

Lesson 18: Counting Problems

Classwork

Opening Exercise

You are about to switch out your books from your locker during passing period but forget the order of your locker combination. You know that there are the numbers 3, 16, and 21 in some order. What is the percent of locker combinations that start with 3?

Locker Combination Possibilities:

3, 16, 21

21, 16, 3

16, 21, 3

21, 3, 16

16, 3, 21

3, 21, 16

Example 1

All of the 3-letter passwords that can be formed using the letters A and B are as follows: AAA, AAB, ABA, ABB, BAA, BAB, BBA, BBB.

a. What percent of passwords contain at least two B's?

b. What percent of passwords contain no A's?

Exercises 1–2

1. How many 4-letter passwords can be formed using the letters A and B?
2. What percent of the 4-letter passwords contain
 - a. No A's?
 - b. Exactly one A?
 - c. Exactly two A's?
 - d. Exactly three A's?
 - e. Four A's?
 - f. The same number of A's and B's?

Example 2

In a set of 3-letter passwords, 40% of the passwords contain the letter B and two of another letter. Which of the two sets below meets the criteria? Explain how you arrived at your answer.

Set 1

BBB	AAA	CAC
CBC	ABA	CCC
BBC	CCB	CAB
AAB	AAC	BAA
ACB	BAC	BCC

Set 2

CEB	BBB
EBE	CCC
CCC	EEE
EEB	CBC
CCB	ECE

Exercises 3–4

3. Shana read the following problem:

“How many letter arrangements can be formed from the word *triangle* that have two vowels and two consonants (order does not matter)?”

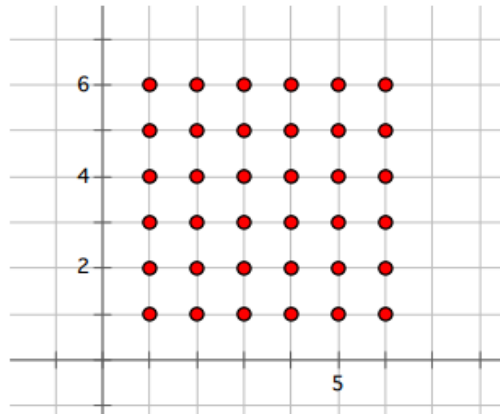
She answered that there are 30 letter arrangements.

Twenty percent of the letter arrangements that began with a vowel actually had an English definition. How many letter arrangements that begin with a vowel have an English definition?

4. Using three different keys on a piano, a songwriter makes the beginning of his melody with three notes, C, E, and G: CCE, EEE, EGC, GCE, CEG, GEE, CGE, GGE, EGG, EGE, GCG, EEC, ECC, ECG, GGG, GEC, CCG, CEE, CCC, GEG, CGC.
- From the list above, what is the percent of melodies with all three notes that are different?
 - From the list above, what is the percent of melodies that have three of the same notes?

Example 3

Look at the 36 points on the coordinate plane with whole number coordinates between 1 and 6, inclusive.



- Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 7.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 6.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 5.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 4.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 3.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 2.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 8.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 9.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 10.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 11.
Draw a line through each of the points which have an x -coordinate and y -coordinate sum of 12.
- What percent of the 36 points have coordinate sum 7?
- Write a numerical expression that could be used to determine the percent of the 36 points that have a coordinate sum of 7.
- What percent of the 36 points have coordinate sum 5 or less?
- What percent of the 36 points have coordinate sum 4 or 10?

Lesson Summary

To find the percent of possible outcomes for a counting problem you need to determine the total number of possible outcomes and the different favorable outcomes. The representation

$$\text{Quantity} = \text{Percent} \times \text{Whole}$$

can be used where the quantity is the number of different favorable outcomes and the whole is the total number of possible outcomes.

Problem Set

- A six-sided die (singular for dice) is thrown twice. The different rolls are as follows:
 1 and 1, 1 and 2, 1 and 3, 1 and 4, 1 and 5, 1 and 6,
 2 and 1, 2 and 2, 2 and 3, 2 and 4, 2 and 5, 2 and 6,
 3 and 1, 3 and 2, 3 and 3, 3 and 4, 3 and 5, 3 and 6,
 4 and 1, 4 and 2, 4 and 3, 4 and 4, 4 and 5, 4 and 6,
 5 and 1, 5 and 2, 5 and 3, 5 and 4, 5 and 5, 5 and 6,
 6 and 1, 6 and 2, 6 and 3, 6 and 4, 6 and 5, 6 and 6.
 - What is the percent that both throws will be even numbers?
 - What is the percent that the second throw is a 5?
 - What is the percent that the first throw is lower than a 6?
- You have the ability to choose three of your own classes, art, language, and physical education. There are three art classes (A1, A2, A3), two language classes (L1, L2), and two P.E. classes (P1, P2) to choose from. The order does not matter and you must choose one from each subject.

A1, L1, P1	A2, L1, P1	A3, L1, P1
A1, L1, P2	A2, L1, P2	A3, L1, P2
A1, L2, P1	A2, L2, P1	A3, L2, P1
A1, L2, P2	A2, L2, P2	A3, L2, P2

Compare the percent of possibilities with A1 in your schedule to the percent of possibilities with L1 in your schedule.

3. Fridays are selected to show your school pride. The colors of your school are orange, blue, and white, and you can show your spirit by wearing a top, a bottom, and an accessory with the colors of your school. During lunch, 11 students are chosen to play for a prize on stage. The table charts what the students wore.

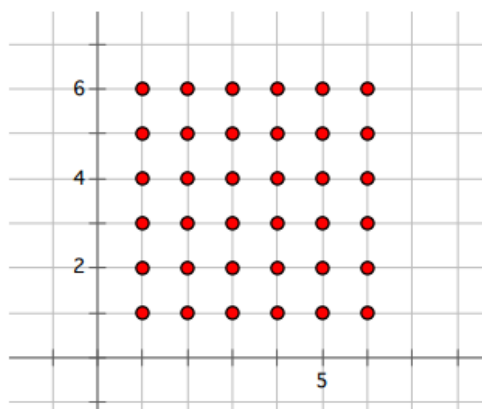
Top	W	O	W	O	B	W	B	B	W	W	W
Bottom	B	O	B	B	O	B	B	B	O	W	B
Accessory	W	O	B	W	B	O	B	W	O	O	O

- What is the percent of outfits that are one color?
 - What is the percent of outfits that include orange accessories?
4. Shana wears two rings (G represents gold, and S represents silver) at all times on her hand. She likes fiddling with them and places them on different fingers (pinky, ring, middle, index) when she gets restless. The chart is tracking the movement of her rings.

	Pinky Finger	Ring Finger	Middle Finger	Index Finger
Position 1		G	S	
Position 2			S	G
Position 3	G		S	
Position 4				S,G
Position 5	S	G		
Position 6	G	S		
Position 7	S		G	
Position 8	G		S	
Position 9		S,G		
Position 10		G	S	
Position 11			G	S
Position 12		S		G
Position 13	S,G			
Position 14			S,G	

- What percent of the positions shows the gold ring on her pinky finger?
- What percent of the positions shows she wears both rings on one finger?

5. Use the coordinate plane below to answer the following questions.



- a. What is the percent of the 36 points whose quotient of $\frac{x\text{-coordinate}}{y\text{-coordinate}}$ is greater than one?
- b. What is the percent of the 36 points whose coordinate quotient is equal to one?