NET (DESCRIPTION): A *net* is a two-dimensional diagram of the surface of a prism.

1. Why are the lateral faces of right prisms always rectangular regions?

2. What is the name of the right prism whose bases are rectangles?

3. How does this definition of right prism include the interior of the prism?
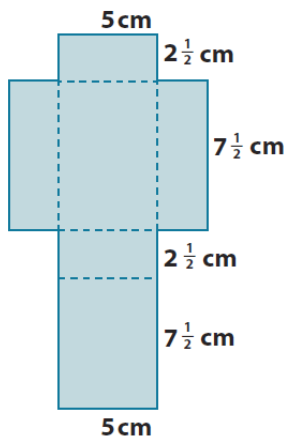
> **Lesson Summary**
>
> The surface area of a right prism can be obtained by adding the areas of the lateral faces to the area of the bases. The formula for the surface area of a right prism is $SA = LA + 2B$, where SA represents surface area of the prism, LA represents the area of the lateral faces, and B represents the area of one base. The lateral area LA can be obtained by multiplying the perimeter of the base of the prism times the height of the prism.
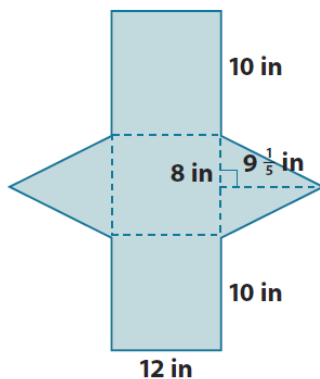
Problem Set

1. For each of the following nets, highlight the perimeter of the lateral area, draw the solid represented by the net, indicate the type of solid, and then find the solid's surface area.

   a.

   5 cm

   $2\frac{1}{2}$ cm

   $7\frac{1}{2}$ cm

   $2\frac{1}{2}$ cm

   $7\frac{1}{2}$ cm

   5 cm

   b.

   10 in

   8 in $9\frac{1}{5}$ in

   10 in

   12 in

2.   Given a cube with edges that are $\frac{3}{4}$ inch long:

   a.   Find the surface area of the cube.

   b.   Joshua makes a scale drawing of the cube using a scale factor of 4. Find the surface area of the cube that Joshua drew.

   c.   What is the ratio of the surface area of the scale drawing to the surface area of the actual cube, and how does the value of the ratio compare to the scale factor?

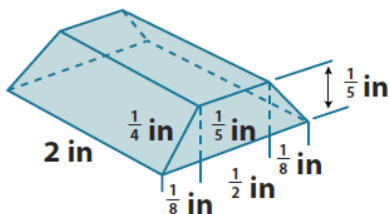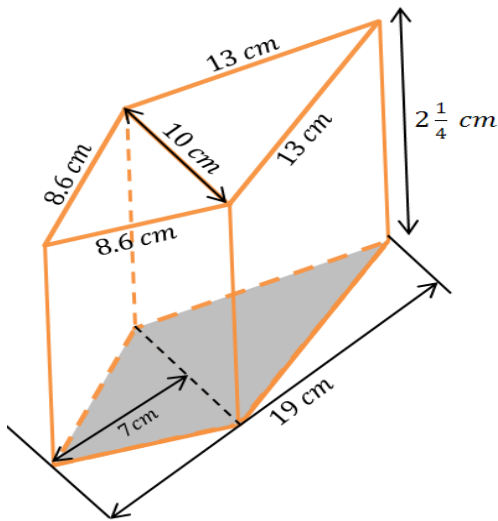3.   Find the surface area of each of the following right prisms using the formula $SA = LA + 2B$.

   a.
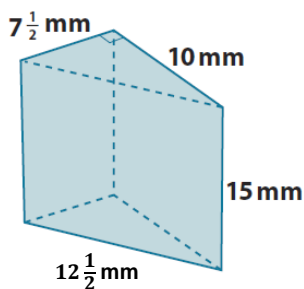
   b.
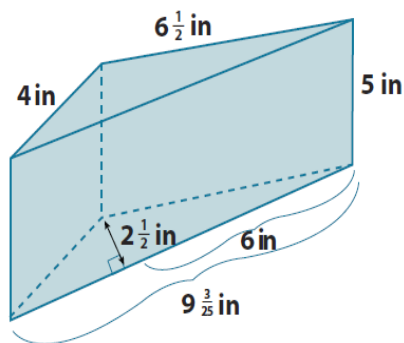
   c.

d.

4. A cube has a volume of 64 m^2. What is the cube's surface area?

5. The height of a right rectangular prism is $4\frac{1}{2}$ ft. The length and width of the prism's base are 2 ft. and $1\frac{1}{2}$ ft. Use the formula $SA = LA + 2B$ to find the surface area of the right rectangular prism.

6. The surface area of a right rectangular prism is $68\frac{2}{3}$ in^2. The dimensions of its base are 3 in. and 7 in. Use the formula $SA = LA + 2B$ and $LA = Ph$ to find the unknown height h of the prism.

7. A given right triangular prism has an equilateral triangular base. The height of that equilateral triangle is approximately 7.1 cm. The distance between the bases is 9 cm. The surface area of the prism is $319\frac{1}{2}$ cm^2. Find the approximate lengths of the sides of the base.